

COMPUTER SCIENCE
SEGEWICK / WAYNE
PART I: PROGRAMMING IN JAVA

Section 3.2
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<http://introcs.cs.princeton.edu>

9. Creating Data Types

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SEGEWICK / WAYNE
PART I: PROGRAMMING IN JAVA

9. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

CS.9.A.CreatingDTs.Overview

Basic building blocks for programming

any program you might want to write

objects

functions and modules

graphics, sound, and image I/O

arrays

conditionals and loops

Math text I/O

primitive data types assignment statements

Ability to bring life to your own abstractions

Object-oriented programming (OOP)

Object-oriented programming (OOP).

- Create your own data types.
- Use them in your programs (manipulate *objects*). An *object* holds a data type value. Variable names refer to objects.

Examples

data type	set of values	examples of operations
Color	three 8-bit integers	get red component, brighten
Picture	2D array of colors	get/set color of pixel
String	sequence of characters	length, substring, compare

CATAGC

An abstract data type is a data type whose representation is *hidden from the client*.

Impact: We can use ADTs without knowing implementation details.

- Previous lecture: how to write client programs for several useful ADTs
- This lecture: how to implement your own ADTs

Implementing a data type

- To **create** a data type, you need provide code that
- Defines the set of values (**instance variables**).
 - Implements operations on those values (**methods**).
 - Creates and initialize new objects (**constructors**).

Instance variables

- Declarations associate variable names with types.
- Set of type values is "set of values".

Methods

- Like static methods.
- Can refer to instance variables.

Constructors

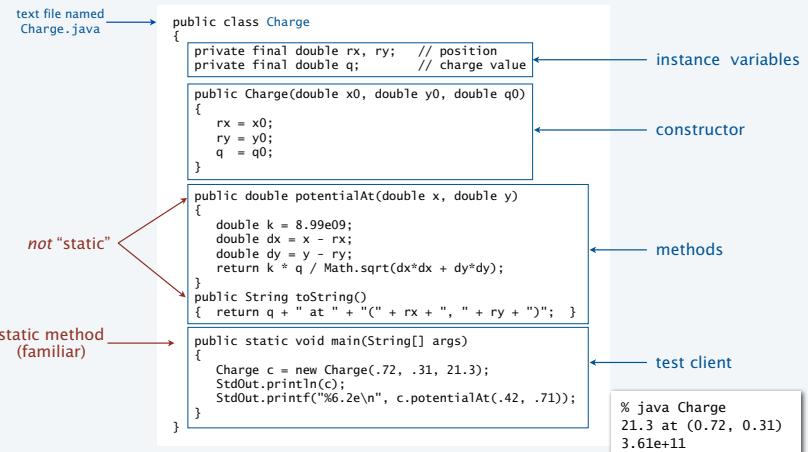
- Like a method with the same name as the type.
- No return type declaration.
- Invoked by new, returns object of the type.

In Java, a data-type implementation is known as a **class**.

A Java class



Anatomy of a Class



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9. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

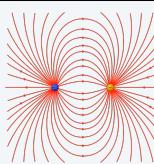
CS.9.A.CreatingDTs.Overview

CS.9.B.CreatingDTs.Charges

ADT for point charges

A **point charge** is an idealized model of a particle that has an electric charge.

An **ADT** allows us to write Java programs that manipulate point charges.



Values	examples	
	position (x, y)	(.53, .63) (.13, .94)
electrical charge	20.1	81.9

public class Charge	
Charge(double x0, double y0, double q0)	
double potentialAt(double x, double y)	<i>electric potential at (x, y) due to charge</i>
String toString()	<i>string representation of this charge</i>

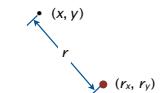
Crash course on electric potential

Electric potential is a measure of the effect of a point charge on its surroundings.

- It **increases** in proportion to the charge value.
- It **decreases** in proportion to the *inverse of the distance* from the charge (2D).

Mathematically,

- Suppose a point charge c is located at (r_x, r_y) and has charge q .
- Let r be the distance between (x, y) and (r_x, r_y)
- Let $V_c(x, y)$ be the potential at (x, y) due to c .
- Then $V_c(x, y) = k \frac{q}{r}$ where $k = 8.99 \times 10^9$ is a normalizing factor.



Q. What happens when multiple charges are present?

A. The potential at a point is the *sum* of the potentials due to the individual charges.

Note: Similar laws hold in many other situations.

Example. N-body (3D) is an inverse square law.

