INTRO À COMPUTAÇÃO QUÂNTICA – PARTE 3

Aula passada:

- Revisão Notação de Dirac e Algelin em $\mathbb{C}^n$
- Evolução de qubits e portas quânticas
- No-Cloning Theorem

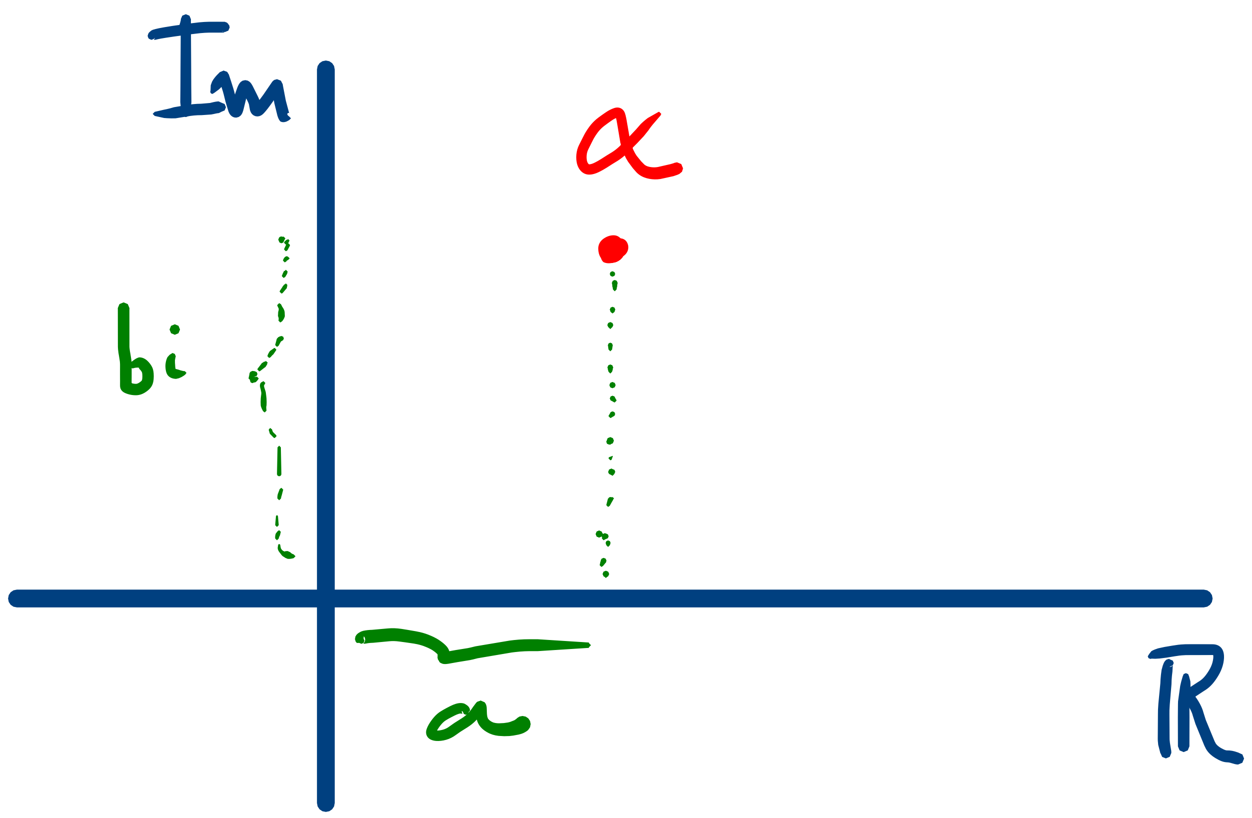
Hoje

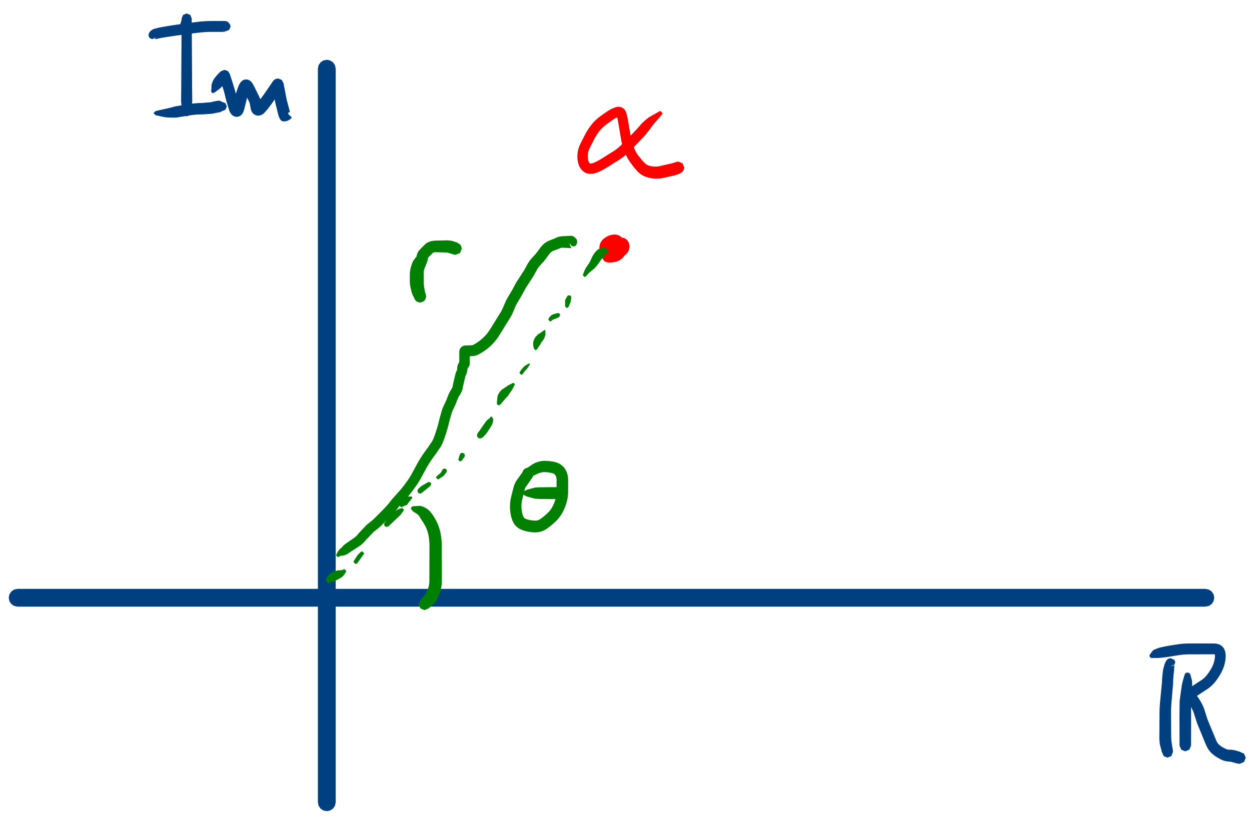
- Representação de 1 qubit – Esfera de Bloch
- Algoritmo de Deutsch
- Algoritmo de Deutsch-Josza

