Workshop

"Young Researchers in Algebra"

IME—USP, São Paulo, Brazil
May 24—25, 2013

List of Participants, Schedule and Abstracts

Workshop

Young Researchers in Algebra

Instituto de Matemática e Estatística
Universidade de São Paulo.
May 24–25, 2013

In the framework of research project

Algebras, representations and applications

supported by FAPESP (Projeto Temático 2010/50347-9).

Organizers

- **Kostiantyn Iusenko**, CMCC, Universidade Federal do ABC;
- **Iryna Kashuba**, Instituto de Matemática e Estatística, Universidade de São Paulo;
- **Alexey Kuzmin**, Instituto de Matemática e Estatística, Universidade de São Paulo.

List of Participants

Abdelmoubine Amar Henni — IMECC–UNICAMP
henni@ime.unicamp.br

Alexey Kuzmin — IME–USP
amkuzmin@ya.ru

Arkady Tsurkov — IME–USP
arkady.tsurkov@gmail.com

Artem Lopatin — IMECC–UNICAMP
artem_lopatin@yahoo.com

Behrooz Mirzaii — ICMC–USP
bmirzaii@icmc.usp.br

Carina Alves — UNESP–Rio Claro
carina@rc.unesp.br

Cecilia Fernanda Saraiva de Oliveira — UERJ
cecilfso@gmail.com

Cristian Ortiz — Universidade Federal do Paranà
cristian.ortiz@ufpr.br

Eduardo Rogério Fàvaro — UFU–Pontal
eduardofavaro@yahoo.com.br

Emerson Ferreira de Melo — UnB
E.F.Melo@mat.unb.br

Evan Andrew Wilson — IME–USP
wilsoneaster@gmail.com

Fatemeh Yeganeh Mokari — IMECC–Unicamp
ra142304@ime.unicamp.br

Felipe Yukihide — IMECC–Unicamp
ra091138@ime.unicamp.br

Flávio Freereira da Rocha — IMPA
flavioartin@gmail.com

Gary Russell Cook — ICMC–USP
garycook82@msn.com

Grasiele Cristiane Jorge — Unicamp
grajorge@gmail.com

Herivelto Borges — ICMC–USP
hborges@icmc.usp.br

Ivan Kaygorodov — IME–USP
kib@ime.usp.br
Izabella Stuhl — IME–USP
stuhlizabella@gmail.com
Javier Sánchez Serdà — IME–USP
jsanchezserda@gmail.com
John H. Castillo — Universidad de Nariño (Colombia)
jhcastillo@gmail.com
John MacQuarrie — UnB
john.macquarrie@googlemail.com
Kisnney Emiliano de Almeida — UEFS
kisnney@gmail.com
Kostiantyn Iusenko — IME–USP
kay.math@gmail.com
Lucio Centrone — UNICAMP
centrone@ime.unicamp.br
Luis Enrique Ramírez — IME–USP
luisenrique317@gmail.com
Marcilio Ferreira dos Santos — Universidade federal de Pernambuco
marciliofds@gmail.com
Marco Geraci — Universitá degli Studi di Palermo
marco.geraci@math.unipa.it
Mario Ruiz — Universidad de El Salvador
marm_marm@hotmail.com
Michael Santos Gonzales Gargate — IMECC–UNICAMP
michael_gg84@hotmail.com
Mykola Khrypchenko — IME–USP
nskhripchenko@gmail.com
Nicola Pace — ICMC–USP
nicolaonline@libero.it
Paula Andrea Cadavid Salazar — IME–USP
paca@ime.usp.br
Paula Olga Gneri — Universidade Tecnológica Federal do Parana
paulitagneri@gmail.com
Renato Alessandro Martins — IME–USP
orenatoalessandro@gmail.com
Stefania Aqué — Universitá degli Studi di Palermo
aque@math.unipa.it

Thiago Castilho de Mello — IME–USP
castilho.thiago@gmail.com
Tiago Duque — UFPE
tdm@dmat.ufpe.br
Tiago Macedo — IMECC–Unicamp
tmacedo@ime.unicamp.br
Vinicius Bittencourt — IME–USP
bittencourt.vs@gmail.com
Wagner de Oliveira Cortes — UFRGS
wocortes@gmail.com

Schedule

9.30-10.20	**Ualbay Umirbayev**	
10.30-11.00	*Coffee Break*	
11.00-11.30	**Javier Sánchez Serdá**	
11.30-12.00	**Artem Lopatin**	
12.00-12.30	**John MacQuarrie**	
12.30-13.00	**Arkady Tsurkov**	
13.00-14.00	*Lunch*	
14.00-14.30	**Ivan Kaygorodov**	**Stefania Aqué**
14.30-15.00	**Izabella Stuhl**	**John H. Castillo**
15.00-15.30	**Emerson Ferreira de Melo**	**Lucio Centrone**
15.30-16.00	**Nicola Pace**	**Thiago Castilho de Mello**
16.00-16.30	*Coffee Break*	
16.30-17.00	**Carina Alves**	**Evan Wilson**
17.00-17.30	**Cecilia Fernanda Saraiva de Oliveira**	**Tiago Macedo**
17.30-18.00	**Eduardo Rogério Fávaro**	**Renato Alessandro Martins**
18.00-18.30	**Grasiele Cristiane Jorge**	**Luis Enrique Ramíres**

Constructing of E_8-lattice from quaternion algebras with small Tamagawa volume

Carina Alves[1] and Jean-Claude Belfiore[2]

