Monochromatic path/cycle partitions

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University of Chile

based on joint work with David Conlon, Richard Lang, Oliver Schaudt

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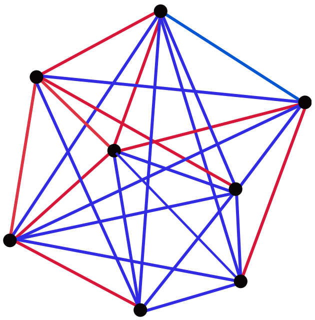
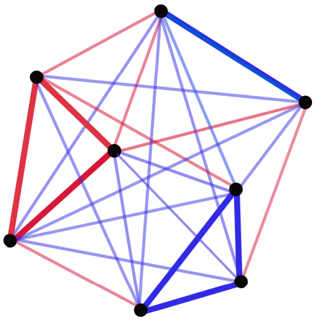
Montevideo
December 11, 2014

OVERVIEW OF THE PROBLEM

Problem

How many disjoint monochromatic cycles do we need to cover any 2-edge-coloured K_n?





Cycles may be empty, singletons, single edges.

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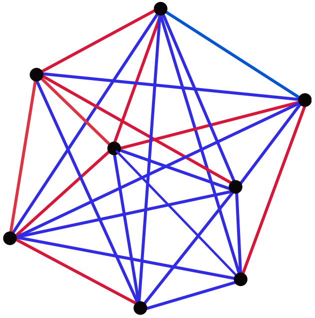
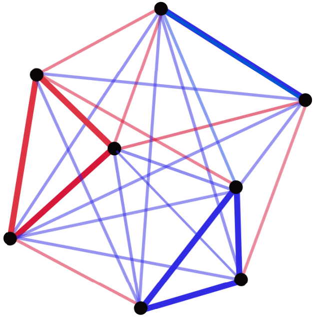
How many disjoint monochromatic cycles do we need to cover any 2-edge-coloured K_n?

function of n?





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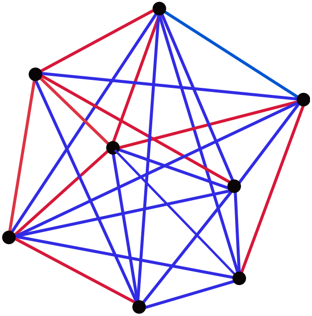
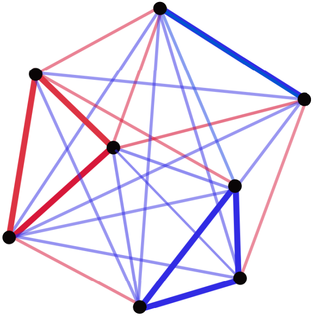
function of n? $\rightarrow$ actually 2





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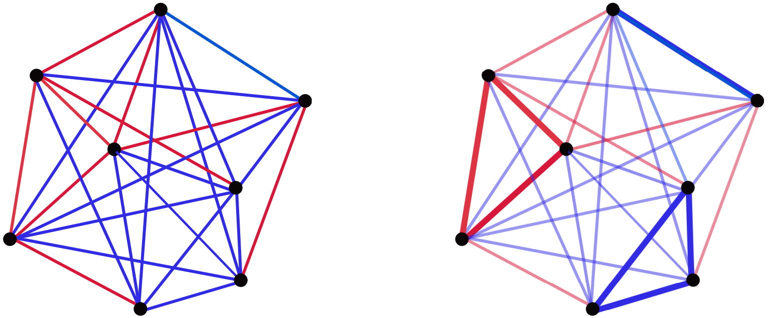

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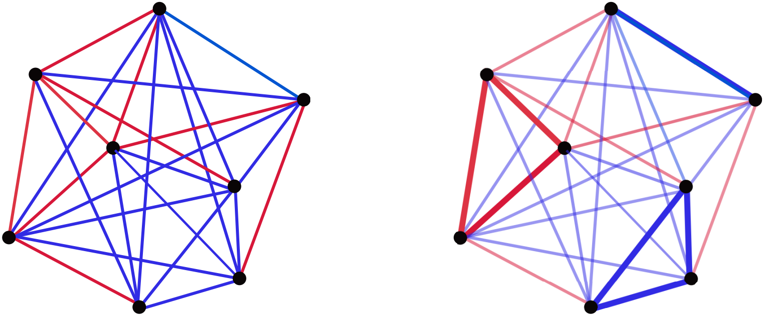
How many disjoint monochromatic cycles do we need to cover any r-edge-coloured K_n?

function of n and r?



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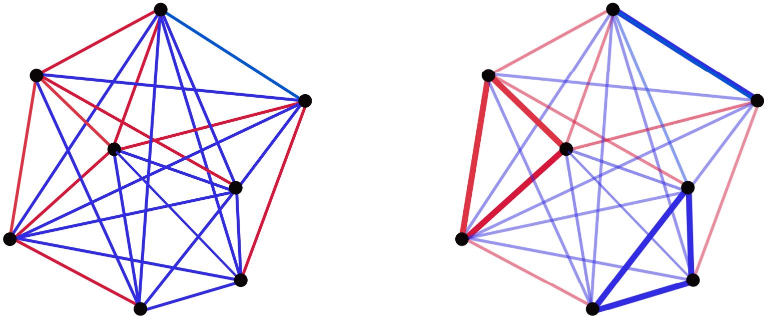

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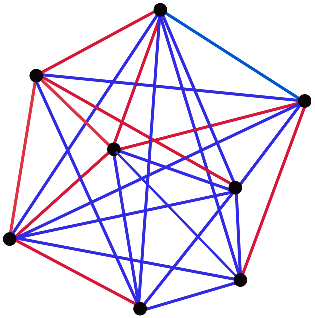

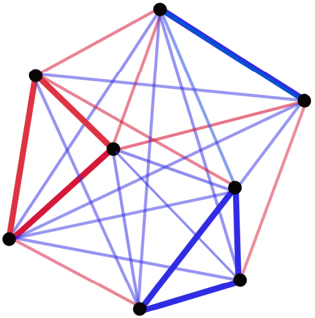

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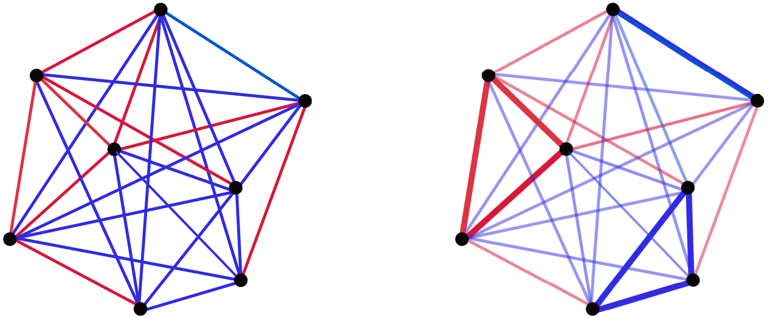

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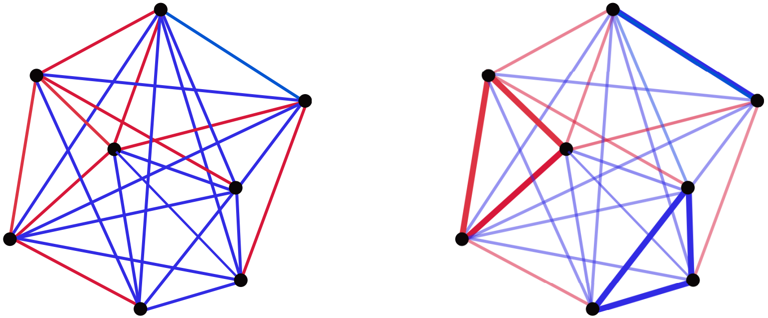

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Problem

How many disjoint monochromatic paths do we need to cover any r-edge-coloured $K_{n,n}$?

