List 2 IBI 5081 – Random variable simulations II.

1. Let $T = \{(x, y) : \ |x| + |y| \leq 1\}$. Vector (X, Y) is uniformly distributed on T, i.e. the joint density $p(x, y)$ is given as

$$p(x,y) = \begin{cases} 0.5 & \text{if } (x,y) \in T; \\ 0 & \text{if } (x,y) \notin T \end{cases}$$

(1) (1 point) Find the marginal densities $p_X(x)$ and $p_Y(y)$; are they independent random variables?
(2) (2 point) Show how to simulate joint values of (X, Y) by part: first simulate X according $p_X(x)$ (by inverse method, show calculations), then given the value $X = x$ simulate Y according conditional distribution (show the calculation of conditional distribution). Write a code and plot 30 (for example) simulated points.
(3) (1 point) Show how to simulate (X, Y) using accept reject method.
(4) (1 point) Suggest how to simulate (X, Y) using changing variables. Try, for example, $x' = x + y, y' = x - y$.

2. Random vector (X, Y) has the following joint distribution

$Y \setminus X$	0	1	3	4
-1	1/6	0	1/3	0
1	0	1/3	0	1/6

(1) (1 point) find marginal distributions of X and Y; are X and Y independent random variables?
(2) (1 point) draw the graph of cumulative distribution functions $F_X(x) = P(X \leq x)$ and $F_Y(y) = P(Y \leq y)$;
(3) (1 point) find variances $\mathbb{V}ar(X)$ and $\mathbb{V}ar(Y)$;
(4) (1 point) simulate separately X and Y using inverse method; show calculus;
(5) (1 point) suggest a method to simulate the vector (X, Y).