[1] *Sao Paulo State University - Dept. of Mathematics -UNESP-Rio Claro, Brazil.*
carina@rc.unesp.br
[2] *TELECOM-ParisTech - Comelec - Paris, France.*
belfiore@telecom-paristech.fr

New advances in wireless communications consider systems with multiple antennas at both the transmitter and receiver ends, in order to increase the data rates and the reliability. The coding problem then becomes more complex and code design criteria for such scenarios showed that the challenge was to construct fully-diverse, full-rate codes, i.e., sets of matrices such that the difference of any two distinct matrices is full rank. This requires new algebraic tools, namely division algebras. Division algebras are non-commutative algebras that naturally yield families of fully-diverse codes, thus enabling to design high rate, highly reliable Space-Time codes [1].
Space-Time Codes based on an order of a quaternion algebra such that the volume of the Dirichlet's polyhedron of the group of units is small, are better suited for decoding using the method of algebraic reduction since the approximation error is smaller [2]. The volume of this Dirichlet's polyhedron is given by the Tamagawa formula and is called the Tamagawa volume [3].
In this work we propose to construct the E_8-lattice as a left ideal of a maximal order of some quaternion algebras with a small Tamagawa volume.

References

[1] Hollanti C., Lahtonen J., Lu H.-f.(F.), *Maximal Orders in the Design of Dense Space-Time Lattice Codes.* IEEE Trans. Inform. Theory, **54** (10) (2008) 4493–4510.

[2] Luzzi L., Othman G. R-B., Belfiore J-C., *Algebraic Reduction for the Golden Code.* Advances in Mathematics of Communications, **6** (1) (2012) 1–26.

[3] Maclachlan C., Reid A. W., *The Arithmetic of Hyperbolic 3-Manifolds.* Springer, 2003.

Cocharacters of bilinear mappings and graded matrices

Stefania Aquè[1] and Antonio Giambruno[2]

[1] *Dipartimento di Matematica ed Informatica, Università di Palermo, Via Archirafi 34, 90123 Palermo, Italy.*
aque@math.unipa.it
[2] *Dipartimento di Matematica ed Informatica, Università di Palermo, Via Archirafi 34, 90123 Palermo, Italy.*
antonio.giambruno@unipa.it

Let $M_k(F)$ be the algebra of $k \times k$ matrices over a field F of characteristic 0. If G is any group, we endow $M_k(F)$ with the elementary grading induced by the k-tuple $(1, \ldots, 1, g)$ where $g \in G$, $g^2 \neq 1$. Then the graded identities of $M_k(F)$ depending only on variables of homogeneous degree g and g^{-1} are obtained by a natural translation of the identities of bilinear mappings (see [1]). We study such identities by means of the representation theory of the symmetric group. We act with two copies of the symmetric group on a space of multilinear graded polynomials of homogeneous degree g and g^{-1} and we find an explicit decomposition of the corresponding graded cocharacter into irreducibles.

References

[1] Bahturin Yu., Drensky V. *Identities of bilinear mappings and graded polynomial identities of matrices.* Linear Algebra and its Applications, **369**, 2003, 95–112.

Lie identities of symmetric elements in group algebras

John H. Castillo

Departamento de Matemáticas y Estadística, Universidad de Nariño, Colombia.
jhcastillo@udenar.edu.co, jhcastillo@gmail.com

Let FG be a group algebra of a group G over a field F of characteristic different from 2. A homomorphism $\sigma : G \to \{\pm 1\}$ is called an *orientation* of the group G. It is possible to define an involution of the group algebra FG as follows

$$\left(\sum_{g\in G} \alpha_g g\right)^* = \sum_{g\in G} \alpha_g \sigma(g) g^{-1}.$$

We denote $(FG)^+ = \{\alpha \in FG : \alpha^* = \alpha\}$ the set of symmetric elements under $*$.

In an associative ring R, the Lie bracket of two elements $x, y \in R$ is defined by $[x, y] = xy - yx$. This definition is extended recursively via $[x_1, \ldots, x_{n+1}] = [[x_1, \ldots, x_n], x_{n+1}]$. Given a nonempty subset S of R, we say that S is Lie nilpotent if there exists an $n \in \mathbb{Z}^+$ such that $[\alpha_1, \ldots, \alpha_n] = 0$, for all $\alpha_i \in S$. For $n \in \mathbb{Z}^+$, we say that S is Lie n-Engel if

$$[\alpha, \underbrace{\beta, \ldots, \beta}_{n \text{ times}}] = 0,$$

for all $\alpha, \beta \in S$.

In this talk we survey some new results and some question about the study of Lie identities of $(FG)^+$.

References

[1] Castillo, J.H. Lie nilpotency indices of symmetric elements under oriented involutions in group algebras. arXiv:1209.6256 (preprint), (2013)

[2] Castillo, J.H. and Polcino Milies, C. Lie properties of symmetric elements under oriented involutions. Comm. Algebra 40 (2012), no. 12, 4404–4419.

[3] Lee, G.T. Group identities on units and symmetric units of group rings. Algebra and Applications, 12. Springer-Verlag London, Ltd., London, 2010.