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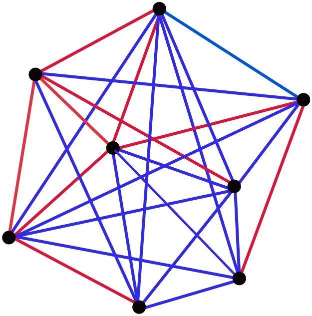

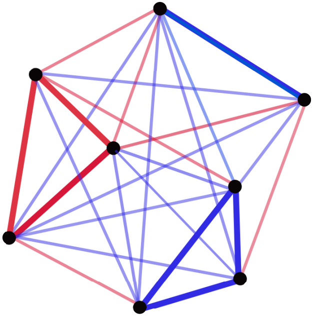

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Problem

How many disjoint monochromatic paths do we need to cover any r-edge-coloured K_n^k?

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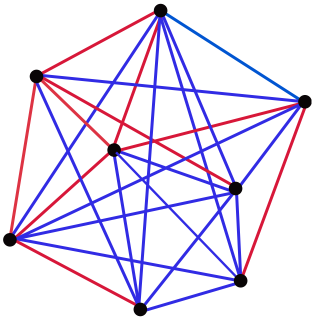

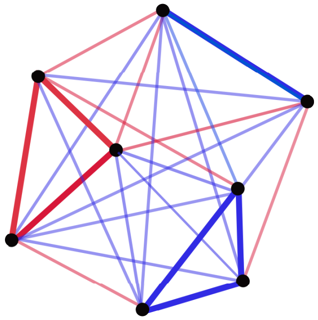

Cycles may be empty, singletons, single edges.

COMPLETE GRAPHS

$E(K_n)$ coloured with 2 colours

An observation of Gerencser and Gyárfás '67:

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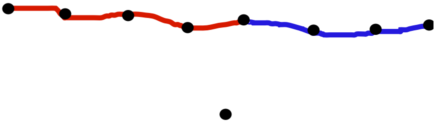

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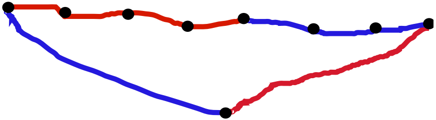

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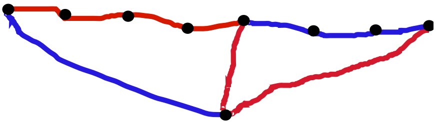

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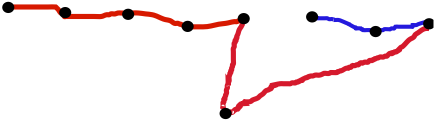

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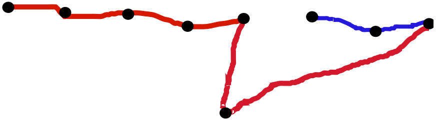

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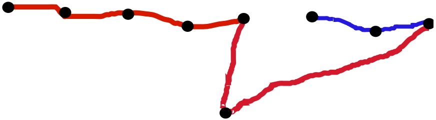

Does it cover all vertices? $\rightarrow$ yes it does!

$\Rightarrow \exists$ partition of K_n into .

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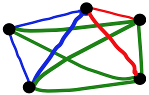
Generalization:

Thm (Conlon, St '14+): The same holds for 2-local colourings.

$E(K_n)$ locally coloured with 2 colours

Thm (Conlon, St '14+): Any 2-local coloured K_n has a partition into 2 cycles.

r-local colouring:
arbitrary many colours, but each vertex sees $\leq r$ colours.



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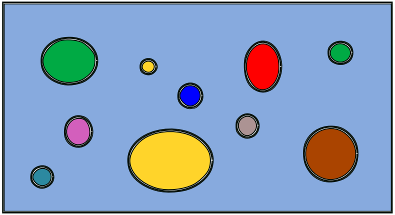
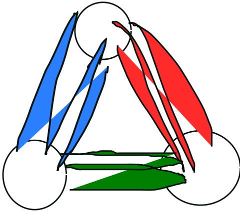
proof idea:

we get one of the following two:



1) $\exists$ colour that sees all vertices



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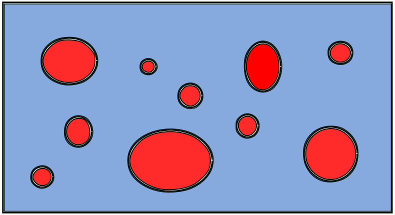

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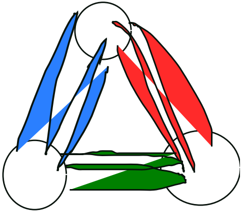

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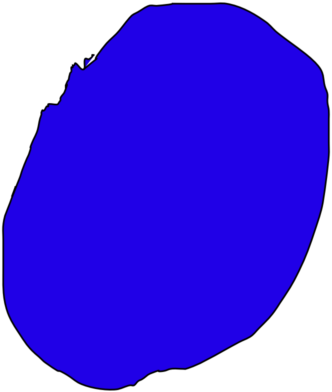
- need at least r paths/cycles

Lower bound: r paths/cycles needed:



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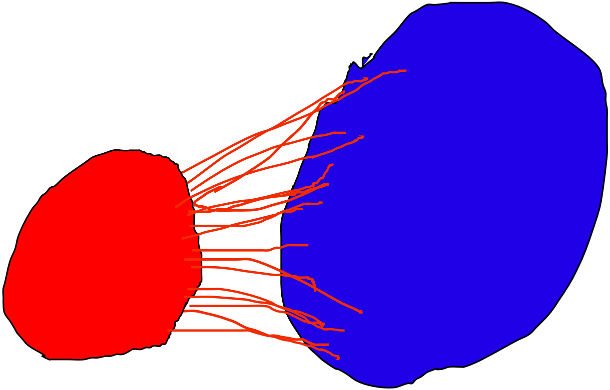

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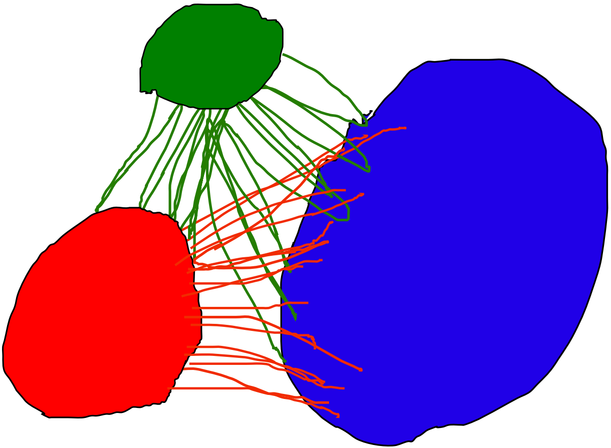