Um qubit genérico tem a forma

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \text{ , onde } \alpha, \beta \in \mathbb{C} \text{ e } |\alpha|^2 + |\beta|^2 = 1$$

Podemos representar números complexos de duas formas





$$\alpha = a + bi$$

$$\alpha = r(\cos\theta + i \operatorname{sen}\theta)$$
$$= re^{i\theta}$$

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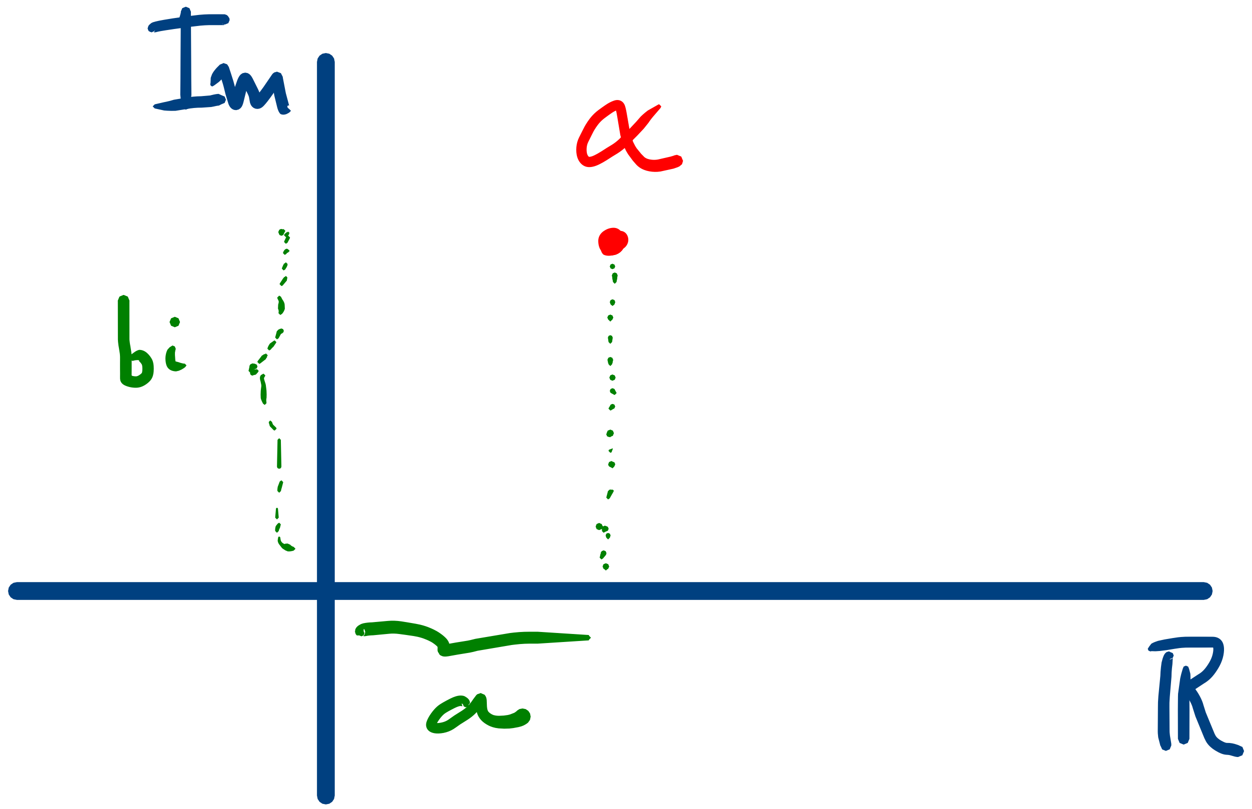

