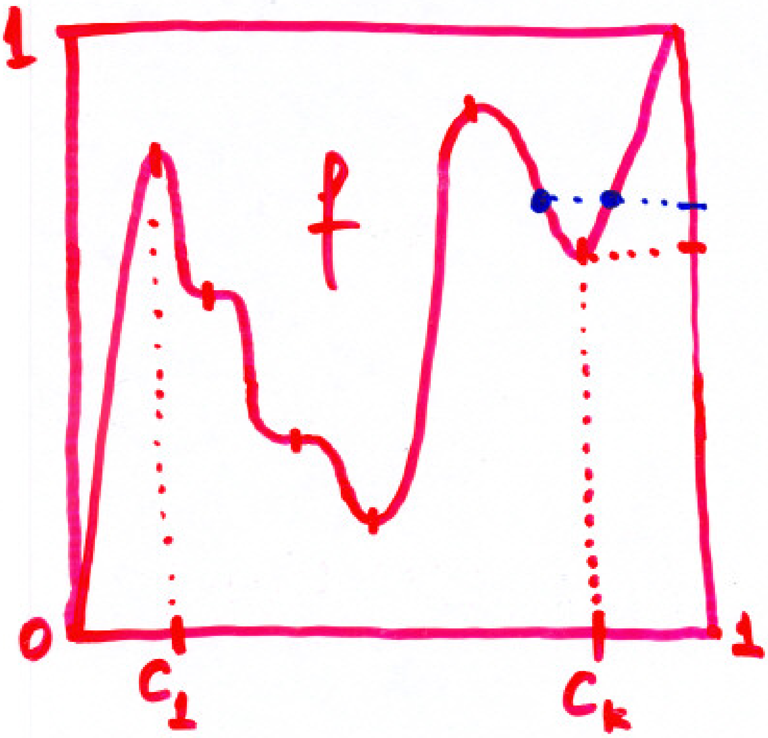
NON-EXISTENCE OF WANDERING INTERVALS FOR MULTIMODAL MAPS

JOINT WITH
VAN STRIEN

MULTIMODAL MAPS



- $f : [0,1] \circlearrowleft$ C^2

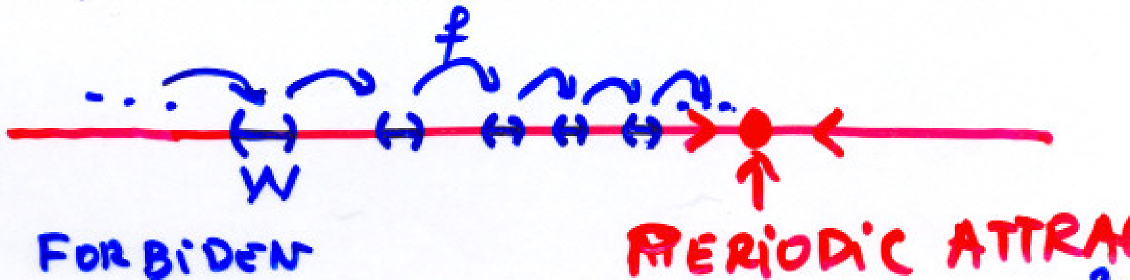
NON-FLAT CRITICAL POINTS $c_1, \dots, c_k$

- REAL POLYNOMIALS

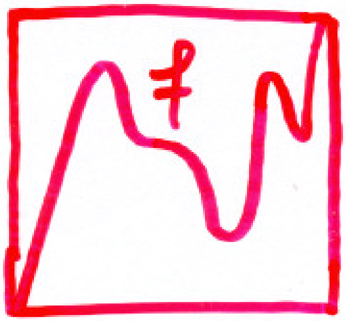
WANDERING INTERVAL W

- $f^m(W) \cap f^n(W) = \emptyset \,;\; m > n \geq 0$

- NOT ASSYMPTOTIC TO A PERIODIC ORBIT

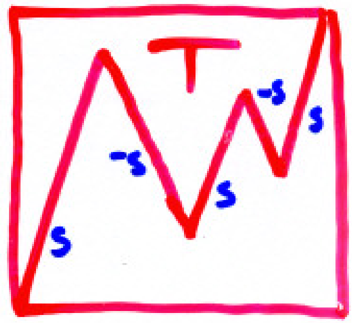


RELEVANCE



$h \circ f = T \circ h$

SEMI - CONJ.



WELL UNDERSTOOD

$s = \exp(h_{top}(f))$

- $h^{-1}(x)$ ONE POINT VERY GOOD
- $h^{-1}(x)$ COMP. OF THE BASIN OF A PERIODIC ATTRACTOR

  INEVITABLE
- $h^{-1}(x)$ PERIODIC INTERVAL

  TREATABLE
- $h^{-1}(x)$ WANDERING INTERVAL

  VERY BAD

THEOREM: IF $f : [0,1] \circlearrowleft C^2$ HAS FINITELY MANY CRITICAL POINTS ALL OF THEM NON-FLAT THEN f HAS NO WANDERING INTERVALS.