Some families of simple derivations and holomorphic foliations without algebraic solutions

CECILIA FERNANDA SARAIVA DE OLIVEIRA

Universidade do Estado do Rio de Janeiro, Rio de Janeiro, Brazil.
cecilfso@gmail.com

Connections between the theory of Holomorphic Foliations and Algebraic Geometry have been under some attention over the years. One of its many interesting problems is the production of generic sets of simple derivations of the complex polynomial ring, whose birth lies on a conjecture answered by J.T.Staffod, [1], in the environment of Weyl Algebras. Our work focuses on the exhibition of families of simple derivations of degree 3 (on whose manipulation and production we made use of a Computer Algebra System, the `Singular`), and foliations of degree greater than 4, having the major support of the famous Jouanolou's Theorem [2].

References

[1] J. T. Stafford, *Non-holonomic modules over the Weyl algebra and enveloping algebras*, Invent. math. **79**

[2] J.P. Jouanolou, *Equations de Pfaff Algébriques*, Springer-Verlag, 1979.

The graded Gelfand-Kirillov dimension of graded PI-algebras

LUCIO CENTRONE

Universidade Estadual de Campinas, Rua Sergio Buarque de Holanda 651, Campinas (SP), Brasil.
centrone@ime.unicamp.br

We consider finitely generated associative graded PI-algebra over a field of characteristic 0. We generalize the notion of Gelfand-Kirillov dimension for graded PI-algebra and we compute it for some important classes of algebras. In particular, let E be the infinite dimensional Grassmann algebra, then we consider the verbally prime algebras $M_n(F)$, $M_n(E)$ and $M_{a,b}(E)$ endowed with their gradings induced by that of Di Vincenzo-Vasilovsky (see [1], [2]) and we compute their graded Gelfand-Kirillov dimensions. We also obtain general teoretical results about the transcendence degree of the relatively free graded algebra of graded prime PI-algebra on its graded center.

References

[1] Di Vincenzo O. M., *On the graded identities of* $M_{1,1}(E)$. Israel J. Math. 80, 323-335., **90**, 323-335.

[2] Vasilovsky S. Y., Z_n*-graded polynomial identities of the full matrix algebra of order* n. Proc. Amer. Math. Soc., **127**, no.12, 3517-3524, (1999).

Agebraic tools for Euclideam Minima in number fields

Eduardo Rogério Fávaro

Universidade Federal de Uberlândia. eduardofavaro@yahoo.com.br

We will describe the trace form for some abelian number fields of prime power conductor with the goal to compute the Euclidean Minima for these fields. In one particular case, the field satisfies the Minkowsky conjecture.

References

[1] Favaro, E.R.; Andrade, A.A.; Shah, T.; *Fields of two power conductor.*Journal of Advanced Research in Applied Mathematics.

[2] Bayer-Fluckiger, E.; Nebe, G; *On the Euclidean minimum of some real number fields*, J. de théorie des nombres de Bordeaux, 17 (2005) 437-454.

[3] Bayer-Fluckiger, E.; *Upper bounds for Euclidean minima of algebraic number fields*, J. Number Theory, 121 (2006), 305-323.

On the Classification problem of small dimensional Lie Superalgebras

Ma Isabel Hernández

Instituto de Matemática e Estatística, IME-USP.
isabellie@gmail.com

An interesting problem in Algebra is to classify small dimensional (super)algebras in different varieties. In this work, we classify real and complex Lie superalgebras $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ having $\mathfrak{g}_0 = \mathfrak{g}_1$ a three dimensional real or complex Lie algebra.

Full diversity rotated lattices

Grasiele C. Jorge[1] and Sueli I. R. Costa[2]

[1] *University of Campinas,* grajorge@gmail.com
[2] *University of Campinas,* sueli@ime.unicamp.br

A *lattice* Λ is a discrete additive subgroup of $\mathbb{R}^n$. Lattices have a range of applications in different areas. In this work we attempt to construct lattices with full rank which may be suitable for signal transmission over both Gaussian and Rayleigh fading channels [2, 3]. For this purpose the lattice parameters that we consider here are packing density, diversity and minimum product distance. Based on algebraic number theory we construct some families of full diversity rotated D_n-lattices via subfields of cyclotomic fields [4, 5]. Closed-form expressions for their minimum product distances are obtained through algebraic properties. We present also a necessary condition for constructing a rotated D_n-lattice via a totally real Galois extension $\mathbb{K}|\mathbb{Q}$. These results are related to an open problem stated in [1]: given a lattice Λ, is there a number field $\mathbb{K}$ such that it is possible to construct Λ via $\mathcal{O}_{\mathbb{K}}$?

References

[1] E. Bayer-Fluckiger, *Lattices and number fields*, Contemporary Mathematics, vol. 241, pp. 69-84, 1999.

[2] E. Bayer-Fluckiger, *Ideal lattices,* Proceedings of the conference Number theory and Diophantine Geometry, Zurich, 1999, Cambridge Univ. Press 2002, pp. 168-184.

[3] E. Bayer-Fluckiger, F. Oggier, E. Viterbo, *New Algebraic Constructions of Rotated $\mathbb{Z}^n$-Lattice Constellations for the Rayleigh Fading Channel*, IEEE Transactions on Information Theory, vol. 50, no. 4, pp. 702-714, 2004.

[4] G.C. Jorge, A.J. Ferrari, S.I.R. Costa, *Rotated D_n-lattices*, Journal of Number Theory, vol. 132, pp. 2397-2406, 2012.

[5] G. C. Jorge, S. I. R. Costa, On rotated D_n-lattices construct via totally real number fields, Archiv der Mathematik, v. 100, p. 323-332, 2013.

This work was partially supported by CNPq.