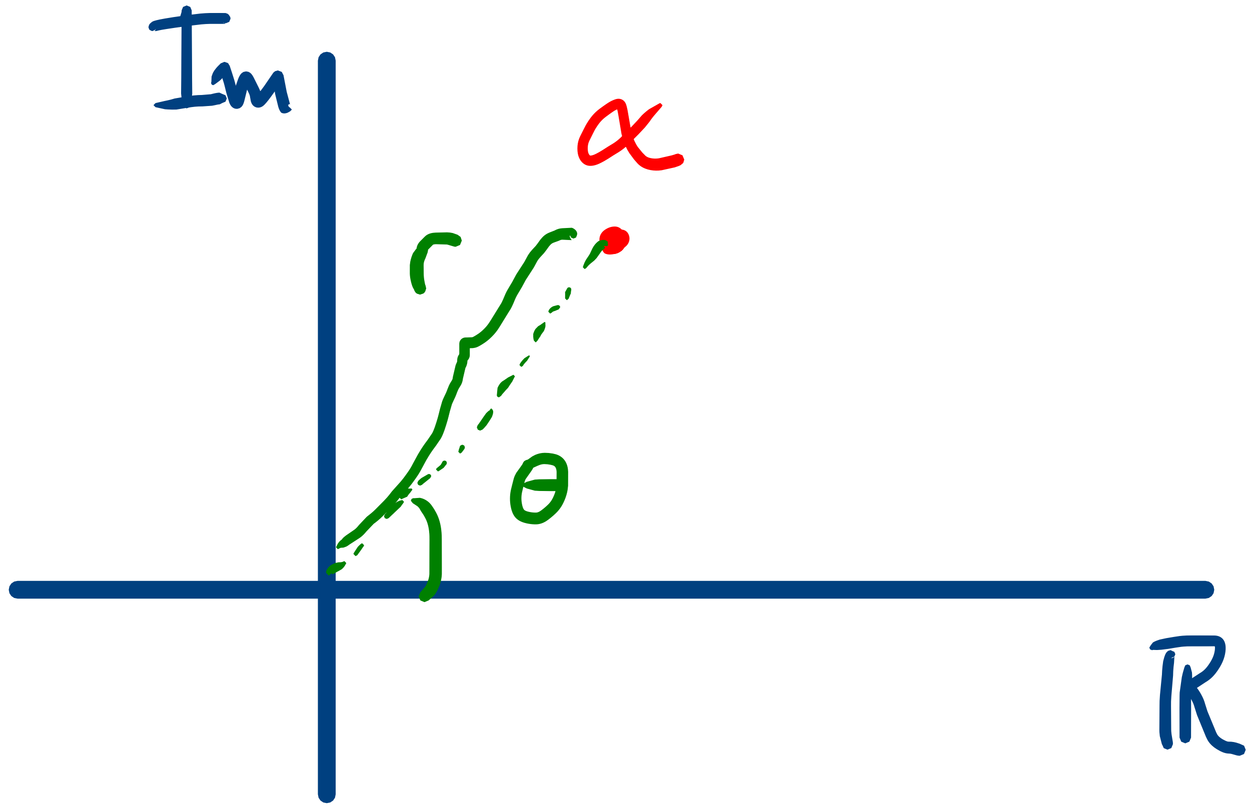

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PROP. Se 2 qubits diferem por uma fase global $\gamma \in \mathbb{C}$ com $|\gamma|=1$, então não há experimento que distingua $|\psi\rangle$ de $\gamma|\psi\rangle$

O que é distinguir 2 qubits?

Se $|\psi_1\rangle = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ e $|\psi_2\rangle = \dfrac{|0\rangle - |1\rangle}{\sqrt{2}}$

Se medirmos $|\psi_1\rangle$ e $|\psi_2\rangle$

$$\begin{tikzcd} & {|\psi_1\rangle} \arrow[dl, "50\%"'] \arrow[dr, "50\%"] & \\ {|0\rangle} & & {|1\rangle} \end{tikzcd} \qquad \begin{tikzcd} & {|\psi_2\rangle} \arrow[dl, "50\%"'] \arrow[dr, "50\%"] & \\ {|0\rangle} & & {|1\rangle} \end{tikzcd}$$

Porém $|\psi_1\rangle$ e $|\psi_2\rangle$ são facilmente distinguíveis após aplicar $H \Rightarrow H|\psi_1\rangle = |0\rangle$, $H|\psi_2\rangle = |1\rangle$

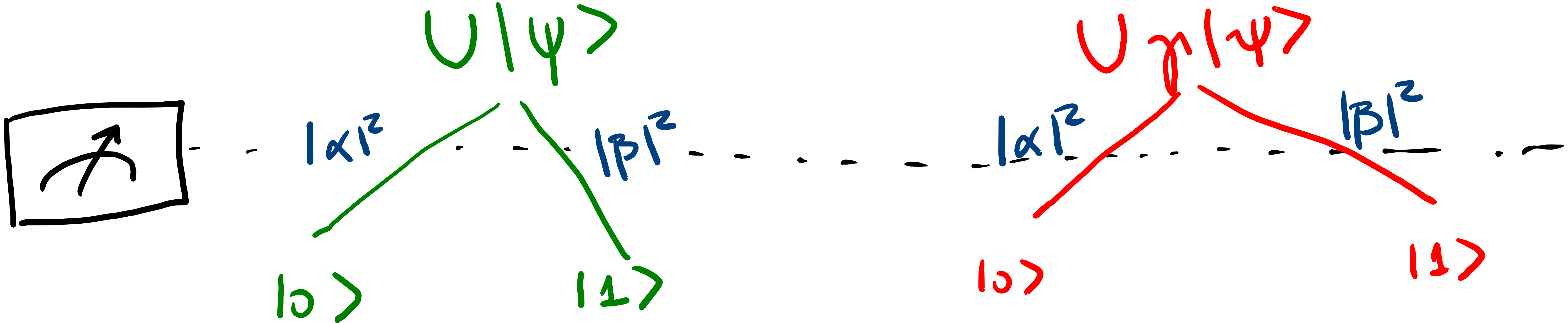
Prop. Se 2 qubits diferem por uma fase global $\gamma \in \mathbb{C}$ com $|\gamma|=1$, então não há experimento que distingua $|\psi\rangle$ de $\gamma|\psi\rangle$

Dem: Seja U uma mudança de base qualquer (matriz unitária)

$$U|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$U\gamma|\psi\rangle = \gamma U|\psi\rangle = \gamma\alpha|0\rangle + \gamma\beta|1\rangle$$

$$\Rightarrow \begin{cases} |\gamma\alpha| = |\gamma||\alpha| = 1|\alpha| = |\alpha| \\ |\gamma\beta| = |\gamma||\beta| = 1|\beta| = |\beta| \end{cases}$$



∎

A esfera de Bloch representa um qubit usando não 4, mas apenas 2 reais.