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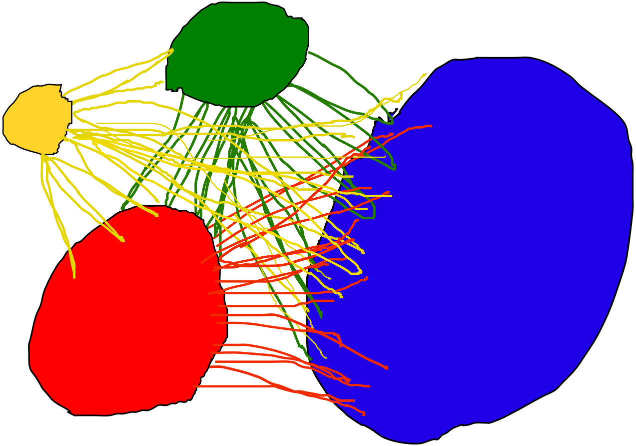

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- they show that $\exists$ partition into $cr^2 \log r$ cycles

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- second conjecture not true

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Bounds for cycles: $E(K_n)$ coloured with r colours

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Partitions into $f(r)$ cycles (for large n):

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Three colours:

• cover all but $o(n)$ vertices with ≤ 3 disjoint cycles (GRSS '12)

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r cycles are not enough:

- counterexample, for all $r \geq 3$ (Pokrovskiy '14)

Cycles: $E(K_n)$ coloured with r colours

Conjecture (EGP '91): There is a partition of all vertices of K_n into r monochromatic cycles.

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(still possible) Conjecture: There is a partition of all but 1 vertices of K_n into r monochromatic cycles.

Cycles: $E(K_n)$ coloured with r colours

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(still possible) Conjecture: There is a partition of all but $o(n)$ vertices of K_n into r monochromatic cycles.

This is known for $r = 3$.

Cycles: $E(K_n)$ coloured with r colours

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(still possible) Conjecture: There is a partition of all vertices of K_n into $r+1$ monochromatic cycles.

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'...into $100r \log r$ cycles' is known [GRSS '06].

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(still possible) Conjecture: There is a partition of all vertices of K_n into $f(r)$ monochromatic cycles.

Conjecture (Gy '89): There is a partition of all vertices of K_n into r monochromatic paths.

true for $r \leq 3$, open for $r > 3$

Partitioning with paths/cycles

paths	K_n
r colours	r ?
2 colours	2
3 colours	3
4 colours	8

cycles	K_n	K_n 'local'
r colours	$100r \log r$	$O(r^2 \log r)$
2 colours	2	2
3 colours	17	

cycles: all but $o(n)$	K_n
r colours	r ?
3 colours	3

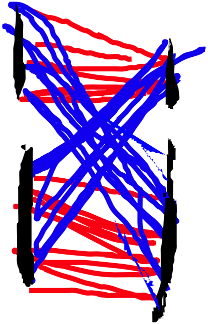
COMPLETE BIPARTITE GRAPHS

Complete bipartite graphs $K_{n,n}$ with 2 colours:

Can we cover $K_{n,n}$ with 2 cycles (or paths)?

Complete bipartite graphs $K_{n,n}$ with 2 colours:

The split colouring:



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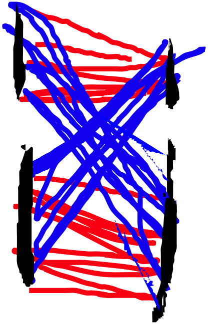

In a split colouring, we might need 3 paths/cycles.

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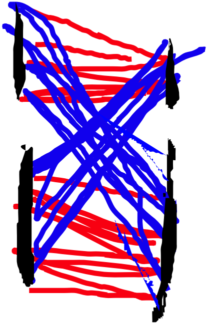

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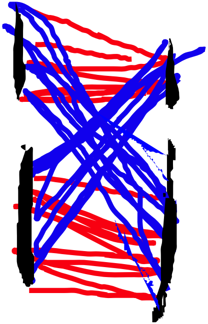

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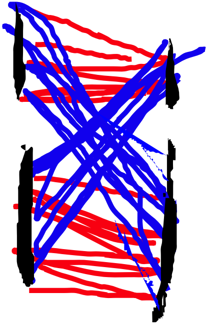

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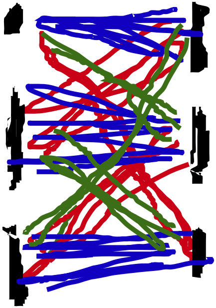
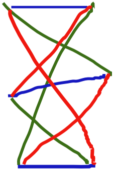
- 3 paths are always enough
- 3 paths suffice also for 2-local colourings (Lang, St. '15+)

Complete bipartite graphs $K_{n,n}$ with r colours:

Complete bipartite graphs $K_{n,n}$ with r colours:

A split colouring for r colours:





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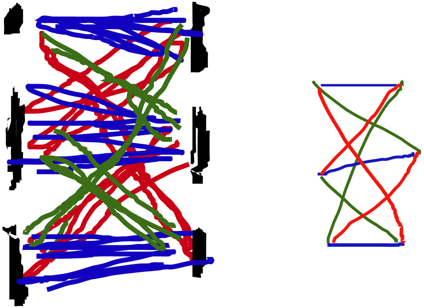

In a such a colouring, we might need $2r - 1$ paths.

Complete bipartite graphs $K_{n,n}$ with r colours:

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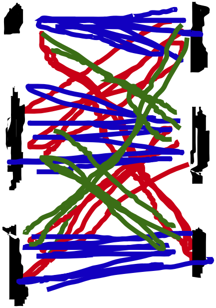

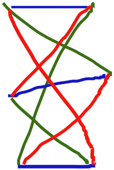

In a such a colouring, we might need $2r - 1$ paths.

Conjecture (Pok '14): $\exists$ a partition into $2r - 1$ paths

Complete bipartite graphs $K_{n,n}$: cycle partitions

<u>with r colours</u>:

Complete bipartite graphs $K_{n,n}$: cycle partitions

with r colours:

- Thm (Haxell '97): $\exists$ partition into $O((r \log r)^2)$ monoχ cycles
- Thm (Peng, Rödl, Ruciński '02): $O(r^2 \log r)$ cycles suffice

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- Cor (Schaudt, St '14+): $\exists$ partition of all but $o(n)$ vertices into 3 monochromatic cycles and a partition of all the vertices into 12 cycles.

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- (improving on Haxell '97: 1695 monochromatic cycles.)