Derived categories of functors

Paulo Olga Gneri[1] and Marcos Jardim[2]

[1]*DAMAT-UTFPR, Av. Sete de Setembro, 3165 - Reboucas, 80230-901 – Curitiba-PR, Brasil.* paulitagneri@gmail.com
[2]*IMECC-UNICAMP, Departemento de Matamática, Caixa Postal 6065, 13083-970, Campinas-SP, Brasil.* jardim@ime.unicamp.br

Let $\mathcal{C}$ be small category and $\mathcal{A}$ an arbitrary category. Consider the category $\mathcal{C}(\mathcal{A})$ whose objects are functors from $\mathcal{C}$ in $\mathcal{A}$ and whose morphisms are natural transformations. Let $\mathcal{B}$ be another category, and again, consider the category $\mathcal{C}(\mathcal{B})$. Now, given a functor $F : \mathcal{A} \to \mathcal{B}$ we construct the induced functor $F_{\mathcal{C}} : \mathcal{C}(\mathcal{A}) \to \mathcal{C}(\mathcal{B})$. Assuming $\mathcal{A}$ and $\mathcal{B}$ to be abelian categories we have that the categories $\mathcal{C}(\mathcal{A})$ and $\mathcal{C}(\mathcal{B})$ are also abelian. We have two main goals: 1) to find a relationship between $D(\mathcal{C}(\mathcal{A}))$ and $\mathcal{C}(D(\mathcal{A}))$; 2) relate the functors $R(F_{\mathcal{C}}))$ and $(RF)_{\mathcal{C}}$ of $\mathcal{C}(D(\mathcal{A}))$ to $\mathcal{C}(D(\mathcal{B}))$. We use this results to prove a version of Mukai's Theorem for Q-coherent sheaves.

Generalized Poisson Algebras

Ivan Kaygorodov[1]

[1]Sobolev Inst. of Mathematics, Novosibirsk, Russia.
[2]IME, Universidade de São Paulo, Brazil.
kaygorodov.ivan@gmail.com

Poisson algebra is a vector space with two multiplications $\cdot$ and $\{,\}$. It is using in mathematical physics, differential geometry and other areas. For example, I. Shestakov and U. Umirbaev used the free Poisson algebra for proof of Nagata conjecture about wild automorphisms. Also, algebraic properties of Poisson algebras were studied in some papers of D. Farkas, L. Makar-Limanov, V. Petrogradsky and others. In particulary, D. Farkas proved that every PI Poisson algebra satisfies some polinomial identity with special type.
Generalized Poisson algebras and superalgebras are generalization of Poisson algebras and superalgebras. V. Kac and N. Cantarini defined generalized Poisson superalgebras as superalgebras with associative-supercommutative multiplication $\cdot$ and super-anticommutative multiplication $\{,\}$ satisfies

$$\{a,\{b,c\}\} = \{\{a,b\},c\}\} + (-1)^{|a||b|}\{b,\{a,c\}\},$$

$$\{a, b\cdot c\} = \{a,b\}\cdot c + (-1)^{|a||b|}b\cdot\{a,c\} - D(a)bc, \text{ where } D(a) = \{a,1\}.$$

If $D = 0$ we have Poisson superalgebra.
Generalized Poisson (super)algebras have many relations with (super)algebras of Jordan brackets. Early, (super)algebras of Jordan brackets was considered by C. Martinez, E. Zelmanov, I. Shestakov, I. Kaygorodov and V. Zhelyabin. For polinomial generalized Poisson algebras was proved an analogue of Farkas Theorem. It is following

Theorem 1 *Every PI generalized Poisson algebra over field with characteristic zero satisfies identity with type*

$$g = \sum_{i=0}^{[m/2]}\sum_{\sigma\in S_m} c_{\sigma,i}\Pi_{k=1}^{i}\langle x_{\sigma(2k-1)}, x_{\sigma(2k)}\rangle \Pi_{k=2i+1}^{m} D(x_{\sigma(k)}),$$

where $\langle x,y\rangle = \{x,y\} - (D(x)y - xD(y))$, $c_{\sigma,i}$ *from basic field.*

It is a joint work with Prof. Ivan Shestakov (USP, Brazil).

From actions of E-unitary inverse monoids to partial actions of groups

Mykola Khrypchenko*

Institute of Mathematics and Statistics, University of São Paulo,
nskhripchenko@gmail.com

It is well-known [1] that partial actions of a group G on a set X are in a one-to-one correspondence with actions of the Exel's inverse monoid $\mathcal{S}(G)$ of G on X. It is easily seen that $\mathcal{S}(G)$ is E-unitary, and moreover, its maximal group image is isomorphic to G. Conversely, having an action of an E-unitary inverse monoid S on a set X, we define a partial action of the maximal group image $\mathcal{G}(S)$ of S on X so that for $S = \mathcal{S}(G)$ this gives the correspondence mentioned above. It turns out that actions of "different" E-unitary inverse monoids may induce "the same" partial actions. More precisely, each epimorphism $\pi : S \to S'$ satisfying certain additional conditions induces an isomorphism $\mathcal{G}(S) \cong \mathcal{G}(S')$, and moreover, for any action of S on X the epimorphism π allows us to construct an action of S' on X, such that the corresponding partial actions of $\mathcal{G}(S)$ and $\mathcal{G}(S')$ on X are the same up to the isomorphism. For example, any action of a semilattice with identity on X induces the (global) action of the trivial group on X.

References

[1] R. Exel, Partial actions of groups and actions of inverse semigroups, *Proc. Amer. Math. Soc.*, **126 (12)** (1998), 3481–3494.