Seja $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, onde $\begin{cases} \alpha = r_1 e^{i\varphi_1} \\ \beta = r_2 e^{i\varphi_2} \end{cases}$

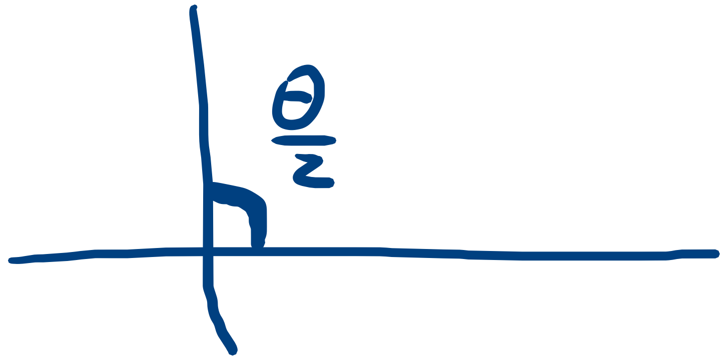
$$|\psi\rangle = r_1 e^{i\varphi_1} + r_2 e^{i\varphi_2}$$
$$= e^{i\varphi_1}\left(r_1|0\rangle + r_2 e^{i(\varphi_2-\varphi_1)}|1\rangle\right)$$

Porém $|e^{i\varphi_1}| = 1$!

$$\Rightarrow |\psi\rangle \overset{\text{IND}}{=} r_1|0\rangle + r_2 e^{i(\varphi_2-\varphi_1)}|1\rangle$$

$$|\psi\rangle = r_1|0\rangle + r_2 e^{i(\varphi_2-\varphi_1)}|1\rangle$$

$\varphi = \varphi_2 - \varphi_1 \Rightarrow \varphi \in [0, 2\pi]$



$$|\psi\rangle = r_1|0\rangle + r_2 e^{i\varphi}|1\rangle$$

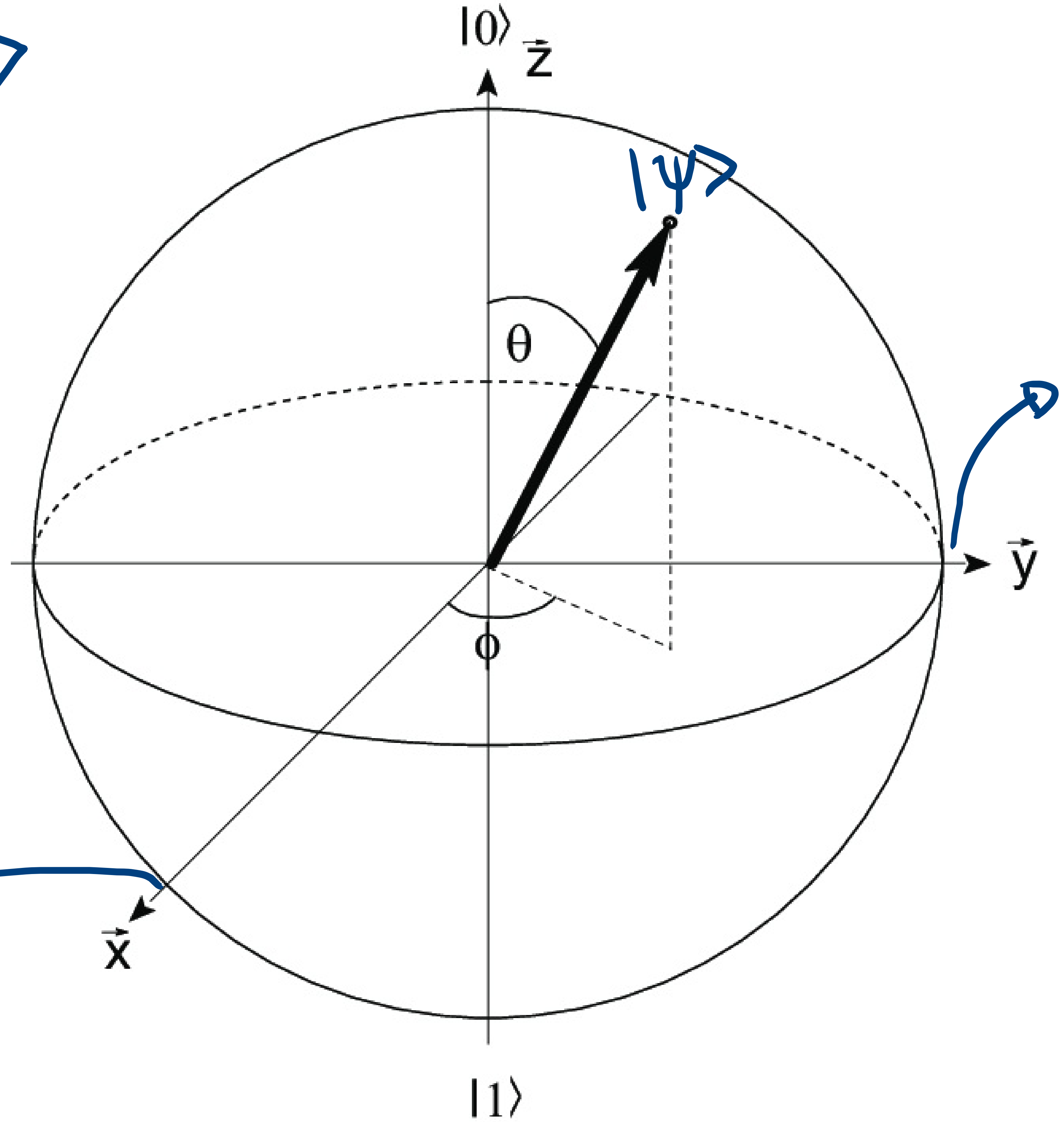
$r_1 = |\alpha|$, $r_2 = |\beta| \Rightarrow r_1^2 + r_2^2 = 1 \Rightarrow \exists\, \theta \in [0, \pi]$ tq $\begin{cases} r_1 = \cos\frac{\theta}{2} \\ r_2 = \text{sen}\frac{\theta}{2} \end{cases}$

