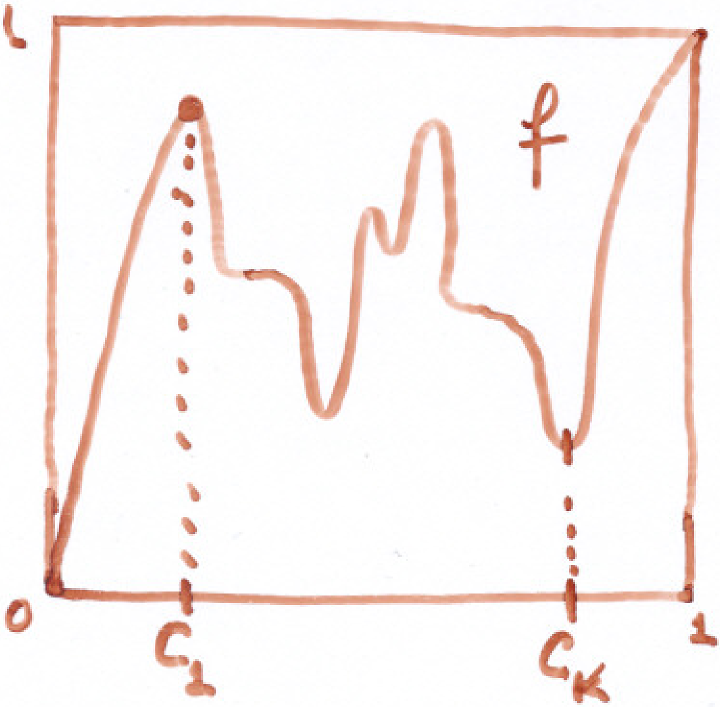
KOEBE PRINCIPLES AND NEGATIVE SCHWARZIAN FOR MULTIMODAL MAPS

CONTROL OF DISTORTION OF HIGH ITERATES

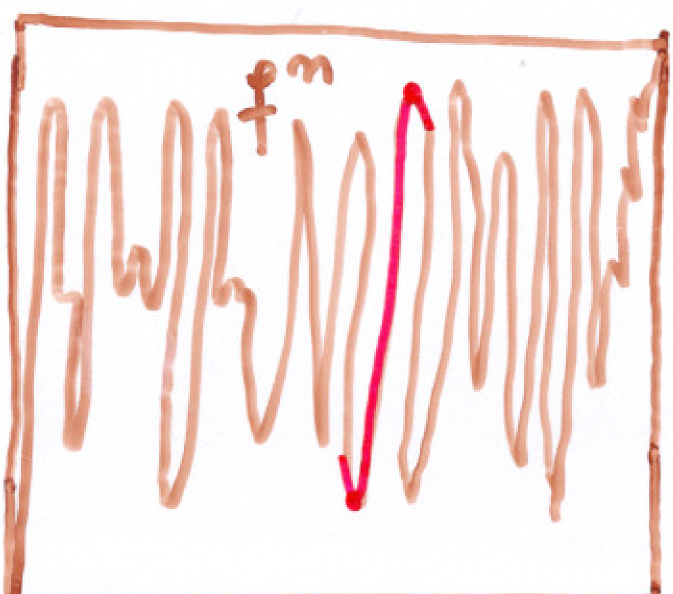
$$\frac{|Df^n(x)|}{|Df^n(y)|} \leq \;?$$