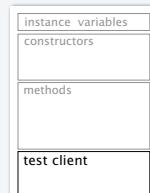
10

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Point charge implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
    Charge c = new Charge(.72, .31, 20.1);
    StdOut.println(c); ← Reminder: automatically invokes c.toString()
    StdOut.printf("%.2e\n", c.potentialAt(.42, .71));
}
```



$$V_c(x, y) = k \frac{q}{r}$$

$$\begin{aligned} r &= \sqrt{(r_x - x)^2 + (r_y - y)^2} \\ &= \sqrt{.3^2 + .4^2} = .5 \\ V_c(.42, .71) &= 8.99 \times 10^9 \frac{20.1}{.5} \\ &= 3.6 \times 10^{11} \end{aligned}$$

```
% java Charge
20.1 at (0.72, 0.31)
3.61e+11
```

← What we expect, once the implementation is done.

Point charge implementation: Instance variables

Instance variables define data-type values.

Values	examples	
	position (x, y)	(.53, .63) (.13, .94)
electrical charge	20.1	81.9

```
public class Charge
{
    private final double rx, ry;
    private final double q;
    ...
}
```



- Modifiers** control access.
- **private** denies clients access and therefore makes data type abstract.
 - **final** disallows any change in value and documents that data type is *immutable*.

stay tuned

Key to OOP. Each *object* has instance-variable values.

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Point charge implementation: Constructor

Constructors create and initialize new objects.

```
public class Charge
{
    ...
    public Charge(double x0, double y0, double q0)
    {
        rx = x0;
        ry = y0;
        q = q0;
    }
    ...
}
```

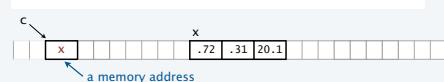
references to instance variables, which
are *not* declared within the constructor

Clients use `new` to invoke constructors.

- Pass arguments as in a method call.
- Return value is reference to new object.



Possible memory representation of
Charge c = new Charge(.72, .31, 20.1);



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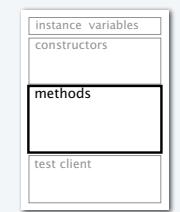
Point charge implementation: Methods

Methods define data-type operations (implement APIs).

public class Charge	
Charge(double x0, double y0, double q0)	
double potentialAt(double x, double y)	<i>electric potential at (x, y) due to charge</i>
String toString()	<i>string representation of this charge</i>

```
public class Charge
{
    ...
    public double potentialAt(double x, double y)
    {
        double k = 8.99e09;           Vc(x, y) = k  $\frac{q}{r}$ 
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }

    public String toString()
    { return q + " at (" + rx + ", " + ry + ")"; }
    ...
}
```



Key to OOP. An instance variable
reference in an instance method
refers to the value for the object
that was used to invoke the method.

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Point charge implementation

text file named
Charge.java

```
public class Charge
{
    private final double rx, ry; // position
    private final double q; // charge value

    public Charge(double x0, double y0, double q0)
    {
        rx = x0;
        ry = y0;
        q = q0;
    }

    public double potentialAt(double x, double y)
    {
        double k = 8.99e09;
        double dx = x - rx;
        double dy = y - ry;
        return k * q / Math.sqrt(dx*dx + dy*dy);
    }

    public String toString()
    { return q + " at (" + rx + ", " + ry + ")"; }

    public static void main(String[] args)
    {
        Charge c = new Charge(.72, .31, 20.1);
        StdOut.println(c);
        StdOut.printf("%.2e\n", c.potentialAt(.42, .71));
    }
}
```

instance variables

constructor

methods

test client

% java Charge
20.1 at (0.72, 0.31)
3.61e+11

Point charge client: Potential visualization (helper methods)

Read point charges from StdIn.

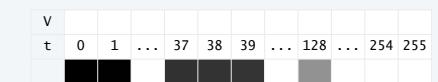
- Uses Charge like any other type.
- Returns an array of Charges.

```
public static Charge[] readCharges()
{
    int N = StdIn.readInt();
    Charge[] a = new Charge[N];
    for (int i = 0; i < N; i++)
    {
        double x0 = StdIn.readDouble();
        double y0 = StdIn.readDouble();
        double q0 = StdIn.readDouble();
        a[i] = new Charge(x0, y0, q0);
    }
    return a;
}
```

Convert potential values to a color.

- Convert V to an 8-bit integer.
- Use grayscale.

