π^T $\quad X_1 \ X_2 \ X_3 \dots X_n \ X_{n+1}$

$$\pi_{n+1} = \begin{pmatrix} \pi_{n+1}(1) \\ \pi_{n+1}(2) \\ \vdots \\ \pi_{n+1}(k) \end{pmatrix}$$

$$\pi_{n+1}(j) = P(X_{n+1}=j) =$$

$$= P(X_{n+1}=j, X_n=1) + P(X_{n+1}=j, X_n=2) + \dots + P(X_{n+1}=j, X_n=k)$$

$$= \sum_{i=1}^{k} P(X_{n+1}=j, X_n=i) = \sum_{i=1}^{k} P(X_{n+1}=j \mid X_n=i)\, P(X_n=i)$$

$$= \sum_{i=1}^{k} \pi_n(i)\, p_{ij}$$

$$P(A \cap B) = P(A \mid B) P(B)$$

$$(\pi_n(1), \dots, \pi_n(k)) \begin{pmatrix} \dots & p_{1j} & \\ & p_{2j} & \\ & \vdots & \\ & p_{kj} & \end{pmatrix} = (\dots \pi_{n+1}(j) \dots)$$

$$\pi_n^T P = \pi_{n+1}^T$$

$$\uparrow$$

$$\pi_{n-1}^T P$$

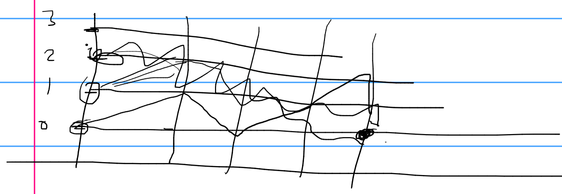
$$\uparrow$$

$$\pi_{n-2}^T P$$

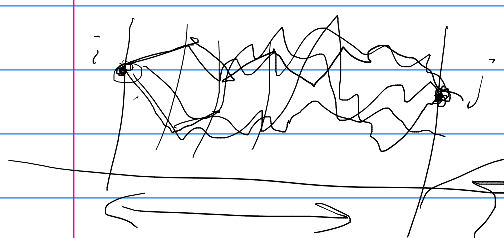
$$\pi_{n-2}^T PPP = \pi_{n+1}^T$$

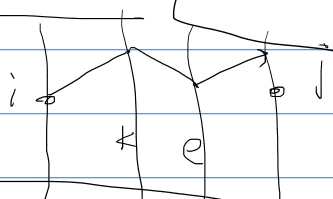
$$\pi_{n-2}^T P^3 = \pi_{n+1}^T$$

$$\pi_0^T P^{n+1} = \pi_{n+1}^T$$



$$\boxed{P(X_{n+k}=j \mid X_n=i) = (P^k)_{ij}}$$





$$\sum_e \sum_k p_{ik}\, p_{ke}\, p_{ej} = (P^3)_{ij}$$

$$(P^n)_{ij}$$