FOR ALL $n \geq 1$

MULTIMODAL MAPS

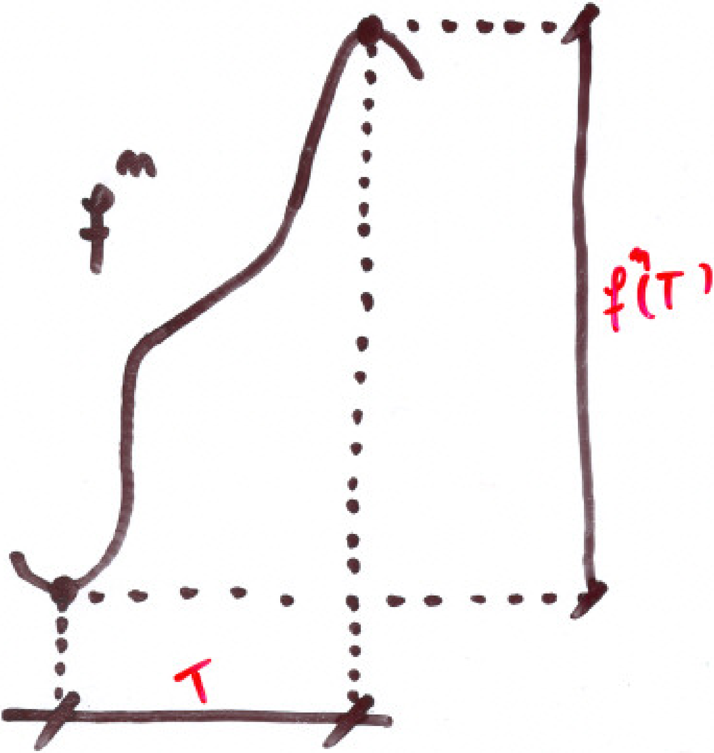


- $f : [0,1] \circlearrowleft$ C^3
  NON-FLAT CRITICAL POINTS $c_1, \dots, c_k$
- REAL POLYNOMIALS



A HIGH ITERATE

ONE BRANCH OF f^m



BIG DISTORTION IN THE MIDLE

SCHWARZIAN DERIVATIVE

$$Sf(x) = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2 \; ; \text{ if } f'(x) \neq 0$$

GOOD

- $S(f \circ g)(x) = Sf(g(x))\left(g'(x)\right)^2 + Sg(x)$

- $Sf(x) < 0 \Rightarrow Sf^n(x) < 0$; ALL $n \geq 1$.