```
public static Color toColor(double V)
{
    V = 128 + V / 2.0e10;
    int t = 0;
    if (V > 255) t = 255;
    else if (V <= 0) t = (int) V;
    return new Color(t, t, t);
}
```

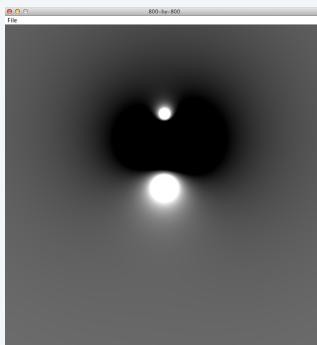


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Point charge client: Potential visualization

```
import java.awt.Color;
public class Potential
{
    public static Charge[] readCharges()
    { // See previous slide. }
    public static Color toColor()
    { // See previous slide. }
    public static void main(String[] args)
    {
        Charge[] a = readCharges();
        int SIZE = 800;
        Picture pic = new Picture(SIZE, SIZE);
        for (int col = 0; col < SIZE; col++)
            for (int row = 0; row < SIZE; row++)
            {
                double V = 0.0;
                for (int k = 0; k < a.length; k++)
                {
                    double x = 1.0 * col / SIZE;
                    double y = 1.0 * row / SIZE;
                    V += a[k].potentialAt(x, y);
                }
                pic.set(col, SIZE-1-row, toColor(V));
            }
        pic.show();
    }
}
```

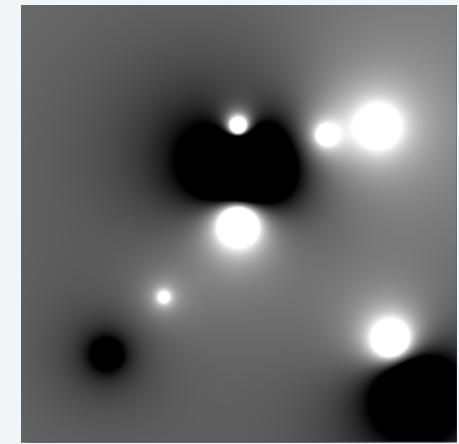
```
% more charges3.txt
3
.51 .63 -100
.50 .50 40
.50 .72 20
% java Potential < charges3.txt
```



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Potential visualization I

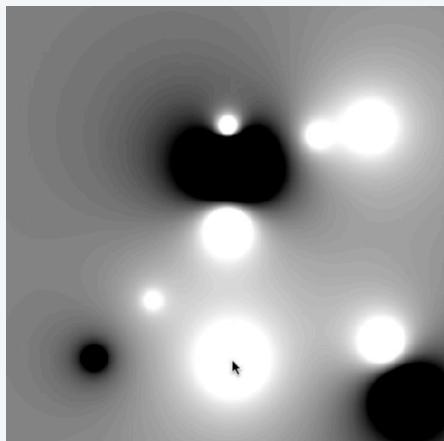
```
% more charges9.txt
9
.51 .63 -100
.50 .50 40
.50 .72 20
.33 .33 5
.20 .20 -10
.70 .70 10
.82 .72 20
.85 .23 30
.90 .12 -50
% java Potential < charges9.txt
```



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Potential visualization II: A moving charge

```
% more charges9.txt
9
.51 .63 -100
.50 .50 40
.50 .72 20
.33 .33 5
.20 .20 -10
.70 .70 10
.82 .72 20
.85 .23 30
.90 .12 -50
% java PotentialWithMovingCharge < charges9.txt
```

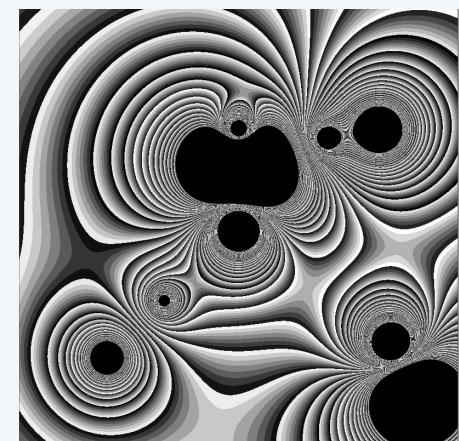


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Potential visualization III: Discontinuous color map

```
public static Color toColor(double V)
{
    V = 128 + V / 2.0e10;
    int t = 0;
    if (V > 255) t = 255;
    else if (V <= 0) t = (int) V;
    t = t*37 % 255
    return new Color(t, t, t);
}
```

V	0	1	2	3	4	5	6	7	8	9	...
t	black	dark gray	medium gray	light gray	white	white	white	white	white	white	...



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CS.9.B.CreatingDTs.Charges

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PART I: PROGRAMMING IN JAVA

9. Creating Data Types

- Overview
- Point charges
- **Turtle graphics**
- Complex numbers

CS.9.C.CreatingDTs.Turtles

ADT for turtle graphics

A **turtle** is an idealized model of a plotting device.

An **ADT** allows us to write Java programs that manipulate turtles.

Values	position (x, y)	(.5, .5)	(.25, .75)	(.22, .12)
position	90°	135°	10°	
orientation				

Seymour Papert
1928–

A small image of a turtle.

API (operations)	public class Turtle		
	Turtle(double x0, double y0, double q0)		
	void turnLeft(double delta)	rotate delta degrees counterclockwise	
	void goForward(double step)	move distance step, drawing a line	

Turtle graphics implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
    Turtle turtle = new Turtle(0.0, 0.0, 0.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
    turtle.goForward(1.0);
    turtle.turnLeft(120.0);
}
```

instance variables
constructors
methods
test client

% java Turtle

Note: Client drew triangle without computing $\sqrt{3}$

A diagram of an equilateral triangle.