Partitioning with paths/cycles

paths	K_n	$K_{n,n}$	$K_{n,n}$ 'local'
r colours	r ?	$2r-1$?	
2 colours	2	3	3
3 colours	3		

cycles	K_n	K_n 'local'	$K_{n,n}$
r colours	$100r\log r$	$O(r^2\log r)$	$O(r^2\log r)$
2 colours	2	2	12
3 colours	17		18

cycles: all but $o(n)$	K_n	$K_{n,n}$
r colours	r ?	$2r-1$??
2 colours	2	3
3 colours	3	5

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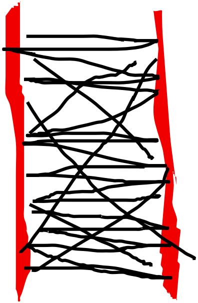
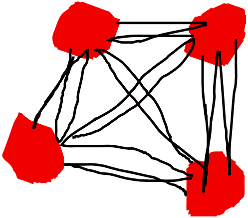
COMPLETE MULTIPARTITE GRAPHS

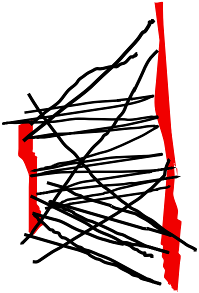
bipartite graph



multipartite graph



bipartite graph



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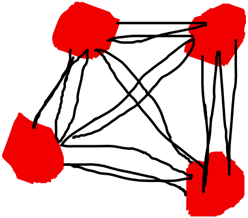
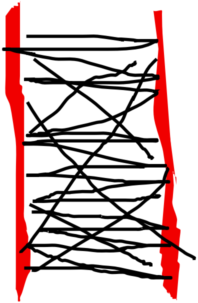
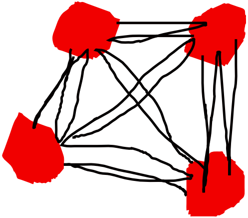


bipartite graph



balanced

multipartite graph



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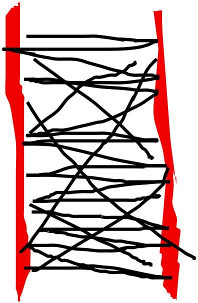

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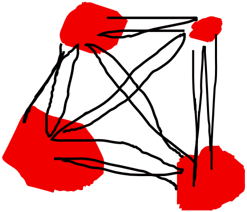
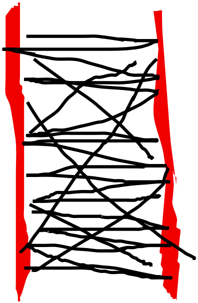


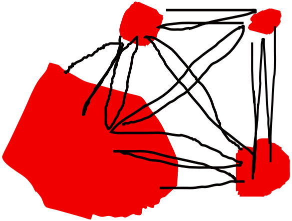
need balanced?

bipartite graph

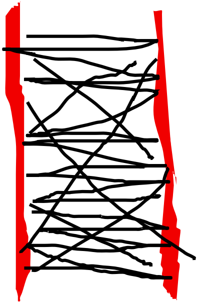


balanced

multipartite graph



bipartite graph



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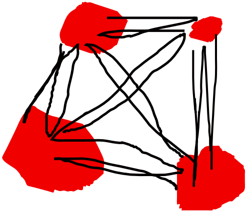

fair

fair multipartite graph: no class has more than half of $V(G)$

Fair complete k-partite graphs, with 2 colours:

Thm (Schaudt, St '14+): $\exists$ partition into two monochromatic paths of distinct colours, if $k \geq 3$.

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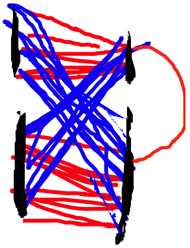

Fair complete k-partite graphs, $k \geq 2$, with 2 colours:

Thm (Schaudt, St '14+):
• can partition all but δn vertices into 2 monoχ cycles of distinct colours, if the colouring is δ-far from a split colouring.

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- can partition all but δn vertices into 2 monoχ cycles of distinct colours, if the colouring is δ-far from a split colouring.
- can partition all but $o(n)$ vertices into 3 monoχ cycles.
- 14 monoχ cycles partition all the vertices.

Partitioning with paths/cycles

paths	K_n	$K_{n,n}$	fair $K_{n_1,\ldots n_\ell}$, $\ell \geq 3$
r colours	r ?	$2r-1$?	r ??
2 colours	2	(2 or) 3	2
3 colours	3		

cycles	K_n	$K_{n,n}$	fair $K_{n_1,\ldots n_\ell}$
r colours	$100r\log r$	$O(r^2\log r)$	$O(r^2\log r)$
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cycles: all but $o(n)$	K_n	$K_{n,n}$	fair $K_{n_1,\ldots n_\ell}$
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Partitioning with paths/cycles

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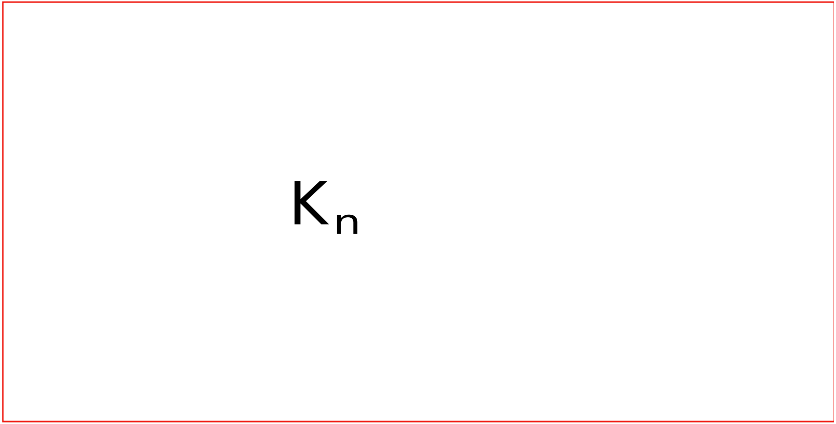
PROOF IDEAS

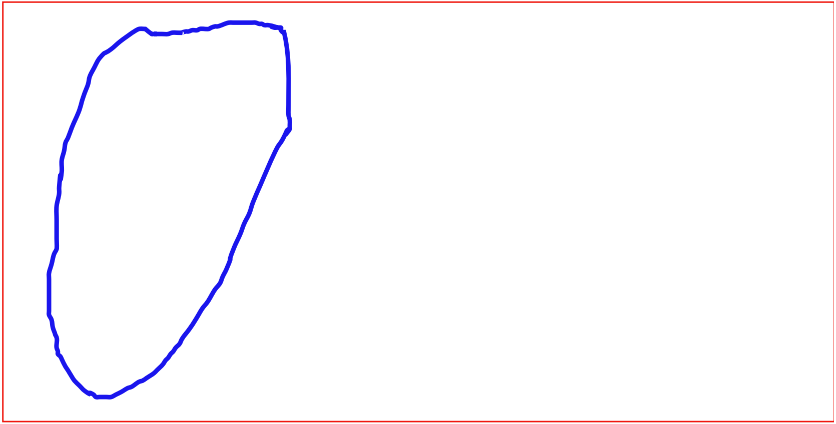
Proof ideas (for cycles):

K_n with r colours



Proof ideas (for cycles):

take out cycles using density



Use: Erdős–Gallai theorem for cycles.

