

List 2 IBI 5081 – Random variable simulations II.

1. Let  $T = \{(x, y) : |x| + |y| \leq 2\}$ . Vector  $(X, Y)$  is uniformly distributed on  $T$ , i.e. the joint density  $p(x, y)$  is given as

$$p(x, y) = \begin{cases} c & \text{if } (x, y) \in T; \\ 0 & \text{if } (x, y) \notin T \end{cases}$$

- (1)  $c = ?$
- (2) find the marginal densities  $p_X(x)$  and  $p_Y(y)$ ;
- (3) are  $X$  and  $Y$  independent random variables?
- (4) find expectation values  $\mathbb{E}(X)$  and  $\mathbb{E}(Y)$ ;
- (5) find variances  $\mathbb{V}ar(X)$  and  $\mathbb{V}ar(Y)$ ;
- (6) simulate separately  $X$  and  $Y$  using inverse method; show calculus;
- (7) simulate joint values of  $(X, Y)$  by part: first simulate  $X$  according  $p_X(x)$ , then given the value  $X = x$  simulate  $Y$  according conditional distribution;
- (8) can you suggest another method (I hope simpler method) of simulation for the vector  $(X, Y)$ ?

2. Random vector  $(X, Y)$  has the following joint distribution

$Y \setminus X$	0	1	3	4
-1	1/6	0	1/3	0
1	0	1/3	0	1/6

- (1) find marginal distribution of  $X$  and  $Y$ ;
- (2) draw the graph of cumulative distribution functions  $F_X(x) = P(X \leq x)$  and  $F_Y(y) = P(Y \leq y)$ ;
- (3) find  $\mathbb{E}(X), \mathbb{E}(Y)$ ;
- (4) find  $\mathbb{E}(X + Y)$ ;
- (5) are  $X$  and  $Y$  independent random variables?
- (6) find variances  $\mathbb{V}ar(X)$  and  $\mathbb{V}ar(Y)$ ;
- (7) simulate separately  $X$  and  $Y$  using inverse method; show calculus;
- (8) suggest a method to simulate the vector  $(X, Y)$ .