What we expect, once the implementation is done.

Turtle implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Turtle
{
    private double x, y; // instance variables
    private double angle; // are not final
    public Turtle(double x0, double y0, double a0)
    {
        x = x0;
        y = y0;
        angle = a0;
    }
    ...
}
```



Values

position (x, y)	(.5, .5)	(.75, .75)	(.22, .12)
orientation	90°	135°	10°
	↙	↙	↙

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Turtle implementation: Methods

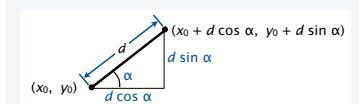
Methods define data-type operations (implement APIs).

```
public class Turtle
{
    ...
    public void turnLeft(double delta)
    { angle += delta; }
    public void goForward(double d)
    {
        double oldx = x;
        double oldy = y;
        x += d * Math.cos(Math.toRadians(angle));
        y += d * Math.sin(Math.toRadians(angle));
        StdDraw.line(oldx, oldy, x, y);
    }
    ...
}
```



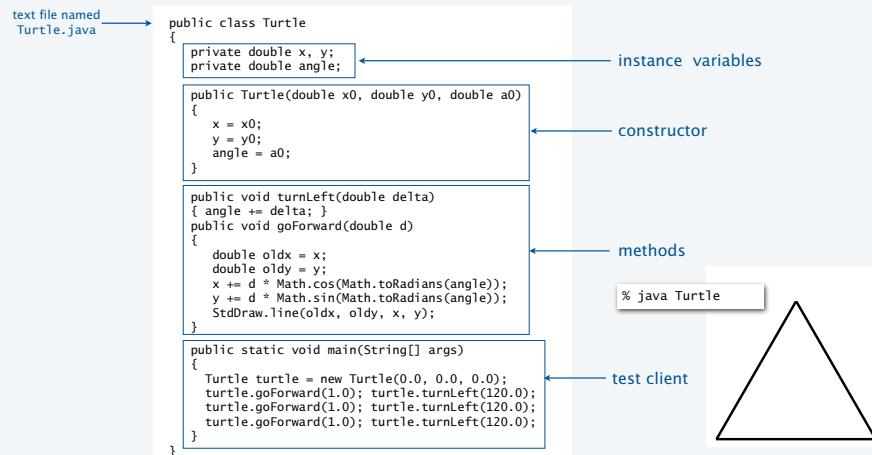
API

```
public class Turtle
{
    Turtle(double x0, double y0, double q0)
    void turnLeft(double delta)
    void goForward(double step)
```



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Turtle implementation

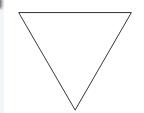


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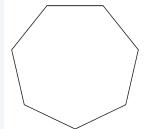
Turtle client: N-gon

```
public class Ngon
{
    public static void main(String[] args)
    {
        int N      = Integer.parseInt(args[0]);
        double angle = 360.0 / N;
        double step = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < N; i++)
        {
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
```

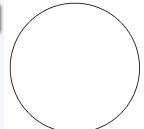
% java Ngon 3



% java Ngon 7



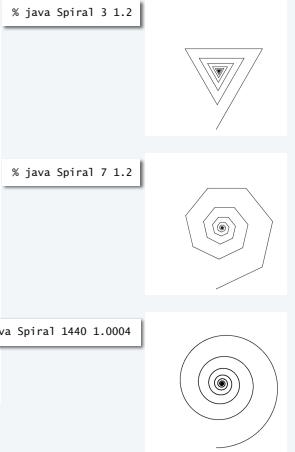
% java Ngon 1440



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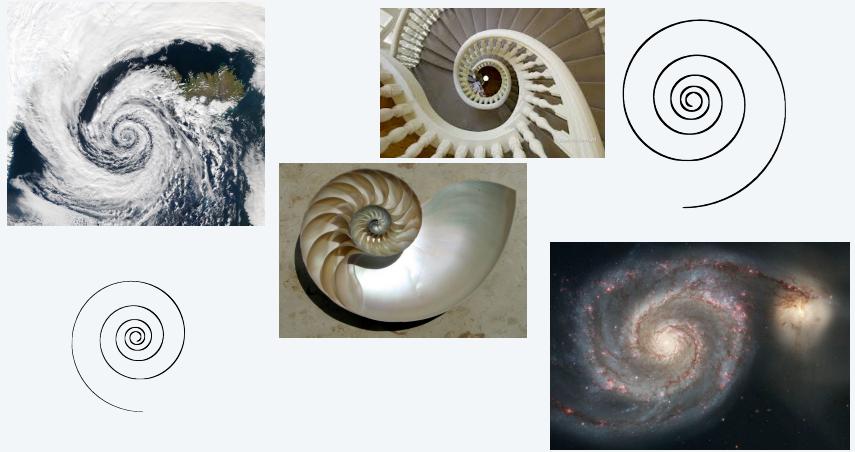
Turtle client: Spira Mirabilis