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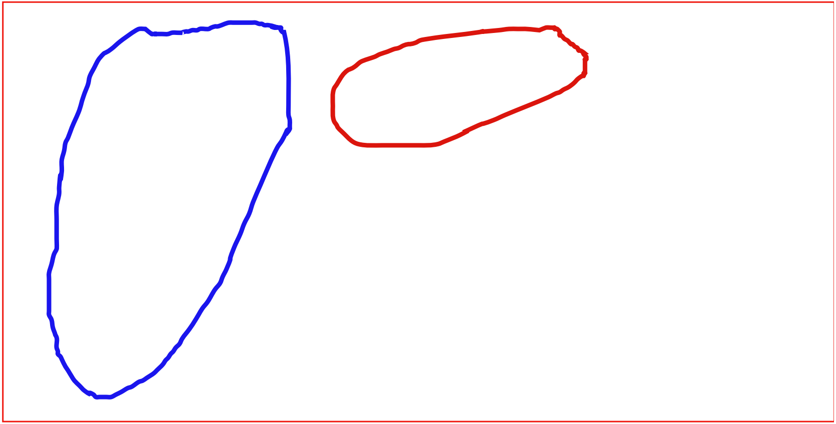

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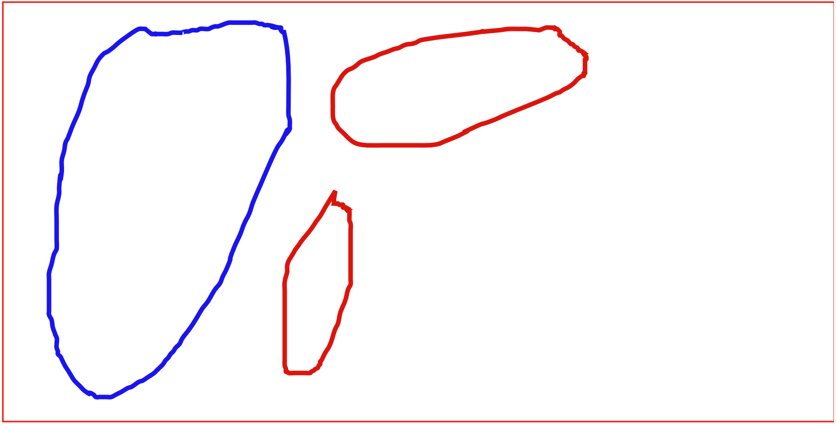

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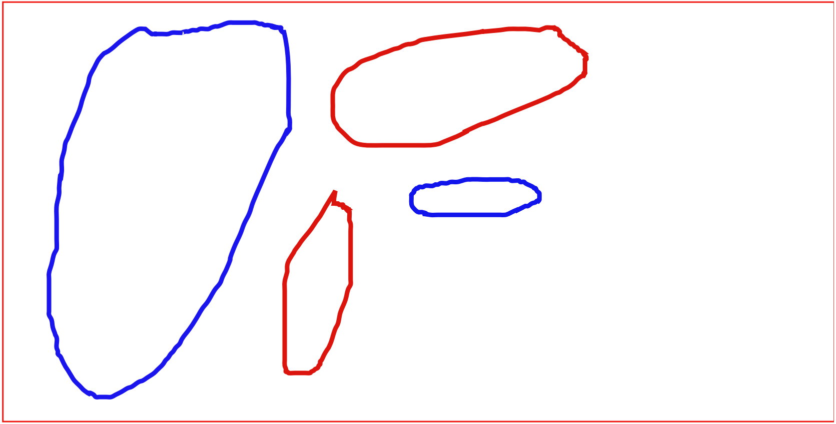

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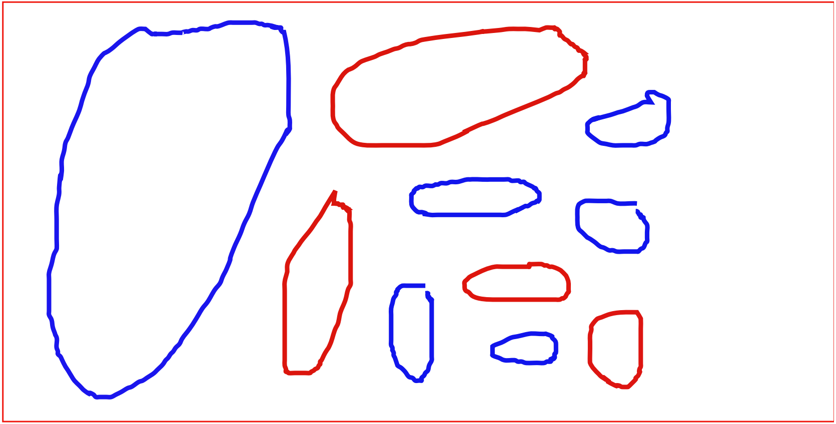
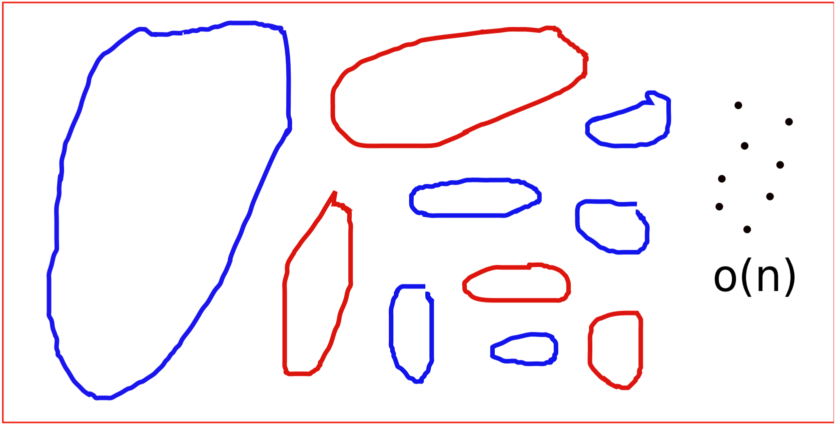


Use: Erdős–Gallai theorem for cycles.

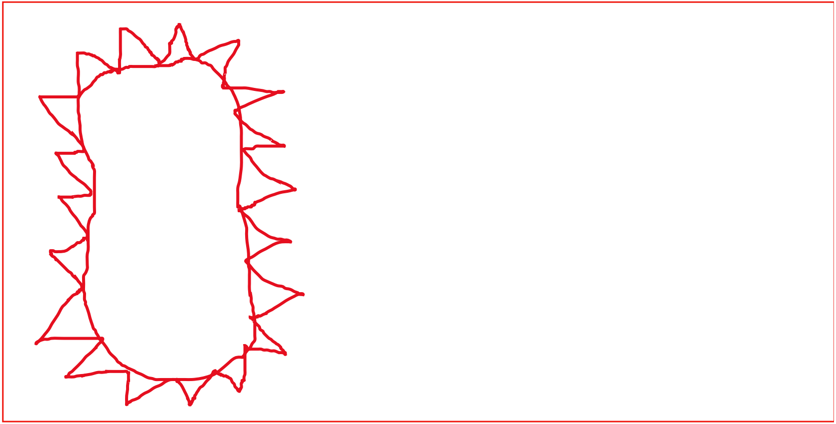
Proof ideas (for cycles):

what to do with the leftover vertices?