Então $|\psi\rangle$ pode ser definido por apenas 2 reais $\begin{cases} \varphi \in [0, 2\pi] \\ \theta \in [0, \pi] \end{cases}$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

Esta representação pode ser visualizada na ESFERA DE BLOCH

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$



$$|i+\rangle$$

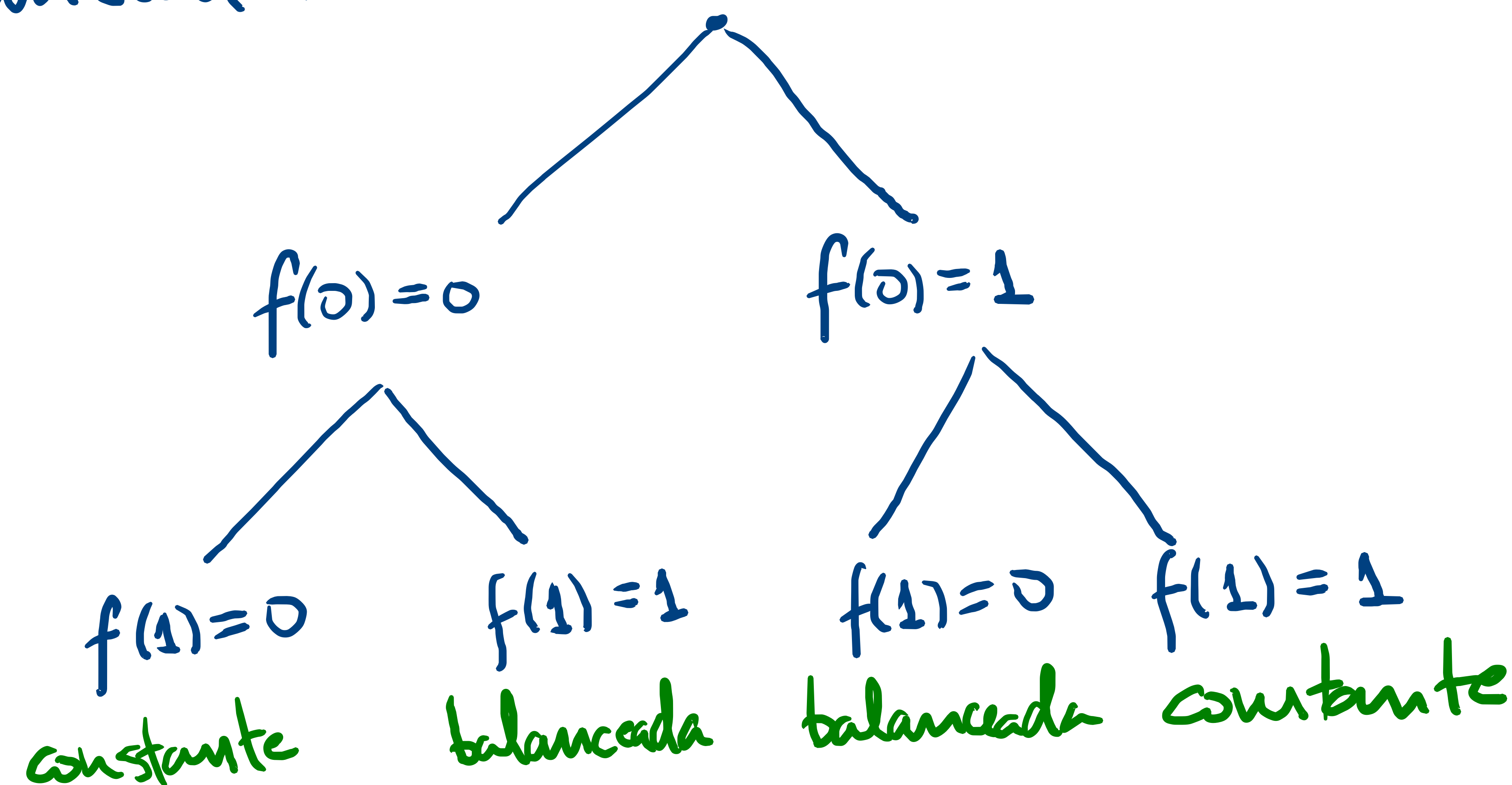
$$\frac{|0\rangle + i|1\rangle}{\sqrt{2}}$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

https://www.researchgate.net/profile/Juergen-Lisenfeld/publication/47041555/figure/fig4/AS:650043006656522@1531993738177/Bloch-sphere-representation-of-the-qubit-state-The-ground-state-0-is-represented-by-a.png

Algoritmo de Deutsch

Seja $f: \{0,1\} \rightarrow \{0,1\}$. f pode ser constante ou balanceada.



· Suponha que temos uma caixa preta que implementa f

→ Queremos um algoritmo que decide se f é constante ou balanceada

- Qualquer algoritmo clássico precisa fazer 2 consultas ao oráculo que implementa f.

