List 4 IBI 5081 – Monte Carlo Method I.

1. In Exercise 3.5 [RC] The value of $\mathbb{P}(Z > 4.5)$ was estimated by Importance Sampling technique. The argument is that the direct Monte Carlo standard normal Z simulation hits a region $[4.5, \infty)$ very rarely – once in about 3 million iterations. We can produce the direct estimation in alternative way using the relation $\mathbb{P}(Z > 4.5) = 1 - \mathbb{P}(Z \leq 4.5)$ and simulating $Z_i \sim N(0,1)$ and estimating $\mathbb{P}(Z \leq 4.5)$.

(1) Construct the standard Monte Carlo estimator for $\mathbb{P}(Z > 4.5)$ using the relation $\mathbb{P}(Z > 4.5) = 1 - \mathbb{P}(Z \leq 4.5)$.
(2) Compare the rate of convergence with Importance Sampling estimator considered in Exercise 3.5 [RC] plotting the absolute error for both estimators. Comment.

Following the scheme of Exercise 3.5, provide the Monte Carlo estimation of the probability $\mathbb{P}(X > 5)$, where X has Cauchy distribution $\mathcal{C}(0, 1)$:

(1) Describe the consequences of choosing standard normal variable as an instrumental variable (Hint: Exercise 3.8).
(2) Suggest an instrumental variable with density g which avoids the problem of previous item.
(3) Using the exact value of the probability $\mathbb{P}(X > 5)$ plot the absolute error of the estimator.

2. [RC1, p.112, Problem 3.10] Compare (in a simulation experiment) the perfomances of the regular Monte Carlo estimator of

$$\int_1^2 \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx = \Phi(2) - \Phi(1)$$

with those of an estimator based on an optimal choice of instrumental distribution.