- GUCKENHEIMER (79): UNIMODAL + NEGATIVE SCHWARZIAN + NON-DEG. C. P.

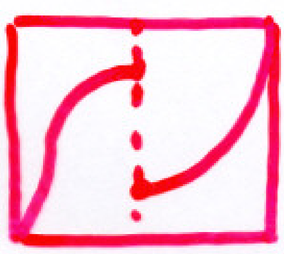
- DE MELO & VAN STRIEN (87): UNIMODAL + NON-FLAT CRITICAL P.

- BLOKH & LYUBICH ( ): MULTIMODAL + NON-FLAT C. P. + NO INFLECTION POINTS

- MARTENS & DE MELO & VAN STRIEN MENDES MULTIMODAL + NON-FLAT C. P.

HISTORY

- DENJOY (1932): DIFF. C^2 ON S^1 HAS NO WAND. INTERVAL
- SCHWARTZ ...
- HALL (81): $\exists$ C^∞ HOMEO. ON S^1 WHICH HAS A WANDERING INTERVAL.
- GUTIERREZ (97): $\exists$ A FINE INTERVAL EXCHANGE WHICH HAS A WANDERING INTERVAL
- DE MELO (87): $\exists$ C^∞ UNIMODAL MAPS WHICH HAS A WANDERING INTERVAL.
- SULLIVAN: HOLOMORPHIC FUNCTIONS ON $\overline{\mathbb{C}}$ HAS NO WAND. DOMAINS.
- LORENZ MAPS



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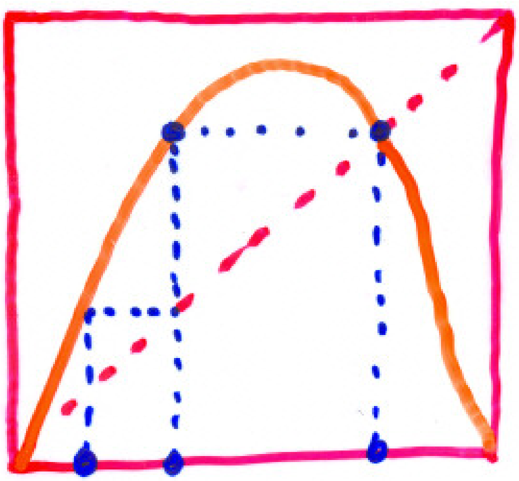
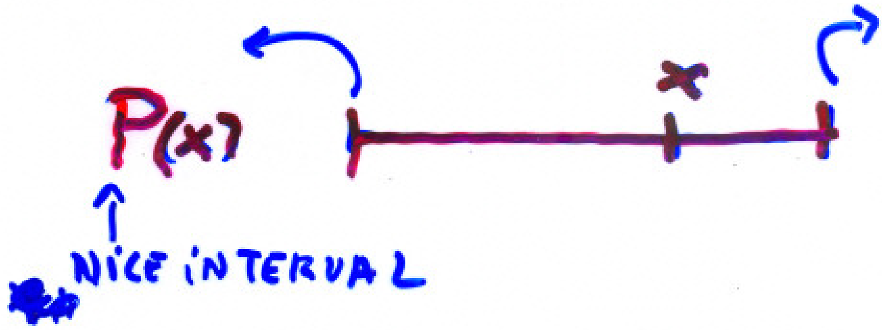
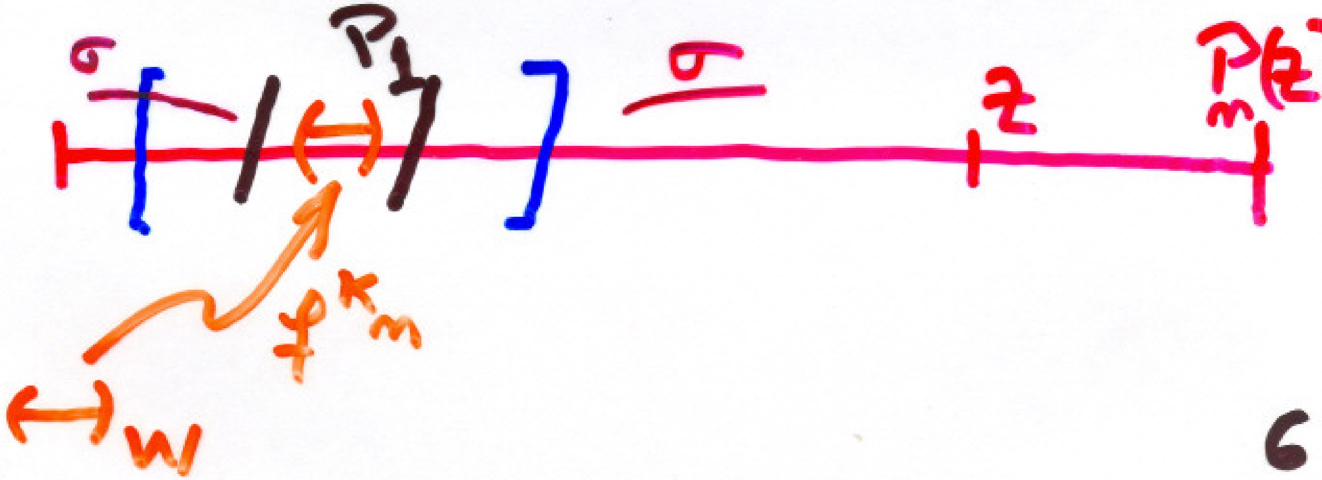
STRATEGY