- Porém, se U_f for um oráculo que responde consultas em superposição, podemos usar interferência para decidir se f é constante ou balanceada com apenas 1 consulta

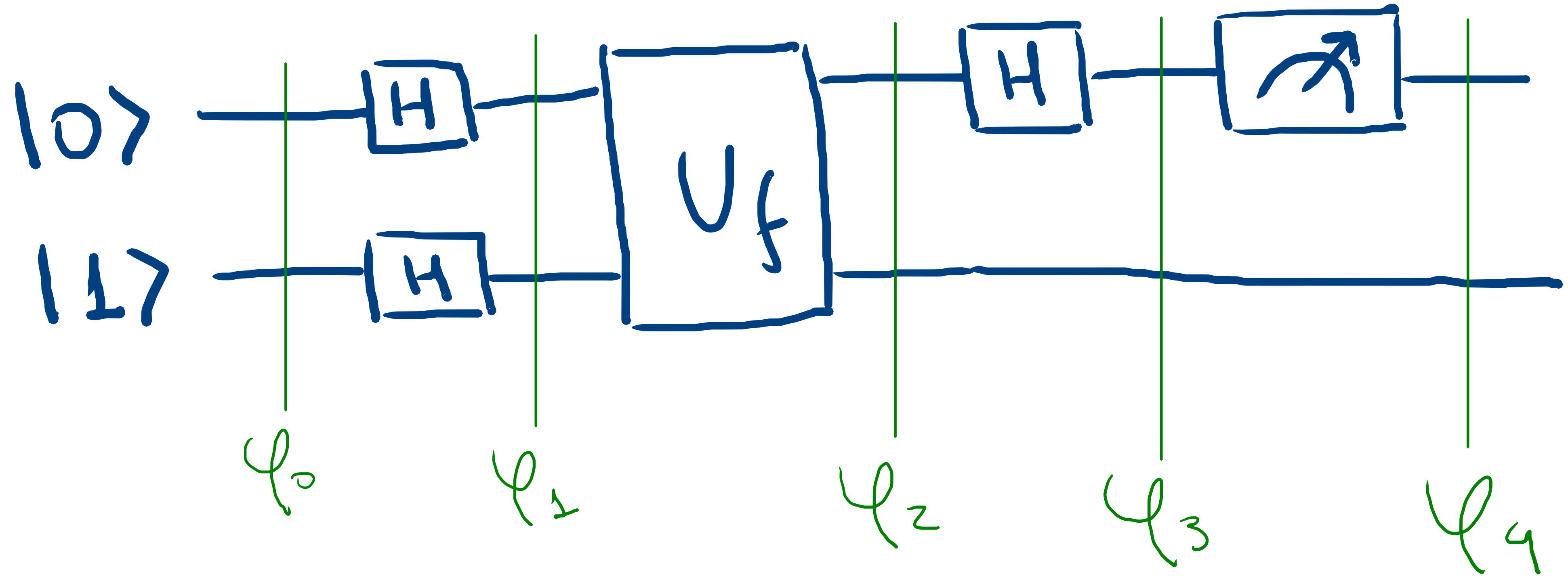
Seja U_f um oraculo tq.

· para $|x\rangle, |y\rangle \in \{|0\rangle, |1\rangle\}$

$$U_f(|x\rangle|y\rangle) \longrightarrow |x\rangle|f(x)\oplus y\rangle$$

<u>IMPORTANTÍSSIMO</u>: Oráculos são definidos pela sua ação nos vetores da base canônica. Se a entrada fosse uma superposição, em geral

$$U_f(|\psi\rangle|\varphi\rangle) \neq |\psi\rangle|\theta\rangle$$



$$\varphi_0 = |0\rangle|1\rangle = |0\rangle \otimes |1\rangle$$

$$\varphi_1 = H|0\rangle \otimes H|1\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$

$$\varphi_2 = \frac{1}{2}\left(U_f|00\rangle - U_f|01\rangle + U_f|10\rangle - U_f|11\rangle\right)$$

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$$U_f\left(|x\rangle|y\rangle\right) = |x\rangle|y \oplus f(x)\rangle \Rightarrow \varphi_2 = \frac{1}{2}\left(|0\rangle|f(0)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|f(1)\rangle - |1\rangle|1\oplus f(1)\rangle \right)$$

CASOS

$f(0)=f(1)=0$: $\varphi_2 = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle \right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

$f(0)=f(1)=1$: $\varphi_2 = \frac{1}{2}\left(|01\rangle - |00\rangle + |11\rangle - |10\rangle \right) = \frac{-1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

$0=f(0)\neq f(1)=1$: $\varphi_2 = \frac{1}{2}\left(|00\rangle - |01\rangle + |11\rangle - |10\rangle \right) = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

$1=f(0)\neq f(1)=0$: $\varphi_2 = \frac{1}{2}\left(|01\rangle - |00\rangle + |10\rangle - |11\rangle \right) = \frac{-1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

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Ou seja, se tomamos somente o 1º qubit

f constante $\Rightarrow \pm\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

f balanceada $\Rightarrow \pm\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

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NOTE ESTES DOIS ESTADOS SÃO DISTINGUÍVEIS NA BASE DE HADAMARD.

LEMBRE

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

f constante $\Rightarrow \pm \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

f balanceada $\Rightarrow \pm \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