Proof ideas (for cycles):

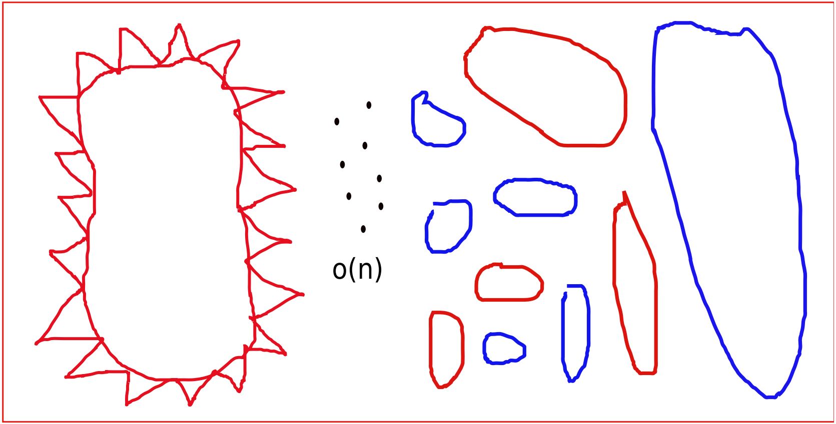
Start again: First, take out a ‘robust hamiltonian’ subgraph



Use: Ramsey number of the triangle cycle.

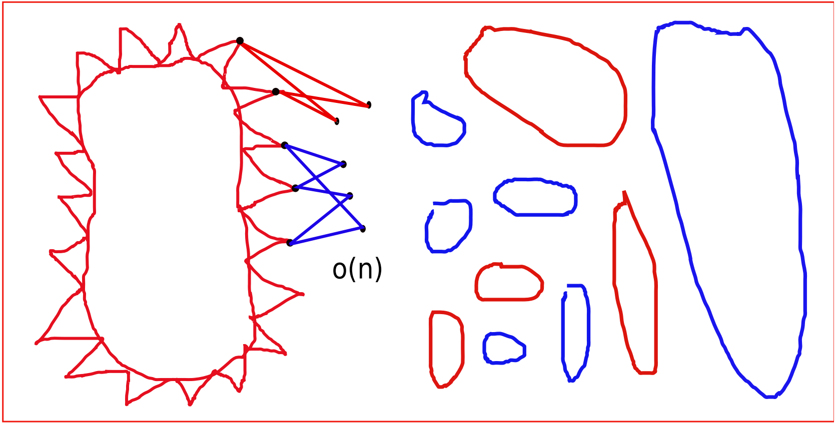
Proof ideas (for cycles):

Take out cycles as before



Proof ideas (for cycles):

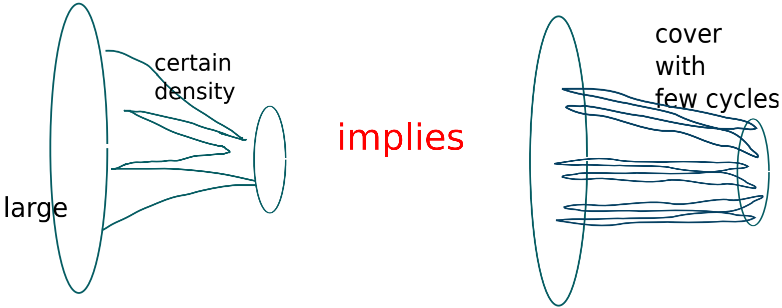
Finish by absorbing the leftover vertices.



Use: 'one sided covering lemma for bipartite graphs'.

Proof ideas (for cycles):

The 'one sided covering lemma':



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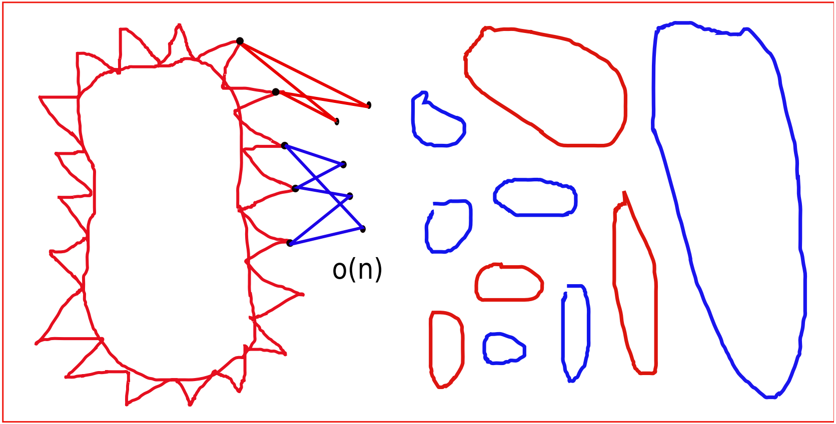

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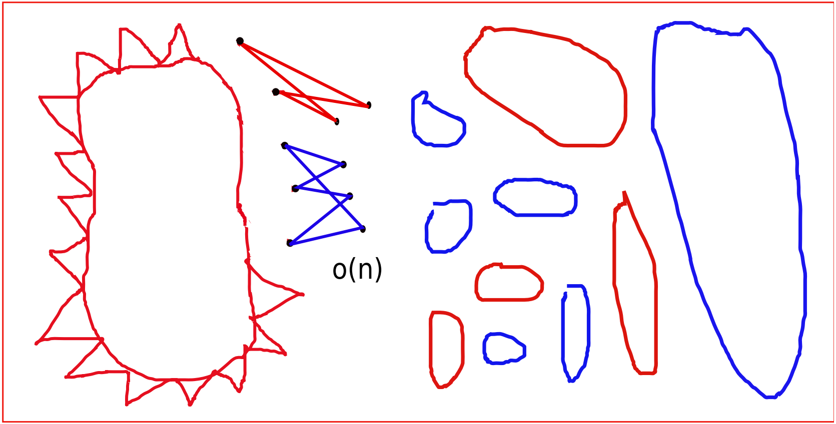