*Supported by FAPESP of Brazil: 2012/01554–7

Identities for matrix invariants of the symplectic group

Artem A. Lopatin

IMECC, Unicamp, Campinas. artem_lopatin@yahoo.com

Assume that the base field $\mathbb{F}$ is infinite of positive characteristic different from two. The identities for matrix invariants of the orthogonal group were described in recent papers [2] and [3]. In this talk we describe identities for matrix invariants of the symplectic group. We also discuss identities for the case of $SO(n)$-invariants, which were computed in [1].

References

[1] A.A. Lopatin, *Invariants of quivers under the action of classical groups*, J. Algebra **321** (2009), 1079–1106.

[2] A.A. Lopatin, *Relations between $O(n)$-invariants of several matrices*, Algebra Repr. Theory, **15** (2012), 855–882.

[3] A.A. Lopatin, *Free relations for matrix invariants in modular cases*, J. Pure Appl. Algebra, **216** (2012), 427–437.

Modular representations of profinite groups

John MacQuarrie

Universidade de Brasília. john.macquarrie@gmail.com

Profinite groups are categorical limits of finite groups (in the category of topological groups). From several perspectives they are natural objects: for instance, they are precisely the Galois groups of algebraic Galois field extensions. Meanwhile, the modular representation theory of a finite group G attempts to understand the category of (finitely generated) kG-modules, where k is a field of characteristic $p > 0$. If k is a finite field and G is a profinite group, there is a natural profinite analogue $k[[G]]$ of the group algebra of G, and we can thus ask about the category of finitely generated (profinite) $k[[G]]$-modules.
I will define everything above and give a quick idea of the sort of foundational results we can use to build a robust modular representation theory of profinite groups. I will talk about work in [2] from a few years ago, which has been continued in [1] and [3].

References

[1] MacQuarrie J.W., *Green correspondence for virtually pro-p groups.* J. Algebra, **323**, no.8, 2203–2208.

[2] MacQuarrie J.W., *Modular representations of profinite groups.* J. Pure Appl. Algebra, **215**, no.5, 753–763.

[3] MacQuarrie J.W. and Symonds, P., *Brauer Theory for profinite groups.* Submitted, available at `http://arxiv.org/abs/1301.5625`.

Characters of Weyl modules for hyper loop algebras

TIAGO MACEDO

IMECC - Unicamp, Campinas - SP - Brazil, 13083-859
tmacedo@ime.unicamp.br

This talk is concerned with the study of certain classes of modules for hyper algebras of current algebras. A hyper algebra is a Hopf algebra associated to a Lie algebra, similar to its universal enveloping algebra, and obtained from it by first choosing a certain integral form and then changing scalars. They provide a way to pass from a category of modules for a Lie algebra over an algebraically closed field of characteristic zero to its analog in positive characteristic. If the underlying Lie algebra is simply laced, we show that local Weyl modules are isomorphic to certain Demazure modules, extending to positive characteristic a result due to Fourier-Littelmann. More generally, we extend a result of Naoi by proving that local Weyl modules admit a Demazure flag, i.e., a filtration with factors isomorphic to Demazure modules. Using this, we prove a conjecture of Jakelić-Moura stating that the character of local Weyl modules for hyper loop algebras are independent of the (algebraically closed) ground field. This is a joint work with A. Moura.

Free field realizations of induced modules for affine Lie algebras

Renato Alessandro Martins and Iryna Kashuba

Instituto de Matemática e Estatística, Universidade de São Paulo, Brasil.
renatoam@ime.usp.br

Classical free field realizations for affine Lie algebras were introduced by M. Wakimoto [1] and B. Feigin and E. Frenkel [2] providing a construction of what is now called *Wakimoto modules*. Generically these modules are isomorphic to irreducible Verma modules.

The goal of this seminar is talk about the construction of a free field realization for new families of irreducible modules for affine Lie algebras recently obtained by V. Bekkert, G. Benkart, V. Futorny and I. Kashuba [3] following the ideas of [4] where realization of imaginary Verma modules was obtained and of [5] where such a realization was obtained for $\mathbb{J}$-imaginary Verma modules. These modules are constructed by parabolic induction from certain (admissible diagonal) irreducible $\mathbb{Z}$-graded module for the Heisenberg subalgebra. The induced modules - generalized loop modules - are irreducible when the central charge is nonzero. Admissible diagonal $\mathbb{Z}$-graded irreducible modules of nonzero level for an arbitrary infinite-dimensional Heisenberg Lie algebra were classified in [3].

References

[1] M. Wakimoto, *Fock representations of the affine Lie algebra $A_1^{(1)}$*, Comm. Math. Phys. **104** (1986), no. 4, 605–609.

[2] B. Feigin, E. Frenkel, *A family of representations of affine Lie algebras*, Uspekhi Mat. Nauk, **43** (1988), 227-228.

[3] V. Bekkert, G. Benkart, V. Futorny, I. Kashuba, *New Irreducible modules for Heisenberg and Affine Lie algebras*, Journal of Algebra (Print), **373** (2013), 284-298.

[4] B. Cox, *Fock Space Realizations of Imaginary Verma modules*, Algebras and Representation Theory **8** (2005), 173-206.

[5] R. A. Martins, *Free field realizations of certain modules for affine Lie algebra* $\widehat{\mathfrak{sl}}_n$, Algebra and Discrete Mathematics, **12** (2011), 28-52.