```
public class Spiral
{
    public static void main(String[] args)
    {
        int N      = Integer.parseInt(args[0]);
        double decay = Double.parseDouble(args[1]);
        double angle = 360.0 / N;
        double step  = Math.sin(Math.toRadians(angle/2.0));
        Turtle turtle = new Turtle(0.5, 0, angle/2.0);
        for (int i = 0; i < 10 * N; i++)
        {
            step /= decay;
            turtle.goForward(step);
            turtle.turnLeft(angle);
        }
    }
}
```



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Spira Mirabilis in the wild



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Pop quiz 1 on OOP

Q. Fix the serious bug in this code:

```
public class Turtle
{
    private double x, y;
    private double angle;
    public Turtle(double x0, double y0, double a0)
    {
        double x = x0;
        double y = y0;
        double angle = a0;
    }
    ...
}
```

Pop quiz 1 on OOP

Q. Fix the serious bug in this code:

```
public class Turtle
{
    private double x, y;
    private double angle;
    public Turtle(double x0, double y0, double a0)
    {
        double x = x0;
        double y = y0;
        double angle = a0;
    }
    ...
}
```

A. Remove type declarations.
They create local variables,
which are *different* from the
instance variables!

Object-oriented programmers pledge. "I will not shadow instance variables"

Every programmer makes this mistake,
and it is a difficult one to detect.

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Image sources

<http://web.media.mit.edu/~papert/>
http://en.wikipedia.org/wiki/Logarithmic_spiral
http://en.wikipedia.org/wiki/Logarithmic_spiral#/media/File:Nautilus_Cutaway_with_Logarithmic_Spiral.png
http://en.wikipedia.org/wiki/File:Low_pressure_system_over_Iceland.jpg

CS.9.C.CreatingDTs.Turtles

9. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers

CS.9.D.CreatingDTs.Mandelbrot

Crash course in complex numbers

A **complex number** is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

Complex numbers are a *quintessential mathematical abstraction* that have been used for centuries to give insight into real-world problems not easily addressed otherwise.



Leonhard Euler
1707–1783

A. L. Cauchy
1789–1857

To perform **algebraic operations** on complex numbers, use real algebra, replace i^2 by -1 and collect terms.

- Addition example: $(3 + 4i) + (-2 + 3i) = 1 + 7i$.
- Multiplication example: $(3 + 4i) \times (-2 + 3i) = -18 + i$.

Example: $|3 + 4i| = 5$

The **magnitude** or **absolute value** of a complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$.

Applications: Signal processing, control theory, quantum mechanics, analysis of algorithms...

ADT for complex numbers

A **complex number** is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

An **ADT** allows us to write Java programs that manipulate complex numbers.

Values	complex number	3 + 4i	-2 + 2i
	real part	3.0	-2.0
	imaginary part	4.0	2.0

API (operations)	public class Complex	
	Complex(double real, double imag)	
	Complex plus(Complex b)	<i>sum of this number and b</i>
	Complex times(Complex b)	<i>product of this number and b</i>
	double abs()	<i>magnitude</i>
	String toString()	<i>string representation</i>

Complex number data type implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
    Complex a = new Complex( 3.0, 4.0);
    Complex b = new Complex(-2.0, 3.0);
    StdOut.println("a = " + a);
    StdOut.println("b = " + b);
    StdOut.println("a * b = " + a.times(b));
}
```



```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

What we expect, once the implementation is done.

Complex number data type implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.

```
public class Complex
{
    private final double re; // instance variables are final
    private final double im;
    public Complex(double real, double imag)
    {
        re = real;
        im = imag;
    }
    ...
}
```



	complex number	3 + 4i	-2 + 2i
Values	real part	3.0	-2.0
	imaginary part	4.0	2.0

Complex number data type implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Complex
{
    ...
    public Complex plus(Complex b) // might also write "this.re"
    {
        double real = re + b.re; // or use Complex a = this
        double imag = im + b.im;
        return new Complex(real, imag);
    }
    public Complex times(Complex b)
    {
        double real = re * b.re - im * b.im;
        double imag = re * b.im + im * b.re;
        return new Complex(real, imag);
    }
    public double abs()
    {
        return Math.sqrt(re*re + im*im);
    }
    public String toString()
    {
        return re + " + " + im + "i";
    }
    ...
}
```