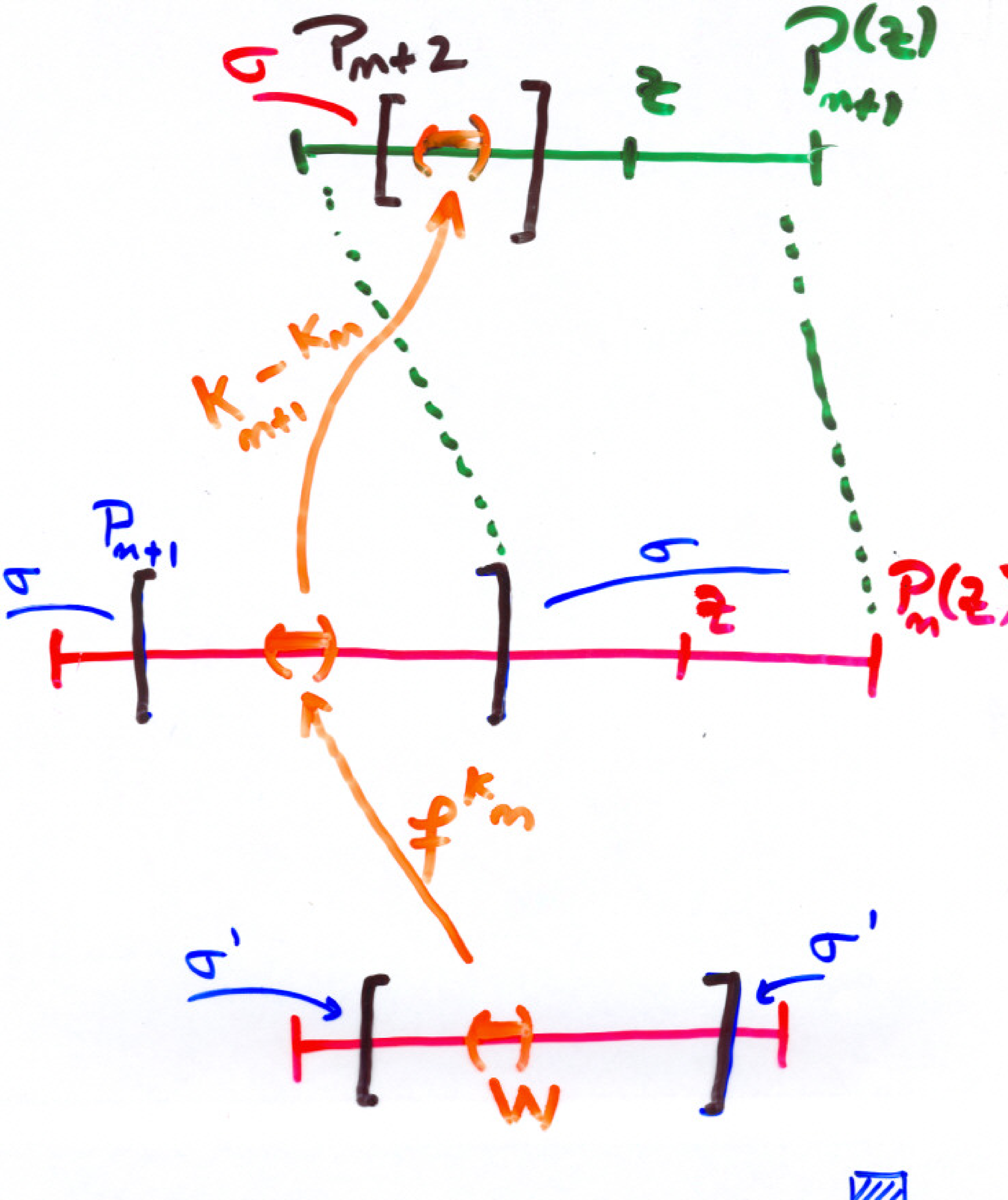
- **FINITE NICE PARTITION:** $f(\mathcal{P}) \subset \mathcal{P}$ AND $\mathcal{P}$ IS A FINITE SET.