φ_2 Pode ser um de 4 estados

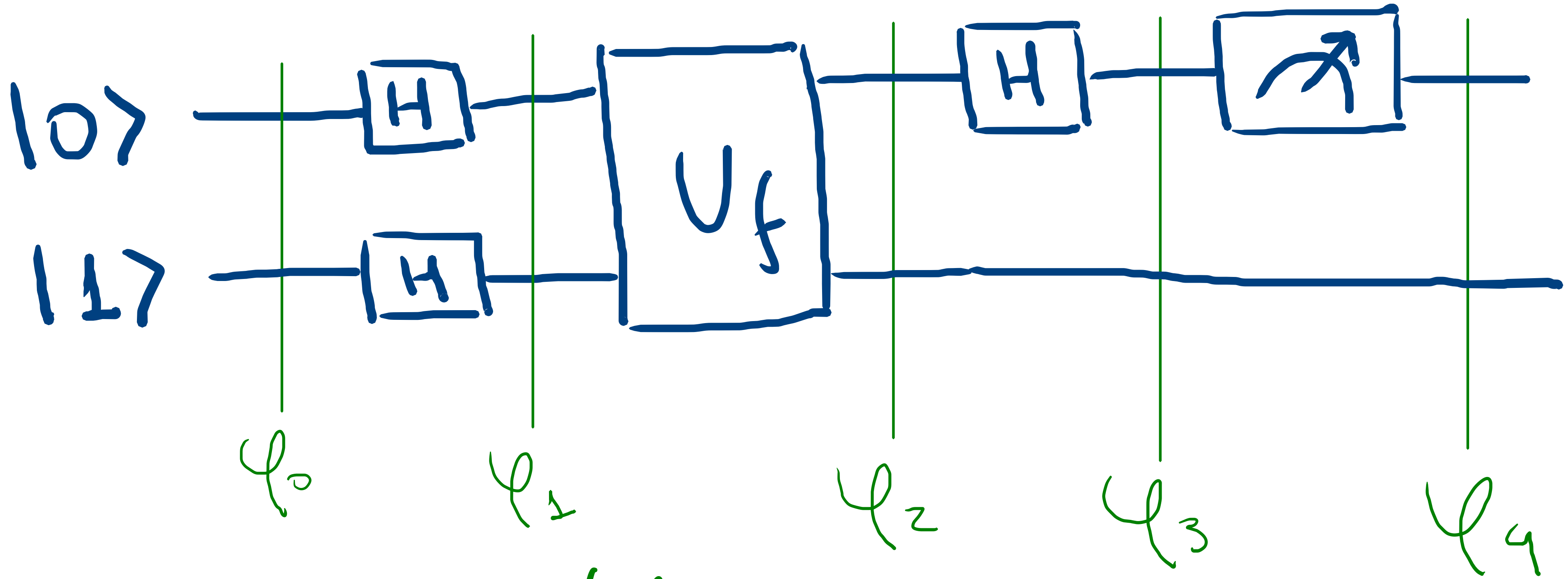
$\varphi_2^{C+} = +\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$\varphi_2^{B+} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$\varphi_2^{C-} = -\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$\varphi_2^{B-} = -\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Vejamos o que acontece com $H\varphi_2$



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$\varphi_3 = H\varphi_2$ Então poderá ser 1 dentre 4.

$$\varphi_3^{C+} = H\varphi_2^{C+} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\\ 0\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$$

$\varphi_3^{C-} = -|0\rangle$; $\varphi_3^{B+} = |1\rangle$; $\varphi_3^{B-} = -|1\rangle$

$$\varphi_2^{C+} = +\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad \varphi_2^{B+} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

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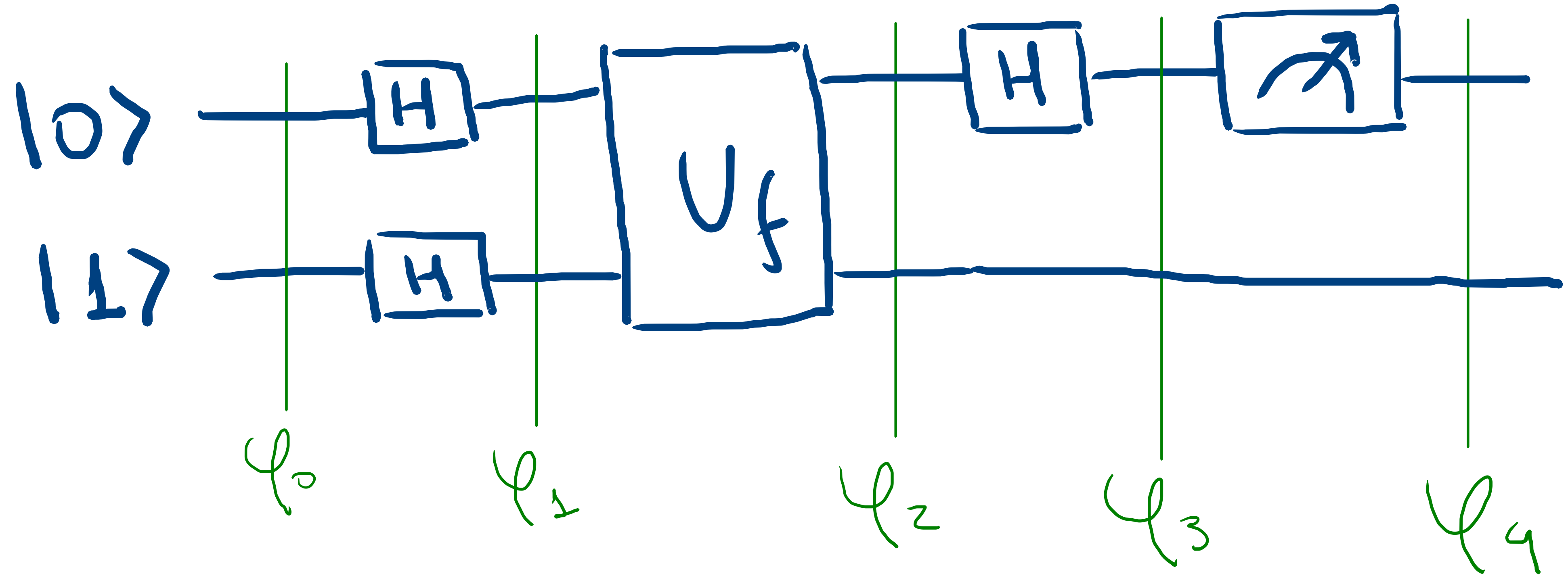
$$\varphi_3^{C-} = H\varphi_2^{C-} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{-1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -|0\rangle$$

$$\varphi_3^{B+} = H\varphi_2^{B+} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = |1\rangle$$

$$\varphi_3^{B-} = H\varphi_3^{B-} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{-1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -|1\rangle$$