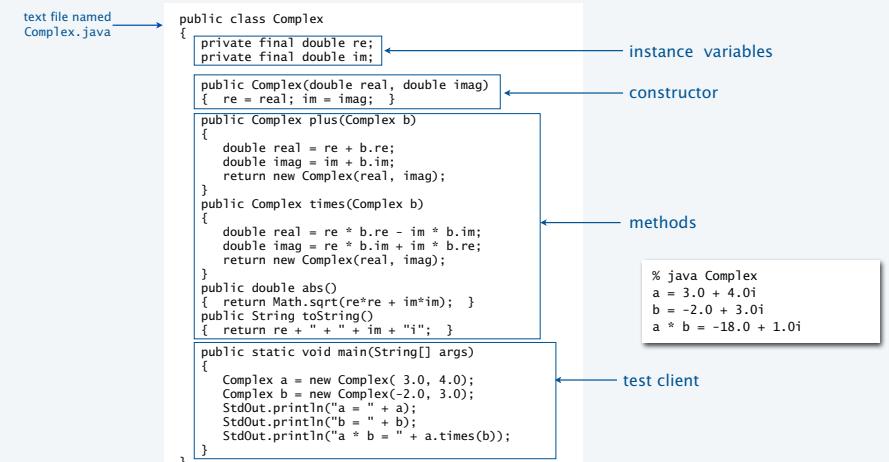
Java keyword "this" is a reference to "this object" and is implicit when an instance variable is directly referenced

$$\begin{aligned} a &= v + wi \\ b &= x + yi \\ a \times b &= vx + vyi + wxi + wyi^2 \\ &= vx - wy + (vy + wx)i \end{aligned}$$

API	public class Complex
Complex(double real, double imag)	
Complex plus(Complex b)	sum of this number and b
Complex times(Complex b)	product of this number and b
double abs()	magnitude
String toString()	string representation



Complex number data type implementation



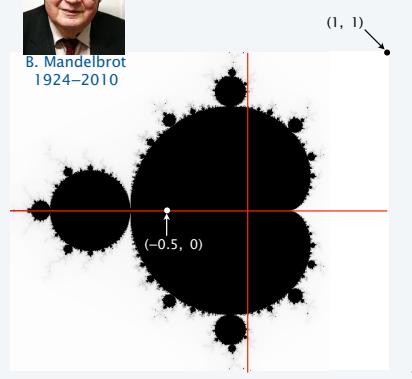
The Mandelbrot set

The **Mandelbrot set** is a set of complex numbers.

- Represent each complex number $x + yi$ by a point (x, y) in the plane.
- If a point is *in* the set, we color it BLACK.
- If a point is *not* in the set, we color it WHITE.



B. Mandelbrot
1924–2010



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Examples

- *In* the set: $-0.5 + 0i$.
- *Not* in the set: $1 + i$.

Challenge

- No simple formula exists for testing whether a number is in the set.
- Instead, the set is defined by an *algorithm*.

Determining whether a point is in the Mandelbrot set

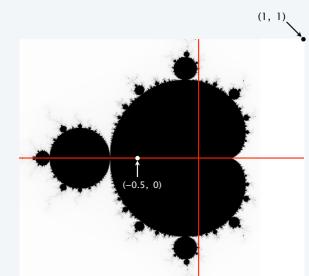
Is a complex number z_0 in the set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, z_0 is *not* in the set.
- If not, z_0 is *in* the set.

t	z_t	t	z_t
0	$-1/2 + 0i$	0	$1 + i$
1	$-1/4 + 0i$	1	$1 + 3i$
2	$-7/16 + 0i$	2	$-7 + 7i$
3	$-79/256 + 0i$	3	$1 - 97i$
4	$-26527/65536 + 0i$	4	$-9407 - 193i$

always between $-1/2$ and 0
 $z = -1/2 + 0i$ is *in* the set

diverges to infinity
 $z = 1 + i$ is *not* in the set



$$(1+i)^2 + (1+i) = 1 + 2i + i^2 + 1 + i = 1+3i$$

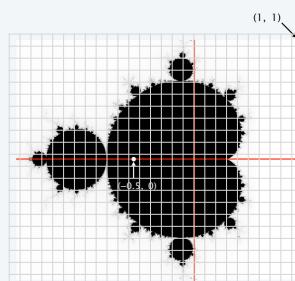
$$(1+3i)^2 + (1+i) = 1 + 6i + 9i^2 + 1 + i = -7+7i$$

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Plotting the Mandelbrot set

Practical issues

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.



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Approximate solution for first issue

- Sample from an N -by- N grid of points in the plane.
- Zoom in to see more detail (stay tuned!).