MAIN LEMMA: $\exists \sigma > 0$ S.T. $\forall$ WAND. INT. W AND $z \in \omega(W)$ $\exists$ F.N.P. $\mathcal{P}_0 \subset \mathcal{P}_1 \subset \dots \subset \mathcal{P}_m \subset \dots$ S.T. $P_{m+1}(f^{k_m}(W))$ IS σ-WELL-INSIDE $P_m(z)$, $k_m \geq 0$ MINIMAL S.T. $f^{k_m}(W) \subset P_m(z)$.




σ
P_{n+2}
P^{(z)}_{n+1}
z
K_{n+1} - K_n
P_{n+1}
σ
σ
z
P_n(z)
f^{K_n}
σ'
σ'
W

TOOLS

• KOEBE: $J \subset I$ INTERVAL S.T.

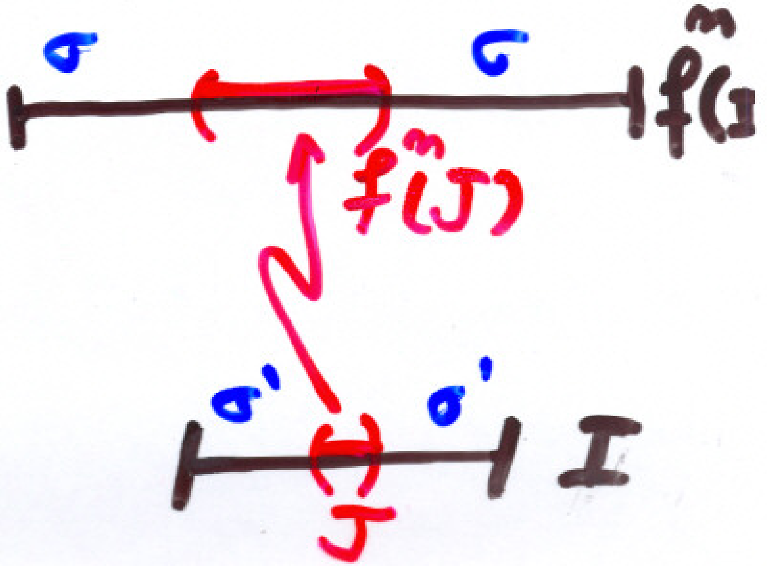
1) $f^n\big|_I$ IS A DIFF.

2) $f^n(J)$ IS σ-WELL-INSIDE $f^n(I)$.

KOEBE SPACE

3) $\sum_{j=0}^{n-1} |f^j(I)| \leq S$ (DISJOINTNES

THEN J IS σ'-WELL-INSIDE I.