- REAL POLYNOMIALS WITHOUT MULTIPLE ROOTS HAVE NEGATIVE SCHWARZIAN

- OPEN

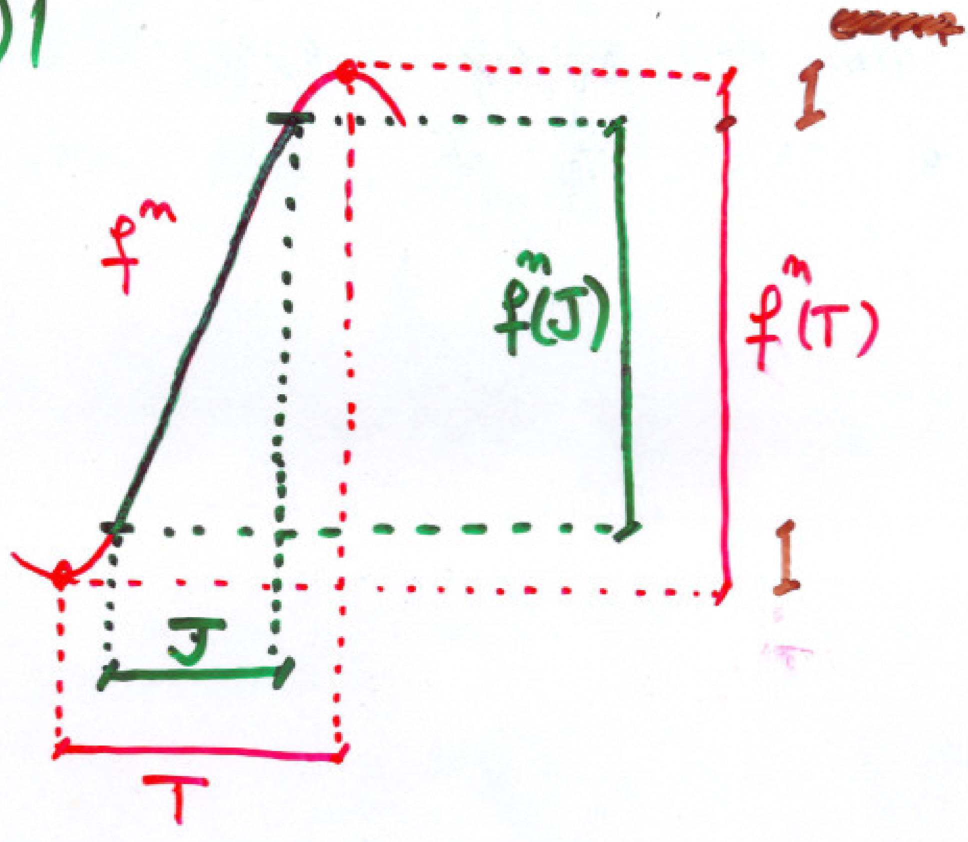
Koebe Principles

IF

- $f^n\big|_T$ is a diffeomorphism
- $f^n(J)$ is γ-well-inside $f^n(T)$
- $Sf < 0$

then for $x, y \in J$

$$\frac{|Df^n(x)|}{|Df^n(y)|} \leq K = K(\gamma)$$



KOEBE PRINCIPLES

IF

- $f^n\big|_T$ is a diffeomorphism
- $f^n(J)$ is γ-well-inside $f^n(T)$
- $Sf < 0$

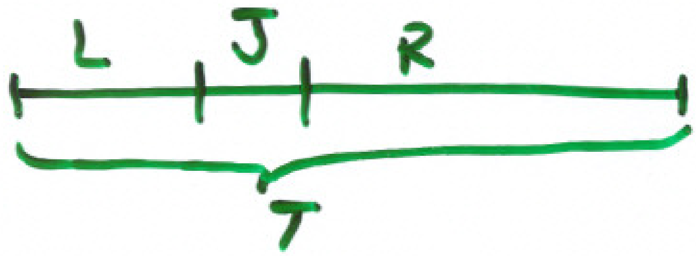
THEN FOR ALL $x, y \in J$

$$\frac{|Df^n(x)|}{|Df^n(y)|} \leq K = K(\gamma)$$

BAD

- MANY NICE MAPS DO NOT HAVE NEGATIVE SCHWARZIAN.
- NEGATIVE SCHWARZIAN IS NOT INVARIANT UNDER SMOOTH CHANGE OF VARIABLES.

CROSS RATIOS



$$A(T,J) = \frac{|T|.|J|}{|L\cup J|.|J\cup R|} \; ; \; B(T,J) = \frac{|T|.|J|}{|L|.|R|}$$

IF $f^n\big|_T$ IS A DIFFEOMORPHISM

$$A(f^n,T,J) = \frac{A(f^n(T), f^n(J))}{A(T,J)} \; ; \; B(f^n,T,J) = \cdots$$

- $Sf < 0 \Rightarrow A(f^n,T,J) > 1$ AND $B(f^n,T,J) > 1$

  EXPANSION OF CROSS RATIOS

- KOEBE PRINCIPLES ARE TRUE IF $Sf < 0$ IS REPLACED BY

  $A(f^n,T,J) \geq C_1 > 0$ OR $B(f^n,T,J) \geq C_1 > 0$

BOUNDS ON THE CONTRACTION OF CROOS RATIOS

$$A(f^n, T, J) \geq \mathrm{Exp}\left(-C\,\varepsilon \sum_{i=0}^{n-1} |f^i(T)|\right)$$

WHERE $\varepsilon = \max\{\, |f^i(T)| \,;\ i=0,\dots,n-1 \}$

- NEED TO BOUND $\sum_{i=0}^{n-1} |f^i(T)|$

IMPROVED KOEBE PRINCIPLES

(JOINT WITH S. VAN STRIEN)

IF

- $f^n\big|_T$ is a diffeomorphism
- $f^n(J)$ is τ-well-inside $f^n(T)$
- either (i) $\sum_{i=0}^{n-1} |f^i(J)| \leq S$ OR

(ii) $f^n(T) \cap B_0(f) = \emptyset$ AND $\mathrm{Dist}(f^i(T), \mathrm{Par}) \geq \delta$

$i = 0, \ldots, n-1$

THEN FOR $x, y \in J$

$$\frac{|Df^n(x)|}{|Df^n(y)|} \leq K = K(\tau, \delta, S)$$

NEGATIVE SCHWARZIAN

(——, S. VAN STRIEN)

FOR EACH c_i NOT IN THE BASIN OF A PERIODIC ATTRACTOR THERE IS A NEIGHBOURHOOD $U_i \ni c_i$ SO THAT IF $f^n(x) \in U_i$ THEN $Sf^{n+1}(x) < 0$.