Important note: Solutions imply significant computation.

Approximate solution for second issue

- Fact: if $|z_t| > 2$ for any t , then z is *not* in the set.
- Pseudo-fact: if $|z_{255}| \leq 2$ then z is "likely" in the set.

Complex number client: Mandelbrot set visualization (helper method)

Mandelbrot function of a complex number.

- Returns WHITE if the number is not in the set.
- Returns BLACK if the number is (probably) in the set.

```
public static Color mand(Complex z0)
{
    Complex z = z0;
    for (int t = 0; t < 255; t++)
    {
        if (z.abs() > 2.0) return Color.WHITE;
        z = z.times(z);
        z = z.plus(z0);
    }
    return Color.BLACK;
}
```

For a more dramatic picture,
return new Color(255-t, 255-t, 255-t)
or colors picked from a color table.

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Complex number client: Mandelbrot set visualization

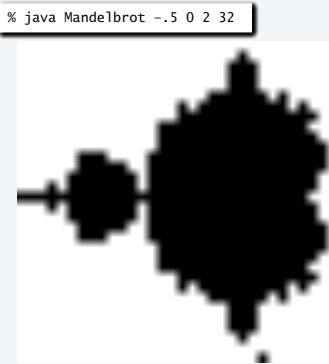
```

import java.awt.Color;
public class Mandelbrot
{
    public static Color mand(Complex z0)
    { // See previous slide. }
    public static void main(String[] args)
    {
        double xc = Double.parseDouble(args[0]);
        double yc = Double.parseDouble(args[1]);
        double size = Double.parseDouble(args[2]);
        int N = Integer.parseInt(args[3]);
        Picture pic = new Picture(N, N);

        for (int col = 0; col < N; col++)
            for (int row = 0; row < N; row++) scale to screen coordinates
            {
                double x0 = xc - size/2 + size*col/N;
                double y0 = yc - size/2 + size*row/N;
                Complex z0 = new Complex(x0, y0);
                Color color = mand(z0);
                pic.set(col, N-1-row, color);
            }
        pic.show();
    }
}

```

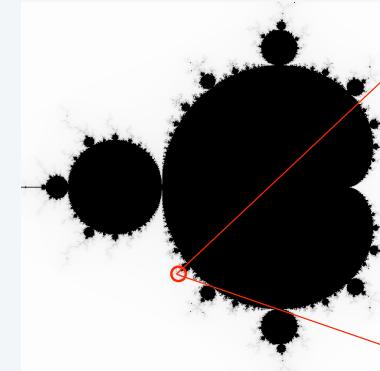
(0, 0) is upper left corner



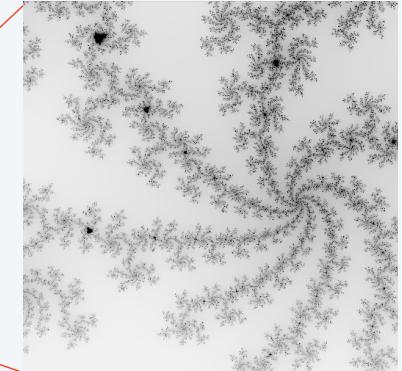
45

Mandelbrot Set

% java GrayscaleMandelbrot -.5 0 2



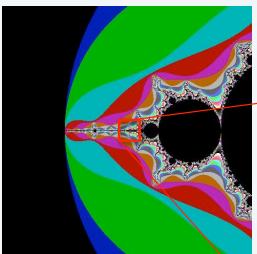
% java GrayscaleMandelbrot .1045 -.637 .01



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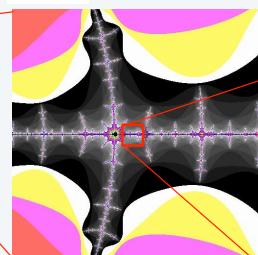
Mandelbrot Set