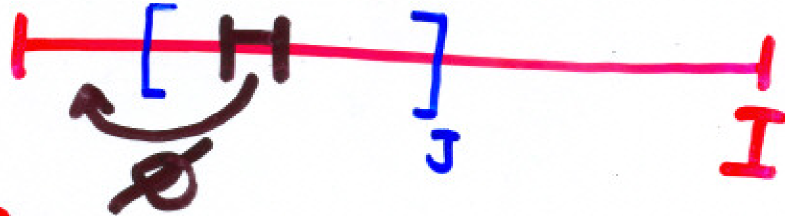


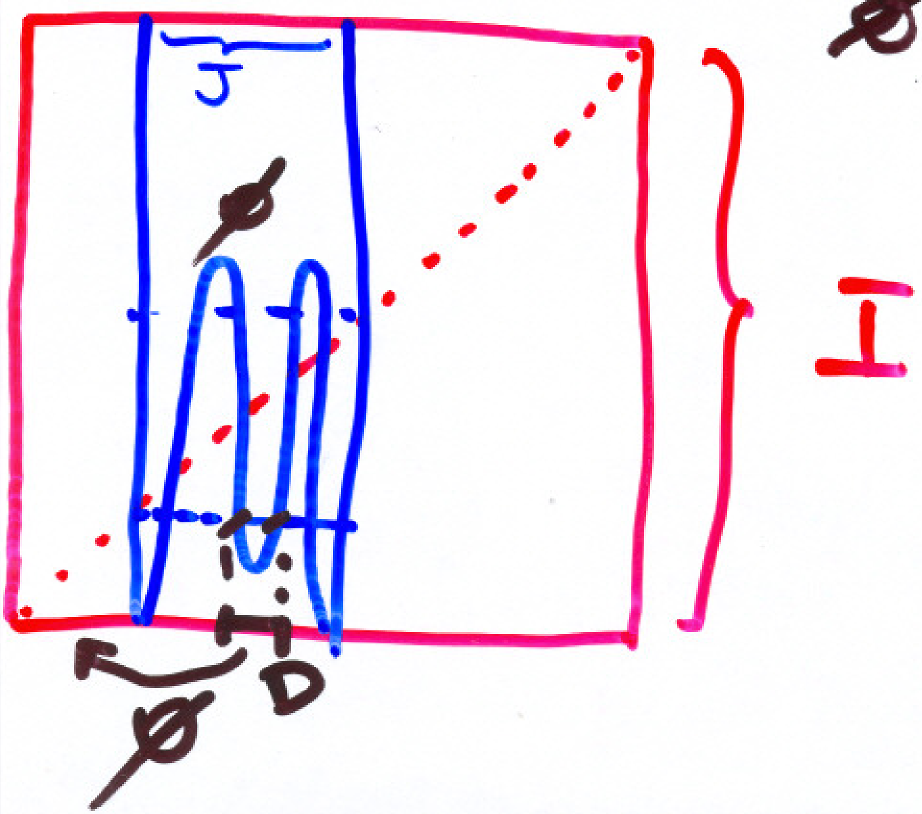
• GENERALIZATION: $f^n = \underbrace{f}_{\text{diff.}} \circ \underbrace{f}_{\text{crit.}} \circ \cdots \circ \underbrace{f^{n_2}}_{\text{diff.}} \circ \underbrace{f}_{\text{crit}}$

NON-FLATNESS

FUNDAMENTAL DOMAIN: $\phi : J \to I$ F.R

$D \subset J$ IS A FUND. DOM. IF D IS MAXIMAL WITH $D, \dots, \phi^{m-1}(D) \subset J$ AND $\phi^{m}(D) \subset I \setminus J$.