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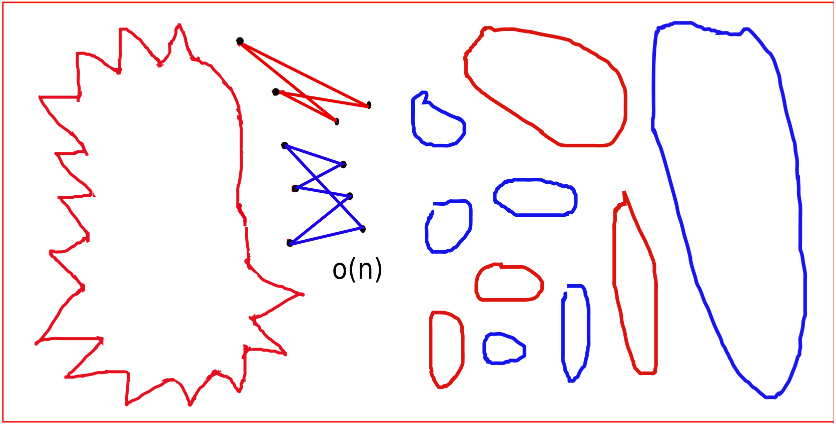
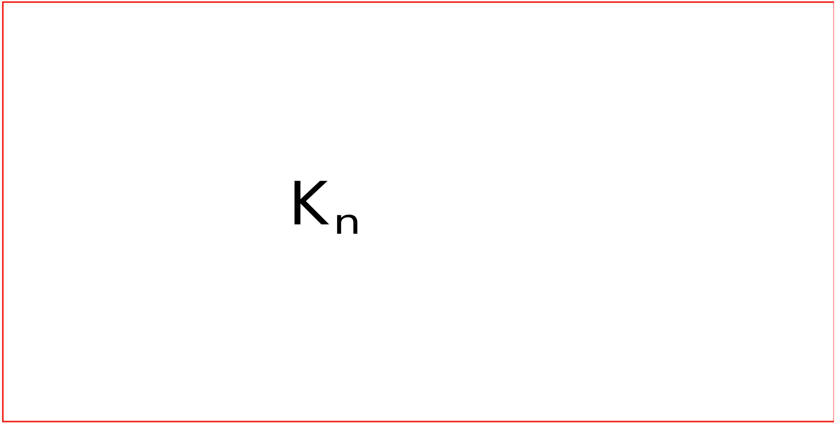


Proof ideas (for cycles):

This gives a bound of $O(r \log r)$ monochromatic cycles.

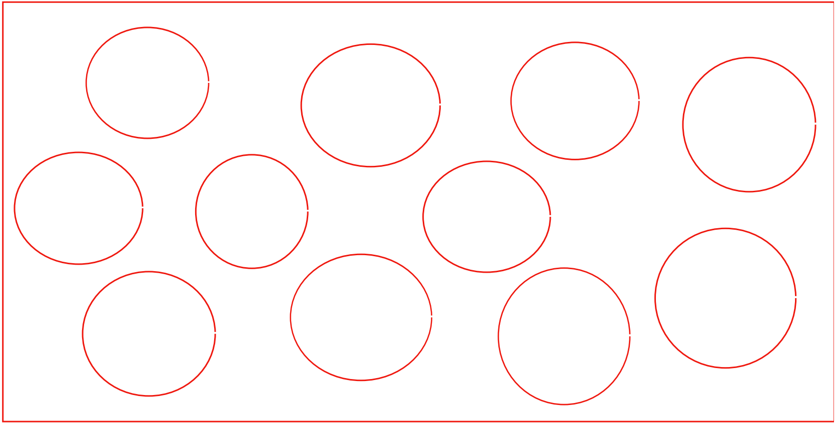
Proof ideas: small r

K_n with three colours



Proof ideas: small r

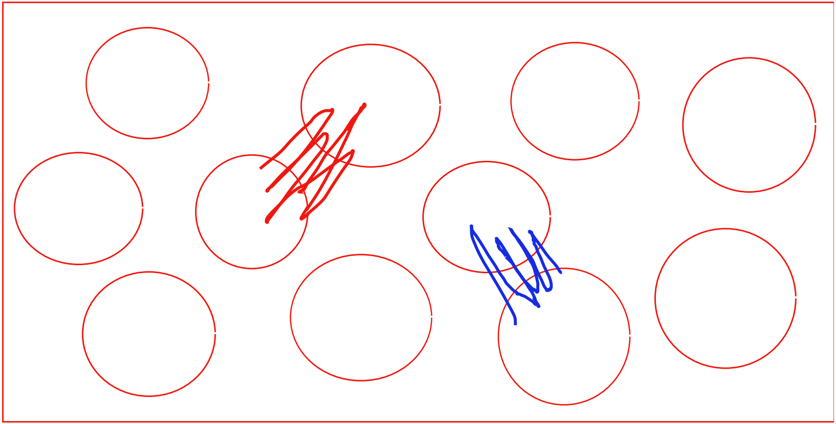
apply regularity lemma



Use: Regularity lemma.

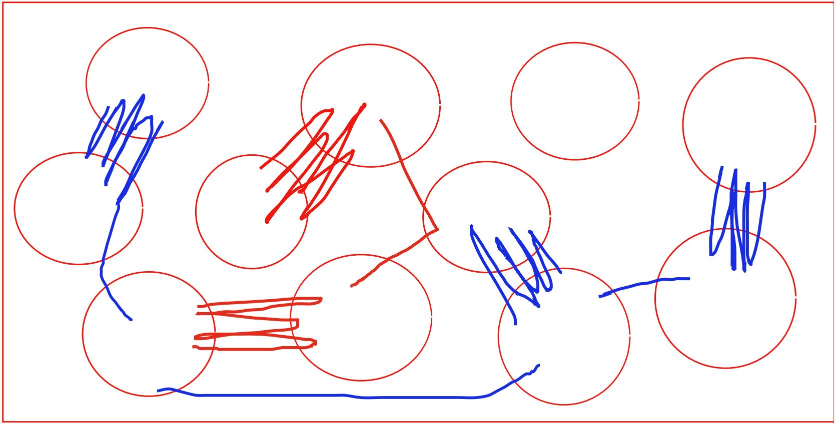
Proof ideas: small r

take majority colouring



Proof ideas: small r

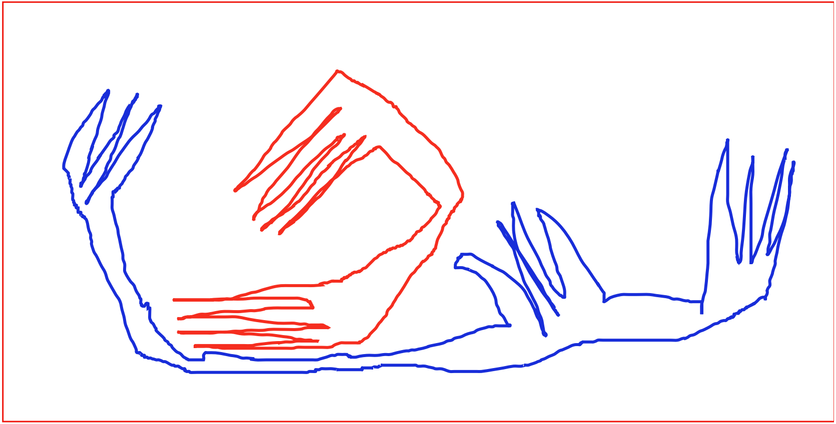
find 3 disjoint monoχ connected matchings covering almost all



Here we have to work.

Proof ideas: small r

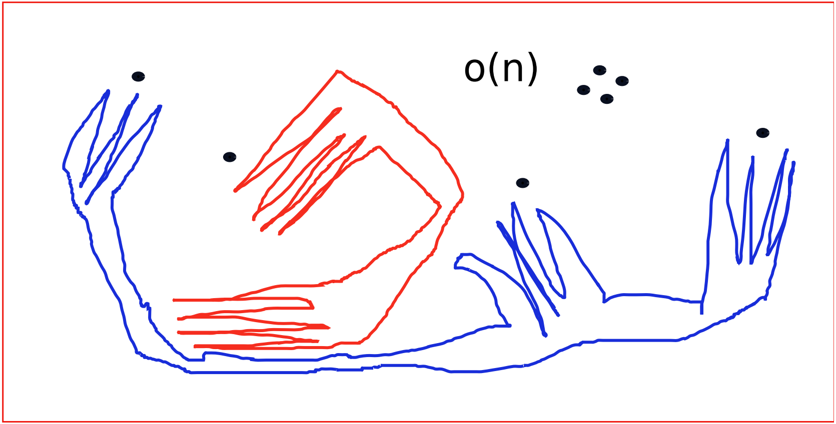
blow them up to get 3 cycles



Łuczak's blow-up technique.

