List 4 IBI 5081 – Monte Carlo Method I.

1. In Aula 4 the value of $\mathbb{P}(Z > 4.5), Z \sim N(0,1)$, was estimated by Importance Sampling (IS) technique using the truncated (at 4.5) exponential distribution.

- **a.** (2 points) Let X be exponentially (with rate 1) distributed random variable. Denote $X^{(4.5)}$ be the random variable X truncated at 4.5: $X^{(4.5)} = X \mid X > 4.5$. Show (using the memoryless property of exponential distribution) that $X^{(4.5)} = 4.5 + Y$, where $Y \sim \exp(1)$, and X and Y are independent.
- **b.** (2 points) Show analytically that the variance of variable used in the IS simulation (the similar calculus is in the Exercise 3.4, see slide of Aula 4 Exercises) can be represented as (check it!)

$$\frac{e^{-4.5}}{\sqrt{2}}\mathbb{P}(Z > 4.25\sqrt{2}).$$

  Calculate numerically this value. Provide the 95% confidence interval for sample size $n = 1000$.

As an alternative IS estimation we will use the variable $Y = 4.5 + |Z|$, where, as before, $Z \sim N(0,1)$. Using this new instrumental variable Y

- **c.** (1 point) construct the IS estimator for the same probability $\mathbb{P}(Z > 4.5)$;
- **d.** (1 points) plot the both IS estimators on the same plot.
- **e.** (2 points) you know that the sampled variance we consider as a realization of a random variable. for the both estimators 100 time simulating IS estimators with 1000 sample size, provide the histograms of variances for both estimators.

2. (2 points) Suggest the method for numerically calculate the following integral

$$\int_0^\infty \frac{(1+\sin^2(x))dx}{(1+\cos(x)+x^2)(1-\sin(x)+\sqrt{x})}.$$

Control the convergence by an error (estimate it). Plot the convergence, and provide a confidence interval with chosen sample size (for example 1000 or another).