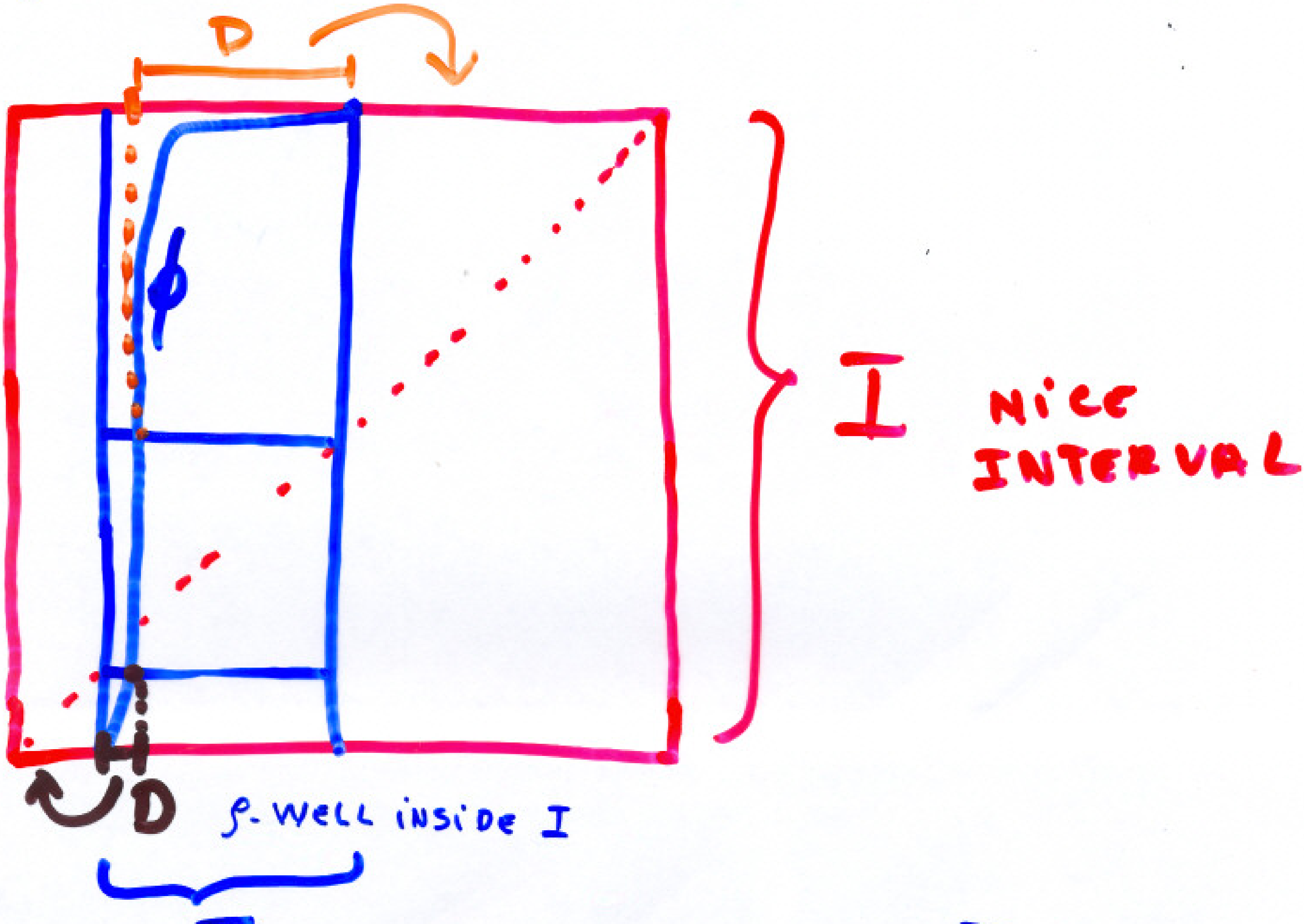




CASE $m = 1$

SMALLEST INTERVAL ARGUMENT

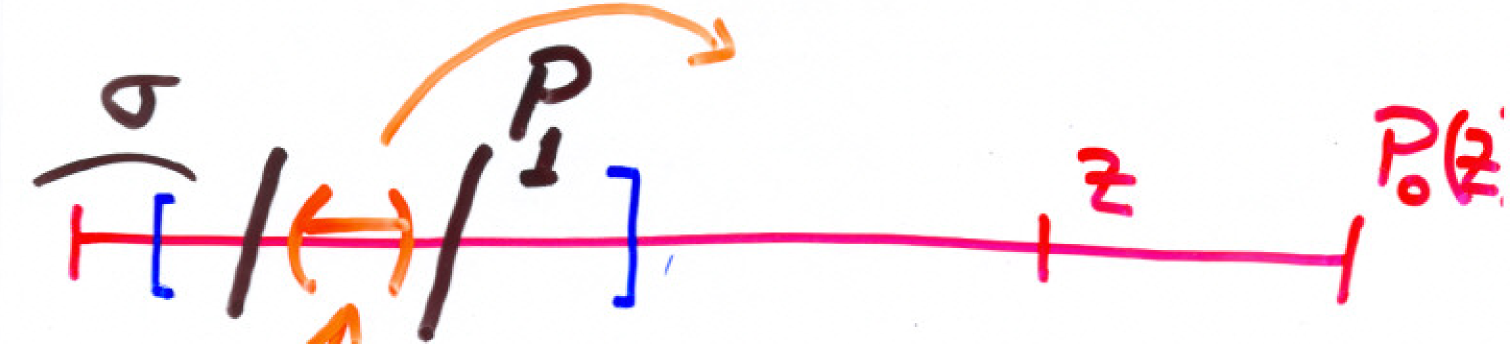
$\exists\, \rho > 0$ S.T. A FUND. DOM. $D \subset J$ IS ρ-WELL-INSIDE I OR $\phi(D)$ IS IN A ρ-BIG SIDE OF J.

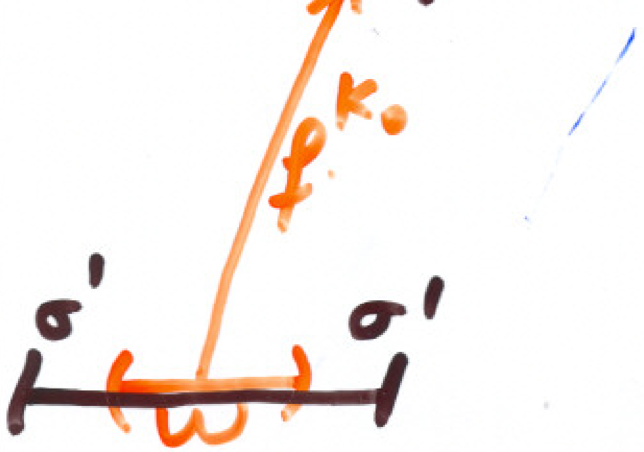


RETURN DOMAIN TO I

PROOF (LEMMA)

