List 7 IBI 5081 – Optimization II.

1. An urn contains N white balls and one red ball. Let $X_i, i = 1, \ldots, n+m$ be i.i.d. variables, where X_i the number of attempts until one removes a red ball. And let $Y_i, i = 1, \ldots, m$ be i.i.d. variables from another experiment: we permit only M trails before the red ball, i.e.

$$Y_i = \begin{cases} X_{n+i}, & \text{if } X_{n+i} \leq M; \\ M+1, & \text{if } X_{n+i} > M. \end{cases}, i = 1, \ldots, m.$$

(1) Write likelihood function for this experiment, and suggest an optimization method for finding the maximum likelihood estimation for the number of white balls, $\hat{N}$.
(2) Suggest the recurrent formula for finding $\hat{N}$ using EM algorithm. Write the complete likelihood function and provide E-step, M-step of EM algorithm.

2. Consider the first exercise about litebulbs from Bernard Flury and Alice Zoppe "Exercises in EM" *The American Statistician*, Vol. 54, No. 3, 2000. Suppose $N = 5$ and the observations from the first experiment are

$$5.37,\ 0.71,\ 2.32,\ 0.69,\ 3.27.$$

For the second experiment let $t = 2$ and $M = 5$. Suppose that $Z = 2$, where Z is the number of litebulbs that are still alive at time $t = 2$.

(1) Numerically find the estimation of the mean $\hat{\theta}$ using EM algorithm.
(2) Write the likelihood function and find numerically the maximum using common optimization method like Newton-Raphson method.