On the Z_n-graded identities of block-triangular matrices

Thiago Castilho de Mello[1] and Lucio Centrone[2]

[1] *IME - USP.* tcmello@ime.usp.br
[2] *IMECC - Unicamp.* centrone@ime.unicamp.br

The algebra of $n \times n$ matrices over the field F, $M_n(F)$, has a natural Z_n-grading (see [4]). The graded identities of such algebra have been described by Di Vincenzo [1] (for n=2) and by Vasilovsky [4] (for any n), when F is a field of characteristic zero.
An important class of subalgebras of $M_n(F)$ are the algebras of block-triangular matrices. It is well known that such algebras satisfy the "factoring property" for its ideal of identities (see [3] section 1.9). Graded identities of such algebras have been studied by Di Vincenzo and La Scala in [2]. They showed that under some conditions on the grading, such algebras have the factoring property.
In this talk, we present some results on the graded identities of the block-triangular matrix algebra with the Di Vincenzo-Vasilovsky grading. It turns out that such algebra satisfies graded monomial identities and do not satisfy the "factoring property". In the case blocks of sizes $n-1$ and 1, we find a description of its graded monomial identities. We also show that the graded identities of such algebra follow from the Vasilovsky identities on $M_n(F)$ and from its monomial identities. As a consequence, and we exhibit a finite basis for its ideal of graded identities.

References

[1] O. M. Di Vincenzo, On the graded identities of $M_{1,1}(E)$, Isr. J. Math. 80 (1992), 323-335.

[2] O. M. Di Vincenzo, R. La Scala, Block-triangular matrix algebras and factorable ideals of graded polynomial identities, J. Algebra 279 (2004), 260-279.

[3] A. Giambruno, M. V. Zaicev, *Polynomial Identities and Asymptotic Methods*, Math. Surveys and Monog. 122 (2005), AMS.

[4] S. Y. Vasilovsky, Z_n-graded polynomial identities of the full matrix algebra of order n, Proc. Amer. Math. Soc. 127(12) (1999), 3517-3524.

Positive laws in finite groups admitting a dihedral group of automorphisms.

Emerson de Melo

Department of Mathematics, University of Brasilia, Brasilia-DF, Brazil.
E.F.Melo@mat.unb.br

Let a group A act by automorphisms on a group G. We denote by $C_G(A)$ the set $C_G(A) = \{x \in G \; ; \; x^a = x$ for any $a \in A\}$, the centralizer of A in G (the fixed-point group). Often properties of $C_G(A)$ have a strong influence over the structure of G. Recently, in [1] and [2] it was proved that if a dihedral group $D = \langle \alpha, \beta \rangle$ generated by two involutions α and β acts on a finite group G in such a manner that $C_G(\alpha\beta) = 1$, then some properties of G should be close to the corresponding properties of $C_G(\alpha)$ and $C_G(\beta)$. In particular, in [1] it was shown that if $C_G(\alpha)$ and $C_G(\beta)$ satisfy a positive law of degree k, then G satisfies a positive law of degree that is bounded solely in terms of k and $|D|$.

References

[1] De Melo E., *Positive laws in finite groups admitting a dihedral group of automorphisms.* J. G. Theory, (2013), DOI 10.1515/jgt-2013-0009.

[2] Shumyatsky P., *The dihedral group as a group of automorphisms.* J. Algebra **375** (2013), 1-12.

Complete arcs in the Desarguesian Plane $PG(2,q)$

Nicola Pace

ICMC - USP, Sao Carlos. nicolaonline@libero.it

Let $PG(2,q)$ be the projective plane over the field $GF(q)$. A *k–arc* in $PG(2,q)$ is a set of k points, no three of which are collinear. A k-arc in $PG(2,q)$ is called *complete* if it is not contained in a $(k+1)$–arc in $PG(2,q)$. A k-arc corresponds to a $[k,3,k-2]$ maximum distance separable code (MDS code); a complete arc corresponds to a code that cannot be extended. Then, coding theory motivates us to the study of complete k–arc in $PG(2,q)$.
A typical approach for finding infinite families of complete k–arcs is considering orbits of a large projectivity group. In this talk, we present some constructions of k–arcs and discuss the problem of determining the size of the smallest complete k–arc in $PG(2,q)$.

Generic $\mathfrak{gl}(n)$-modules

Luis Enrique Ramírez

IME-USP, Rua do Matão 1010. Cidade Universitaria, São Paulo, SP.
luiser@ime.usp.br

In [2] I. Gelfand and M. Tsetlin construct bases and explicit formulas for the action of $\mathfrak{gl}(n)$ for every irreducible finite dimensional $\mathfrak{gl}(n)$-module. The bases are given by combinatoric objects called Gelfand-Tsetlin tableaux. In [1] the authors use the same kind of bases to construct some infinite dimensional $\mathfrak{gl}(n)$-modules called *generic* modules.

Generic modules are not irreducible in general. In this talk we will discuss those constructions and give the description of generic irreducible modules for $n = 3$.

References

[1] Y. Drozd,S. Ovsienko, V. Futorny, *Harish-Chandra subalgebras and Gelfand-Zetlin modules.* Math. and Phys. Sci., **424** (1994), 72–89.

[2] I. Gelfand, M. Tsetlin, *Finite-dimensional representations of the group of unimodular matrices.* Doklady Akad. Nauk SSSR (N.s.), **71** (1950), 825–828.

[3] V. Mazorchuk, *Tableaux realization of generalized Verma modules.* Can. J. Math., **50** (1998), 816–828.