$$\mathcal{P}_0 = f^{-1}(\{\text{FIXED POINTS}\})$$





* MOSTRAR QUE EXISTE P_1 σ-WELL INSIDE $P_0(z)$ AND NICE

$$\underset{\text{NICE}}{P_1} \longrightarrow \mathcal{P}_1$$

$$P_1(z)$$

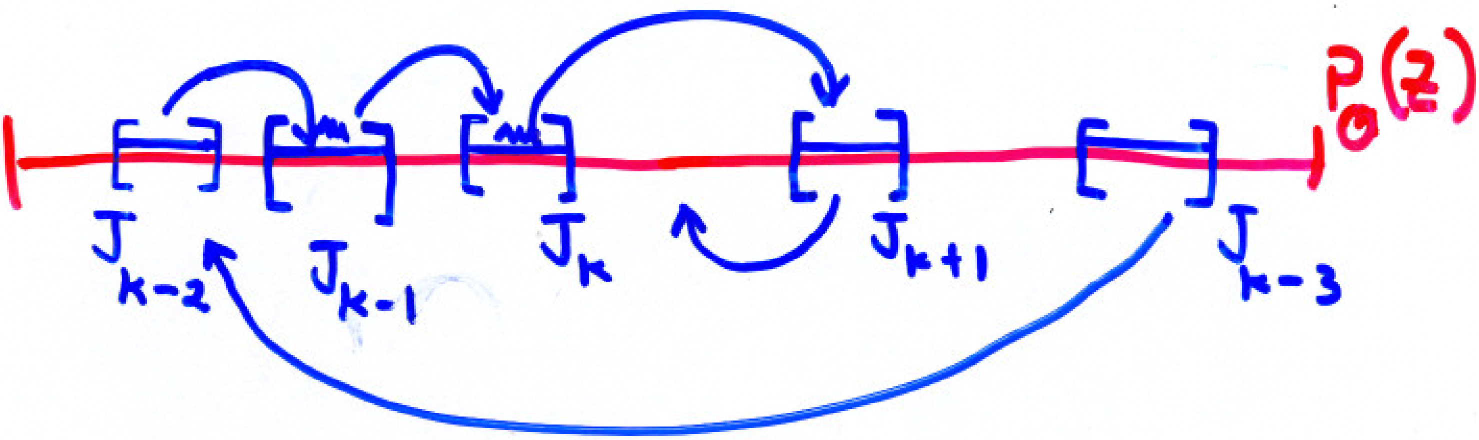
$$\vdots$$

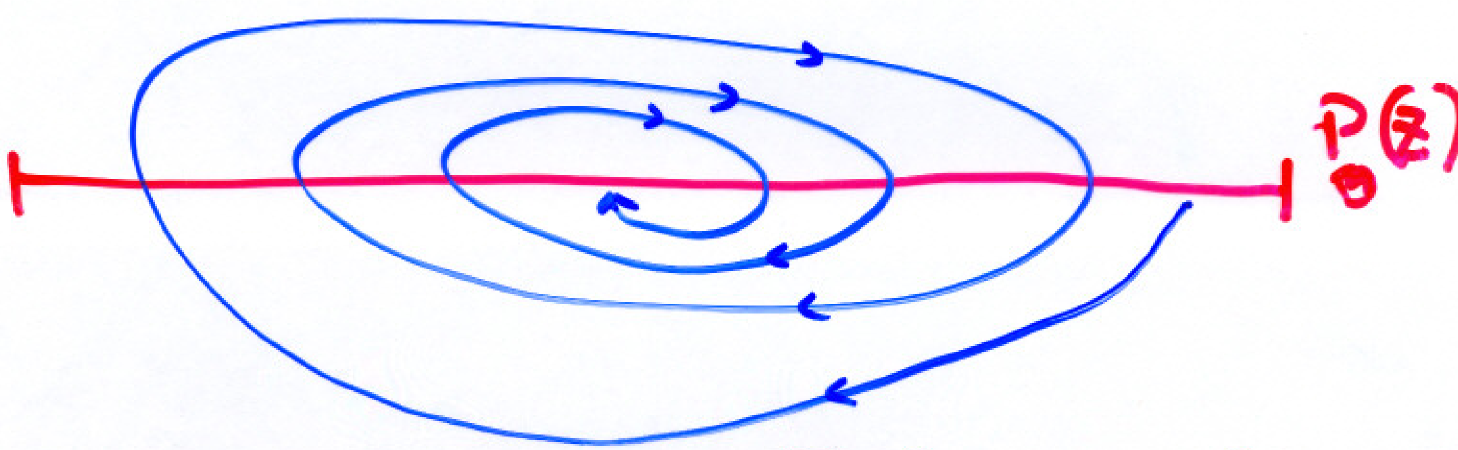
NEW IDEAS

POSITION OF 1º RETURN DOMAINS TO $P_0(z)$ VISITED BY W

SPIRAL: $J_1, J_2, \dots$ DOM. VISITED BY W SATISFY $(J_k, J_{k+1}) \cap \left(\bigcup_{i=1}^{k-1} J_i\right)^{\circ} = \emptyset$.