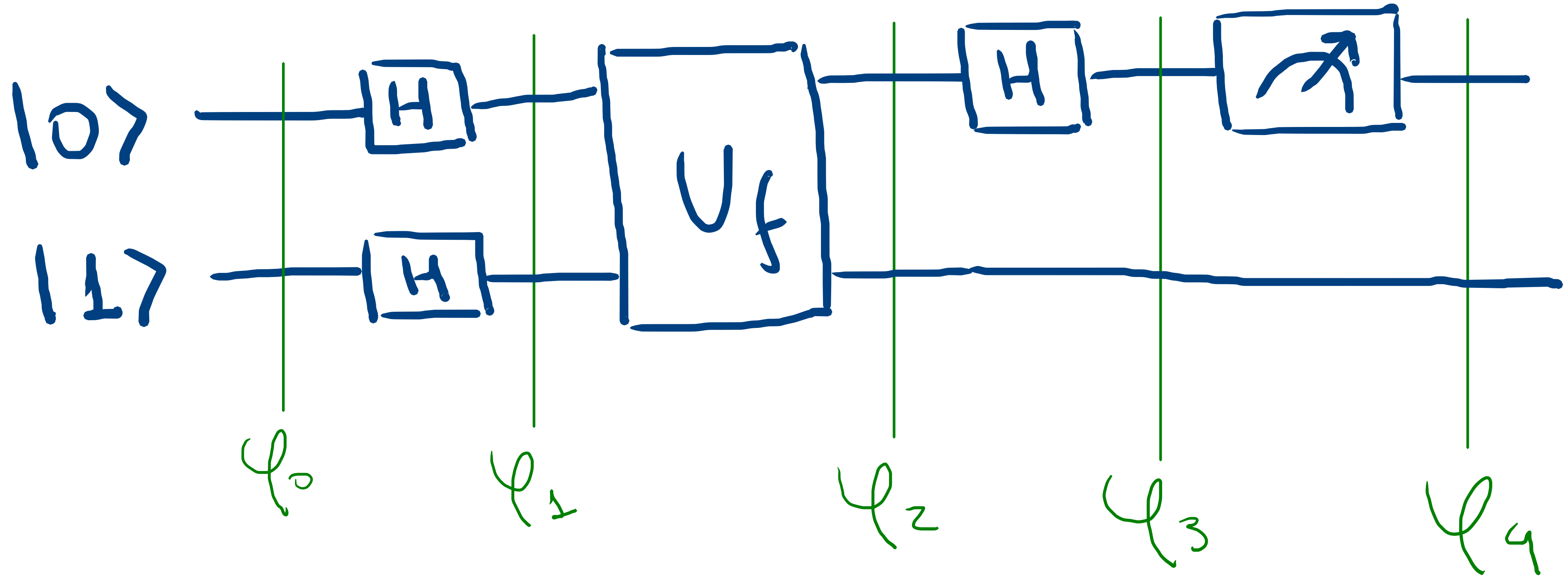
Agora é feita uma medição e temos

Se $\varphi_3 = \varphi_3^{C+} = |0\rangle \Rightarrow \varphi_4 = |0\rangle$ com prob $1^2 = 1$

Se $\varphi_3 = \varphi_3^{C-} = -|0\rangle \Rightarrow \varphi_4 = |0\rangle$ com prob $(-1)^2 = 1$

Se $\varphi_3 = \varphi_3^{B+} = |1\rangle \Rightarrow \varphi_4 = |1\rangle$ com prob $= 1$

Se $\varphi_3 = \varphi_3^{B-} = -|1\rangle \Rightarrow \varphi_4 = |1\rangle$ com prob $= (-1)^2 = 1$



Ou Seja

f constante $\Rightarrow \varphi_4 = |0\rangle$

f balanceada $\Rightarrow \varphi_4 = |1\rangle$

IMPLEMENTANDO O ALGORITMO EM QISKIT

- Precisamos definir os oráculos de funções constantes e balanceadas

$$|x\rangle|y\rangle \xmapsto{f} |x\rangle|y \oplus f(x)\rangle$$

São 4 possibilidades

- $f_{00} \Rightarrow f(0) = f(1) = 0$
- $f_{01} \Rightarrow 0 = f(0) \neq f(1) = 1$
- $f_{10} \Rightarrow 1 = f(0) \neq f(1) = 0$
- $f_{11} \Rightarrow 1 = f(0) = f(1)$

f_{00}

$$\begin{cases} |x\rangle|0\rangle \rightarrow |x\rangle|0\rangle \\ |x\rangle|1\rangle \rightarrow |x\rangle|1 \oplus 0\rangle = |x\rangle|1\rangle \end{cases}$$

$$|00\rangle \longmapsto |00\rangle$$

$$|01\rangle \longmapsto |01\rangle$$

$$|10\rangle \longmapsto |10\rangle$$

$$|11\rangle \longmapsto |11\rangle$$

$$U_{f_{00}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$U_{f_{00}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$U_{f_{01}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|00\rangle \mapsto |00\rangle$$
$$|01\rangle \mapsto |01\rangle$$
$$|10\rangle \mapsto |11\rangle$$
$$|11\rangle \mapsto |10\rangle$$

$$U_{f_{10}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|00\rangle \mapsto |01\rangle$$
$$|01\rangle \mapsto |00\rangle$$
$$|10\rangle \mapsto |10\rangle$$
$$|11\rangle \mapsto |11\rangle$$

$$U_{f_{11}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|00\rangle \mapsto |01\rangle$$
$$|01\rangle \mapsto |00\rangle$$
$$|10\rangle \mapsto |11\rangle$$
$$|11\rangle \mapsto |10\rangle$$

Note uma propr. global das f's balanceadas

EXEMPLO EM QISKIT