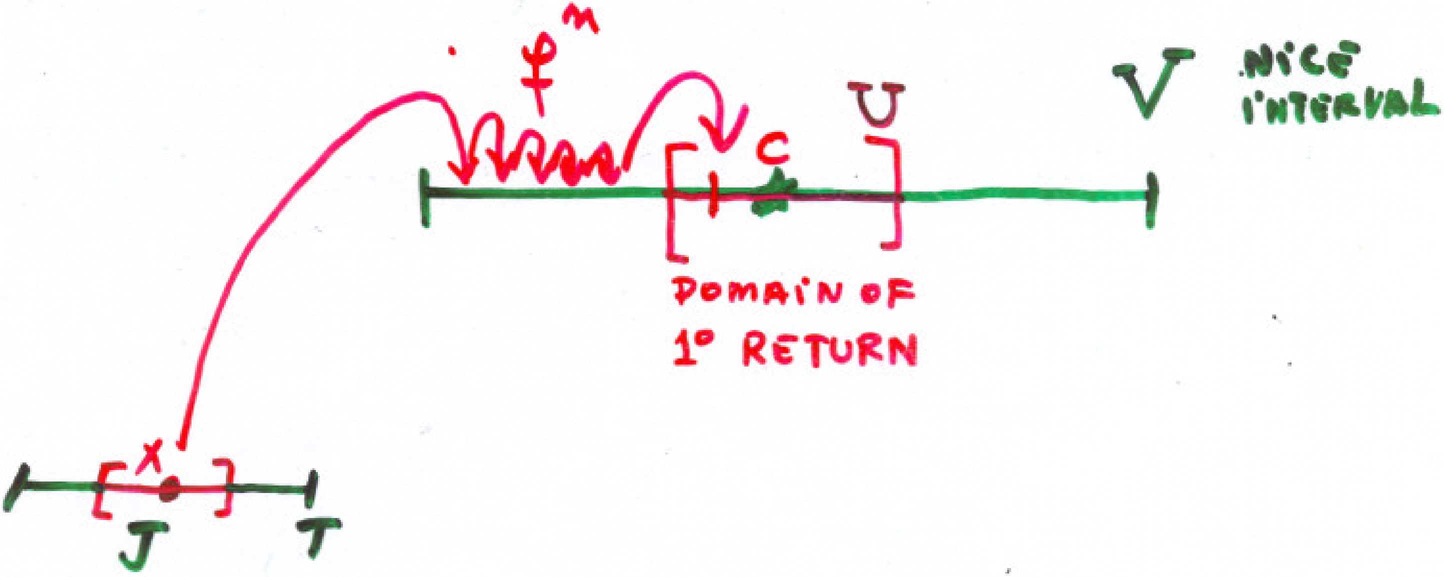
NEGATIVE SCHWARZIAN EVERYWHERE (GRACZYK - SANDS)