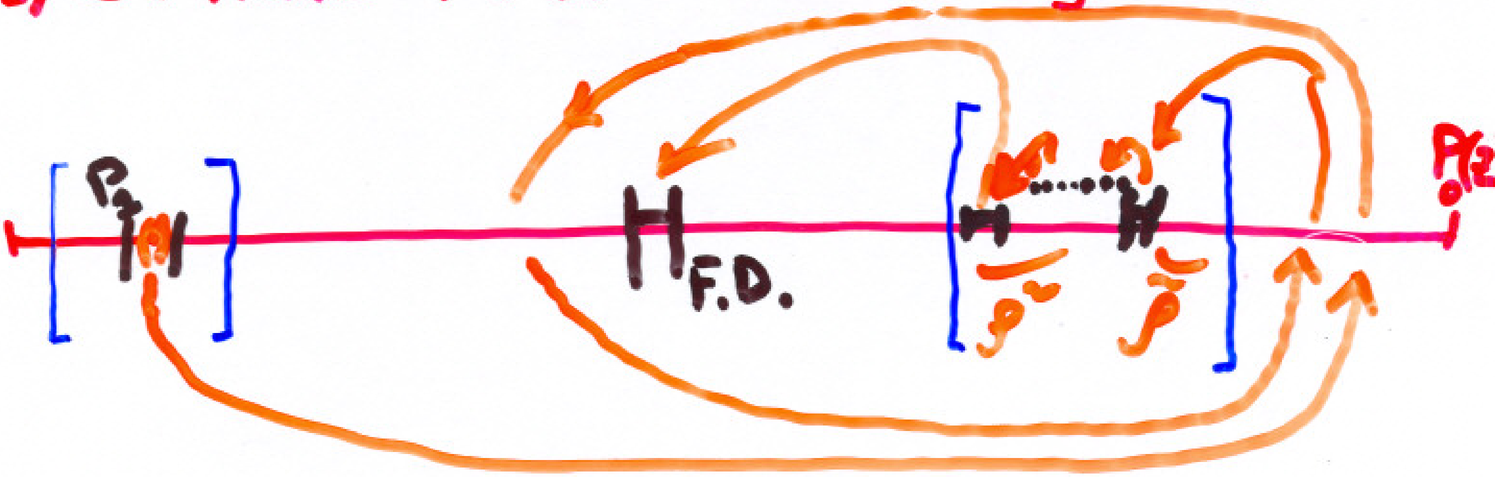
SPIRAL ⇒ PULL-BACK DISJOINT



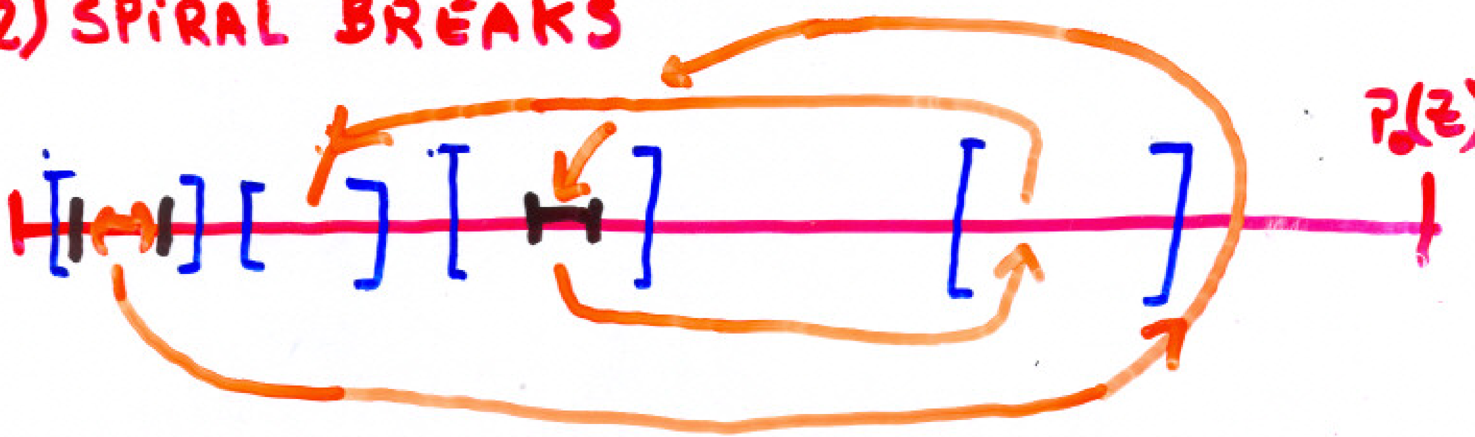


SPIRAL BREAKS ⇒ ∃ KOEBE SPACE

1) SPIRAL + FUND. DOMAIN ρ_WELL INSID



2) SPIRAL BREAKS



3) SPIRAL + A UNIQUE DOMAIN (REN)

4) SPIRAL + CYCLE BETWEEN 2 DOMAINS
(REDUCES)