On the Structure of the Burnside Semigroups

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For a fixed set of generators A and for $n \geq 1$ and $m \geq 1$ integers, the free associative semigroup that satisfies the equation $x^{n+m} = x^n$ is called a Burnside semigroup. If we allow $n = 0$ we have a Burnside group and it can be infinite for m large enough.

We present some structural results on the Burnside semigroups.

In order to classify the structure of these semigroups we have to split them in three cases: $n = 1$, $n = 2$ and $n \geq 3$. Our main interest here is the case $n = 2$ whose structure was practically unknown until now. We have a fairly complete set of results for the case $n = 2$ but here we restrict ourselves to the statement of two facts: one old and one new.

It was known that the semigroups are infinite (with infinitely many $\mathcal{J}$-classes) when $n \geq 2$ for more than one generator, while the semigroup is finite for m small enough when $n = 1$. Anyway, there are only finitely many $\mathcal{J}$-classes for $n = 1$ and any m.

The maximal subgroups of these semigroups are proved to be free Burnside groups satisfying $x^m = 1$ whose set of generators is associated with a special graph. The cardinality of the generating set is the cyclomatic number of this graph. When $n \geq 3$ this special graph is a cycle and the maximal subgroups are cyclic groups of order m. We have examples with an arbitrary number of generators both for $n = 1$ and for $n = 2$. Hence we might have infinite maximal subgroups for m large enough. Some of these results in the case $n = 1$ have been known.