Division rings of fractions for crossed products of a division ring by the universal enveloping algebra of a free Lie algebra.

JAVIER SÁNCHEZ

Departamento de Matemática, Instituto de Matemática e Estatística, Universidade de São Paulo, São Paulo, SP
jsanchez@ime.usp.br

Let K be a division ring and G a free group. Consider a crossed product KG of K by G. J. Lewin proved that the division subring generated by KG inside the Malcev-Neumann series ring $K((G))$ is the universal division ring of fractions of KG [3].
As part of a joint work with Dolors Herbera, we prove an analogous result for the crossed product $KU(L)$ of a field K by the universal enveloping algebra $U(L)$ of a free Lie algebra L [2].
More precisely, let K be a division ring, L be a free Lie algebra and $U(L)$ be its universal enveloping algebra. P. Cohn (and later A. Lichtman), showed that $KU(L)$ can be embedded in a division ring, denoted by $\mathfrak{D}(L)$ [1]. Consider also the embedding $KU(L) \hookrightarrow K((L))$, where $K((L))$ is a certain ring of series constructed by A. Lichtman [4]. We show that the universal division ring of fractions of $KU(L)$, the division ring $\mathfrak{D}(L)$ and the division ring generated by $KU(L)$ inside $K((L))$ are isomorphic as division rings of fractions of $KU(L)$. This result was proved by A. Lichtman in the case that $KU(L)$ is $U(L)$, the universal enveloping algebra of L [4].

References

[1] Cohn P., *Skew Fields.* Encyclopedia of Mathematics and its Applications, vol. 57 Cambridge University Press, 1995.

[2] Herbera D., Sánchez J. *The inversion height of the free field is infinite.* arXiv:1303.5287.

[3] Lewin J., *Fields of fractions for group algebras of free groups.* Trans. Amer. Math. Soc., **192** (1974), 339–346.

[4] Lichtman A., *On universal fields of fractions for free algebras.* J. Algebra, **231** (2000), no.2, 652–676.

Singular locus of torsion-free instanton sheaves of rank 2 on $\mathbb{P}^3$

MICHAEL SANTOS GONZALES GARGATE

Universidade Estadual de Campinas, IMEEC
michael_gg84@hotmail.com

Consider to monad

$$\mathcal{O}_{\mathbb{P}^3}(-1)^c \xrightarrow{\alpha} \mathcal{O}_{\mathbb{P}^3}^{r+2c} \xrightarrow{\beta} \mathcal{O}_{\mathbb{P}^3}(1)^c$$

with α injective and β surjective. Let $E = \frac{ker\,\beta}{Im\,\alpha}$ be to cohomology of the monad with $rank(E) = r$ and charge c.

In this poster we present the study of singular loci $Sing(E)$ of rank 2 torsion-free instanton sheaves E on $\mathbb{P}^3$. Recall

$$\begin{aligned} Sing(E) &= \{x \in \mathbb{P}^3 \,/\, E_x \textit{ is not free over } \mathcal{O}_x\} \\ &= \{x \in \mathbb{P}^3 \,/\, \alpha(x) \textit{ is not injective}\} \end{aligned}$$

We show that:

1. $Sing(E)$ has pure dimension 1.
2. E^{**} is a locally free instanton sheaf.

Both are false if $r \geq 3$.

We will make a construction based in the definition of degeneration data for instanton showing that this singular loci in general case is not connected.

References

[1] Jardim, M., *Instanton Sheaves on Complex Projective Spaces*, Collect. Math. 57, 2006.

[2] Hartshorne, R., *Algebraic Geometry*, Springer-Verlag New York, 1977.

[3] Hartshorne, R., *Stable Reflexive Sheaves* , Math. Ann. 254, 121-176, 1980.

[4] Okonek, C., Schneider M., Spindler, H., *Vector Bundles on Complex Projective Spaces,* Birkhäuser Verlag 1988.

[5] Kobayashi, S., *Differential Geometry of Complex Vector Bundles*, Princeton University, 1987.

[6] Roggero, M., *On the rank 2 reflexive sheaves and the Subcanonical Curves in* $\mathbb{P}^3$, Comm. Alg. 16(9), 1988.

Free diassociative loops of exponent 2

ALEXANDER GRISHKOV[1], MARINA RASSKAZOVA[2], AND IZABELLA STUHL[3]

[1]*Institute of Mathematics and Statistics, Sao Paulo, Rua do Matao, 1010*
grishkov@ime.usp.br
[2]*Omsk Institute of Consumer Service Technology, Omsk, Str. Pevzova 13*
marras123@gmail.com
[3]*Institute of Mathematics and Statistics, Sao Paulo, Rua do Matao, 1010*
izabella@ime.usp.br

Diassociative loops of exponent 2 form a variety that is coincides with the variety of all Steiner loops, which are in one-to-one correspondence with Steiner triple systems (see in [1]).

We can operate with free objects on this variety and use the term free Steiner triple system for the combinatorial object corresponding to the free Steiner loop.

We give a construction of free diassociative loops, determine their multiplication group, which is a useful knowledge for loops (see [3], Section 1.2) and focus our attention on automorphisms. The automorphism group of a Steiner triple system coincides with the automorphism group of the associated Steiner loop. In [2] is proved that any finite group is the automorphism group of a Steiner triple system.

The main results are that all automorphisms of the free diassociative loops of exponent 2 are tame. Furthermore, the group of automorphisms of the free Steiner triple system cannot be finitely generated when the loop is generated by more than 3 elements. Finally, in the case of 3-generated free diassociative loop of exponent 2 we specify the (triples of) generators of the automorphism group.

