List 1 (IBI 5081) Random variable simulations I.

1. Random variable X has the following cumulative distribution function $F(x)$

$$F(x) = \begin{cases} 0, & \text{if } x < 0; \\ x^5, & \text{if } x \in [0,1]; \\ 1, & \text{if } x > 1. \end{cases}$$

(1) Using inverse method, show how to simulate the random variable X.
(2) Show how to simulate X using accept-reject method. What is the probability to accept the value?
(3) Perform the both simulations in computer (use R or another programming software): construct the histograms for each method and compare the graphs with true (theoretical) density of X.

2*. Let us try the following (toy) version of accept-reject method:
(1) generate two uniforms U_1 and U_2;
(2) if $U_2^2 > U_1$ then we accept U_2, and we accept U_1 in the contrary case.
Find the distribution of such simulated random variable.

3. One indirect way to simulate standard normal distribution is the using of the central limit theorem. Indeed, if we have e sequence of i.i.d. random variable $(X_i, i = 1, 2, \ldots)$ with expectation zero, $\mathbb{E}(X_i) = 0$, and with the finite second moment $\mathbb{E}(X_i^2) < \infty$, then the following scaled variable $Z_n = \frac{\sqrt{n}\overline{x}_n}{s_n}$, where $\overline{x}_n$, s_n are n-sampled average and sampled deviation respectively, has approximately the standard normal distribution. This approximation is better as sample size n is greater.
(1) Let X_i are i.i.d. uniformly distributed on $[-1, 1]$ random variables. Generate in computer 20 random Z_{20} numbers, construct the histogram and compare with true density profile of standard normal distribution.
(2) Repeat the previous item for 20 random Z_{100}. Comment if you found some difference with results from the previous item.
(3) The central limit theorem works if we consider discrete X_i. Repeat the two previous itens for the case when X_i have Bernoulli distribution with success probability $p = 0.3$. Observe, that in this case instead of $Z_n = \frac{\sqrt{n}\overline{x}_n}{s_n}$ we should consider the another $Z'_n = \frac{\sqrt{n}(\overline{x}_n - 0.3)}{s_n}$.