IF ALL PERIODIC POINTS OF f ARE HYPERBOLIC AND REPELLING THEN f IS DIFFERENTIABLY

APPLICATION: ERGODICITY OF UNIMODAL MAPS W.R. TO THE LEBESGUE MEASURE

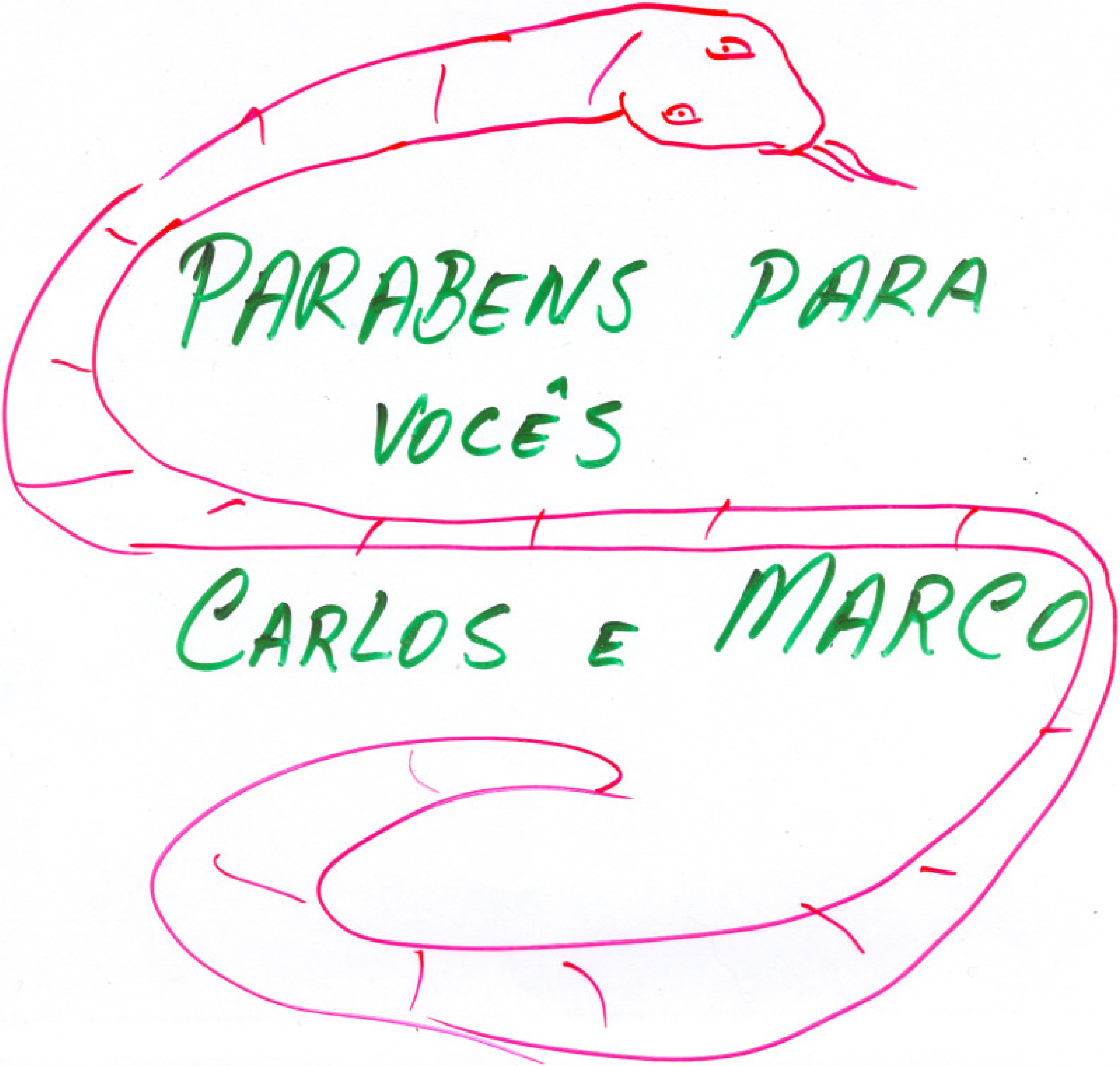
WITHOUT PERIODIC ATTRACTOR

- IF $f^{-1}(X) = X$ THEN $|X| = 1$ OR $|X| = 0$



- U δ-WELL INSIDE $V \Rightarrow f^n|_J$ HAS B. D.
- THE ARE PAIRS $U \subset\subset_{\delta} V$ ARBIT. SMALL
- IF $|X| > 0 \Rightarrow \dfrac{|X \cap U|}{|U|} \longrightarrow 1 \Rightarrow$

$\Rightarrow |X^c| = 0.$


PARABENS PARA
VOCÊS
CARLOS E MARCO