References

[1] Ganter B., Pfüller U., *A remark on commutative di-associative loops.* Algebra Universalis, **21**, (1985), 310–311.

[2] Mendelsohn E., *On the Groups of Automorphisms of Steiner Triple and Quadruple Systems.* J. of Combinatorial Theory, **25** Ser.A, (1978), 97–104.

[3] Nagy P. T., Strambach K., *Loops in Group Theory and Lie Theory, Expositions in Mathematics 35.* Walter de Gruyter, Berlin–New York (2002).

Automorphic equivalence of many-sorted algebras

Arkady Tsurkov

Institute of Mathematics and Statistics, Sao Paulo, Rua do Matao, 1010
arkady.tsurkov@gmail.com

Universal algebras H_1, H_2 of the variety Θ are geometrically equivalent if they have same structure of the algebraic closed sets. Automorphic equivalence of algebras is a generalization of this notion. We can say that universal algebras H_1, H_2 of the variety Θ are geometrically equivalent if the structures of the algebraic closed sets of these algebras coincides up to changing of coordinates defined by some automorphism of the category Θ^0. Θ^0 is a category of the free finitely generated algebras of the variety Θ. The quotient group $\mathfrak{A}/\mathfrak{Y}$ determines the difference between geometric and automorphic equivalence of algebras of the variety Θ, where $\mathfrak{A}$ is a group of the all automorphisms of the category Θ^0, $\mathfrak{Y}$ is a group of the all inner automorphisms of this category.
The method of the verbal operations was worked out in: B. Plotkin, G. Zhitomirski, *On automorphisms of categories of free algebras of some varieties*, 2006 - for the calculation of the group $\mathfrak{A}/\mathfrak{Y}$. By this method the automorphic equivalence was reduced to the geometric equivalence in: A. Tsurkov, *Automorphic equivalence of algebras*, 2007. All these results were true for the one-sorted algebras: groups, semigroups, linear algebras...
Now we reprove these results for the many-sorted algebras: representations of groups, actions of semigroups over sets and so one.

Group identities on symmetric units in regard to oriented group involutions

Alexander Holguín Villa[1] and César Polcino Milies[2]

[1] *Instituto de Matemática e Estatística, IME-USP, R. do Matão, 1010 Cidade Universitária - São Paulo - SP, Brasil*
aholguin@ime.usp.br
[2] *Instituto de Matemática e Estatística, IME-USP, R. do Matão, 1010 Cidade Universitária - São Paulo - SP, Brasil*
[2] *Centro de Matemática, Computação e Cognição, CMCC-UFABC, R. Santa Adélia 166, Bairro Bangu - Santo André - SP, Brasil*
polcino@ime.usp.br; polcino@ufabc.edu.br

Some time ago, Brian Hartley made the following conjecture: Let G be a torsion group and $\mathbb{F}$ a field. If $\mathcal{U}(\mathbb{F}G)$ satisfies a group identity, then $\mathbb{F}G$ satisfies a polynomial identity, [3].
Given an involution $*$ in the group G extended $\mathbb{F}$-linearly to a ring involution $* : \mathbb{F}G \to \mathbb{F}G$, it is possible to show that if the set of symmetric elements in the unit group $\mathcal{U}(\mathbb{F}G)$ satisfies a group identity then either a group identity holds in $\mathcal{U}(\mathbb{F}G)$ or then $\mathbb{F}G$ satisfies a polynomial identity, see [1, 2, 3].
Let $\mathbb{F}G$ denote the group algebra of a group G over the field $\mathbb{F}$ with $char(\mathbb{F}) \neq 2$, and let $\circledast : \mathbb{F}G \to \mathbb{F}G$ denote the involution defined by $\alpha = \Sigma\alpha_g g \mapsto \alpha^{\circledast} = \Sigma\alpha_g \sigma(g) g^*$, where $\sigma : G \to \{\pm 1\}$ is a group homomorphism (called an orientation) and $*$ is an involution of G. We prove, under some assumptions, that if the set of $\circledast$-symmetric units of $\mathbb{F}G$ satisfies a group identity then $\mathbb{F}G$ satisfy a polynomial identity, i.e., we obtain an affirmative answer to Hartley's Conjecture in this setting. Moreover, in case when the prime radical $\eta(\mathbb{F}G)$ of $\mathbb{F}G$ is nilpotent we characterize the groups for which the symmetric units $\mathcal{U}^+(\mathbb{F}G)$ do satisfy a group identity.

References

[1] Dooms A. and Ruiz M., *Symmetric units satisfying a group identity.* J. Algebra, **308** (2007):742-750.

[2] Giambruno A., Polcino Milies C. and Sehgal S. K., *Group identities on symmetric units.* J. Algebra, **322** (2009):2801-2815.

[3] Giambruno A., Sehgal S. K. and Valenti A., *Symmetric units and group identities* Manuscripta Math., **96** (1998):443-461.

Tensor product decomposition of $\widehat{\mathfrak{sl}}(n)$-modules

Evan Wilson

IME-USP, São Paulo, wilsonea@ime.usp.br

In this talk, we discuss recent work with Kailash Misra on decomposing the tensor product of level one $\widehat{\mathfrak{sl}}(n)$-modules using a realization of such modules in terms of extended Young diagrams. We also used character formulas to compute the outer multiplicities of irreducible submodules, leading to some generating function identities.