Proof ideas: small r

only $o(n)$ vertices left

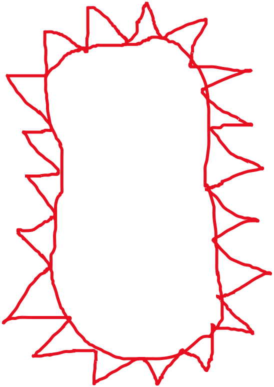


Proof ideas: small r

This gives a partition of all but $o(n)$ vertices into 3 monochromatic cycles.

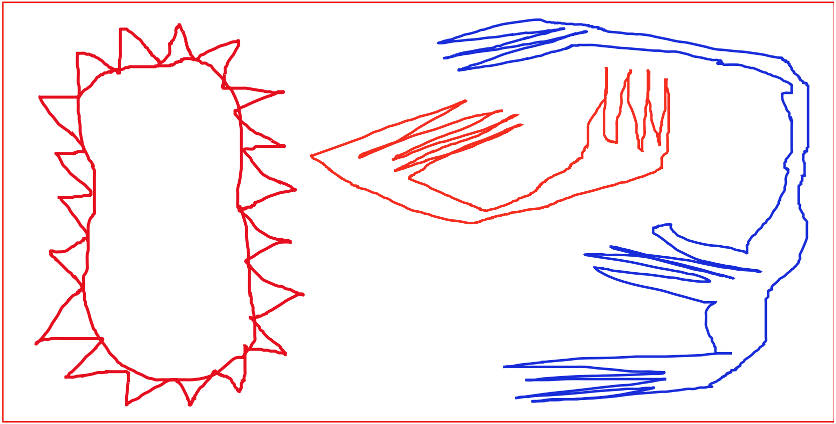
Proof ideas: small r

find a large 'robustly hamiltonian' monoχ graph



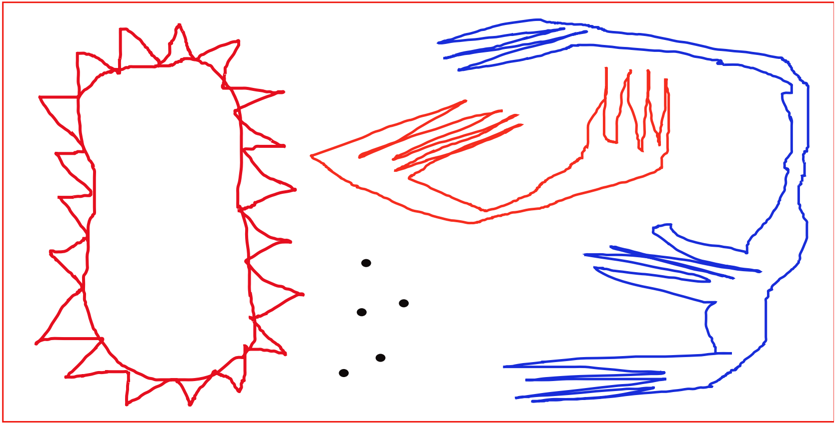
Proof ideas: small r

in the rest, apply the previous result



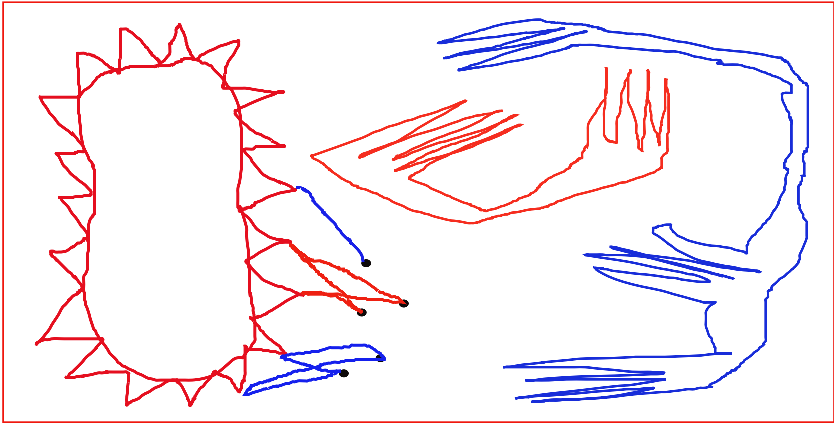
Proof ideas: small r

leftover very small compared to the first subgraph

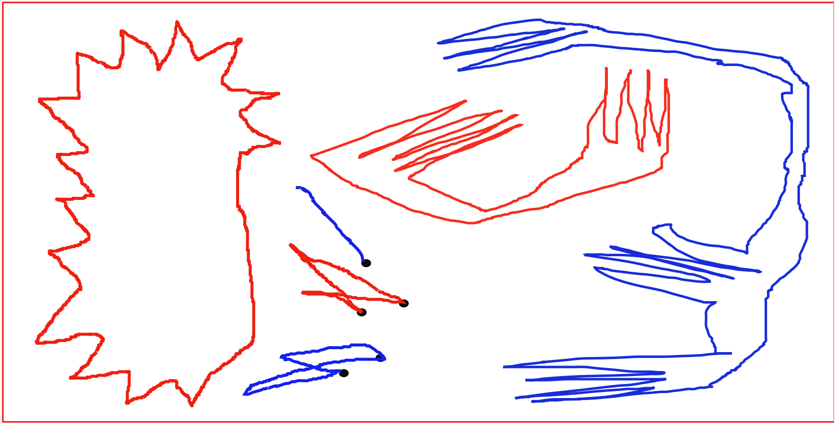


Proof ideas: small r

absorb the leftover



Proof ideas: small r



Proof ideas: small r

This gives a partition of all vertices into 17 monochromatic cycles.

Proof ideas: small r

Problems:

-how to find the connected monochromatic matchings
-how to find the path partitions
-how to deal with local colourings.

OTHER DIRECTIONS

OTHER DIRECTIONS

- partitions into trees [EGP '91 conjecture $r-1$ trees, Haxell, Kohayakawa '97 prove r]
- partitions into k-regular graphs and isolated vertices $\to$ function $f(k,r)$ [Sárközy, Selkow '99]
- partitions into members of an ∞ family of bounded degree graphs $\to$ function $f(\Delta, r)$ [Grinshpun, Sárközy '14+]
- partitions of arbitrary graphs G instead of K_n, $K_{n,n}$ $\to$ function $f(\alpha(G), r)$ [Sárközy '11, Balogh, Bárát, Gerbner, Gyárfás, Sárközy '13+]
- improvements for r-local colourings
- r-mean colourings
- covers instead of partitions [Gyárfás '83,...]

Thank you!