% java ColorMandelbrot -.5 0 2 < mandel.txt

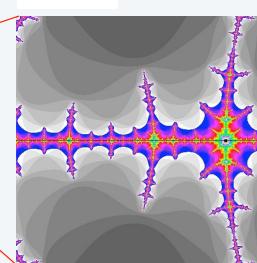


-1.5 0 2

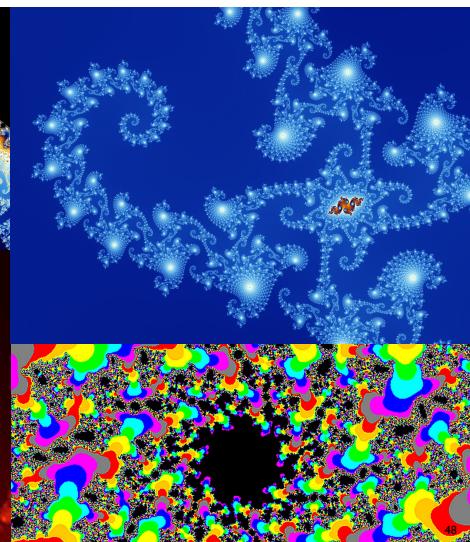
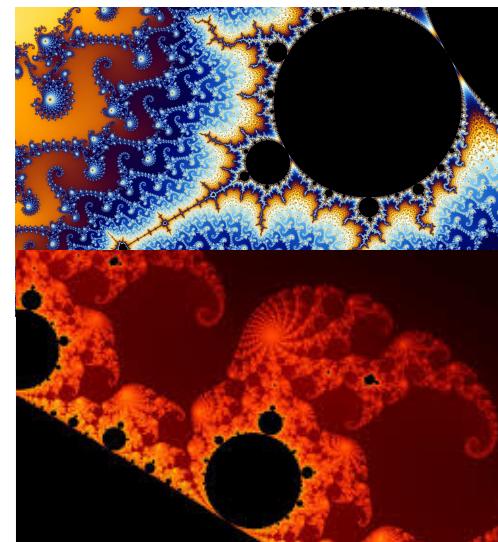
color map



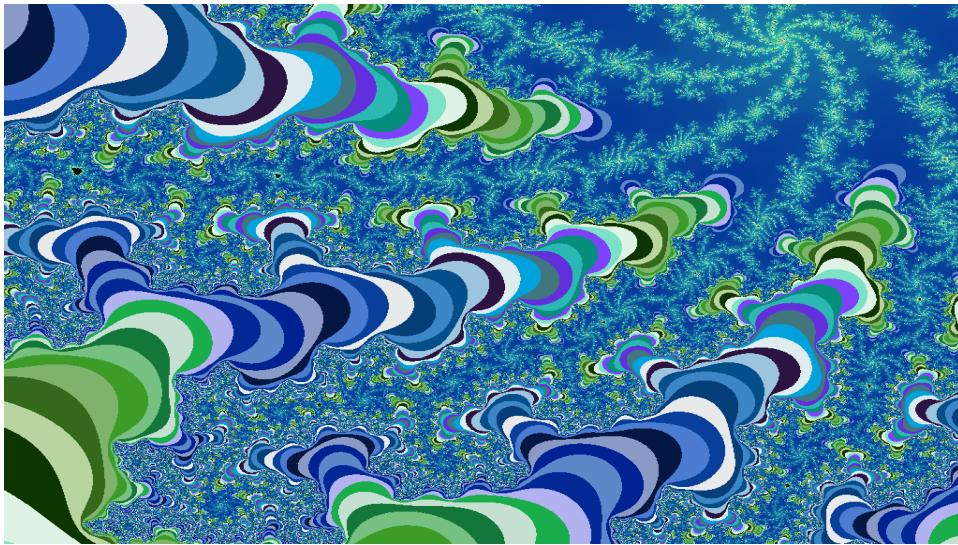
-1.5 0 .002



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OOP summary

Object-oriented programming (OOP)

- Create your own data types (sets of values and ops on them).
- Use them in your programs (manipulate *objects*).



OOP helps us simulate the physical world

- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Examples: charged particle, color, sound, genome....



T A G A T G T G C T A G C

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You have come a long way

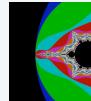
any program you might want to write

```
public class HelloWorld
{
    public static void main(String[] args)
    {
        System.out.println("Hello, World");
    }
}
```

objects



functions and modules



graphics, sound, and image I/O

T A G A T G T G C T A G C

arrays

conditionals and loops



Math

text I/O

primitive data types

assignment statements

Course goal. Open a *whole new world* of opportunity for you (programming).



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Image sources

http://en.wikipedia.org/wiki/Leonhard_Euler#/media/File:Leonhard_Euler.jpg
http://en.wikipedia.org/wiki/Augustin-Louis_Cauchy
http://upload.wikimedia.org/wikipedia/commons/e/e9/Benoit_Mandelbrot_mg_1804-d.jpg
http://upload.wikimedia.org/wikipedia/commons/f/fc/Mandel_zoom_08_satellite_antenna.jpg
<http://upload.wikimedia.org/wikipedia/commons/1/18/Mandelpart2.jpg>
http://upload.wikimedia.org/wikipedia/commons/f/fb/Mandel_zoom_13_satellite_seahorse_tail_with_julia_island.jpg
http://upload.wikimedia.org/wikipedia/commons/4/44/Mandelbrot_set_à_la_Pop_Art_-_Wacker_Art_Fractal_Generator.jpg

CS.9.D.CreatingDTs.Mandelbrot

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9. Creating Data Types