MAT 2110 - Cálculo Diferencial e Integral I para Química

Terceira Provinha - 13 de junho de 2008

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Nota	

Questão 1: Usando que $\int \sec^3 x\,dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$, calcule $\int_0^{\pi/4} \sec^5 x\,dx$.

$$\int_0^{\pi/4} \underbrace{\sec^3 x}_{u}\ \underbrace{\sec^2 x\,dx}_{dv} = \sec^3 x\, tg\,x \Big|_0^{\pi/4} - 3\int_0^{\pi/4} \sec^3 x\, tg^2 x\,dx$$

$du = 3\sec^2 x \sec x\, tg\,x$ $\qquad \sec \pi/4 = \sqrt{2} \quad tg\,\pi/4 = 1$

$v = tg\,x$ $\qquad tg^2 x = \sec^2 x - 1$

$$= 2^{3/2} - 3\int_0^{\pi/4} (\sec^5 x - \sec^3 x)\,dx =$$

$$= 2^{3/2} - 3\int_0^{\pi/4} \sec^5 x\,dx + 3\int_0^{\pi/4} \sec^3 x\,dx$$

$$= 2^{3/2} - 3\int_0^{\pi/4} \sec^5 x\,dx + \frac{3}{2}\left(\sec x\, tg\,x + \ln|\sec x + tg\,x|\right)\Big|_0^{\pi/4}$$

$$= 2^{3/2} + \frac{3}{2}\Big[\sqrt{2} + \ln(\sqrt{2}+1) - \underset{0}{\underset{\shortparallel}{\ln 1}}\Big] - 3\int_0^{\pi/4} \sec^5 x\,dx$$

$$\therefore\ 4\int_0^{\pi/4} \sec^5 x\,dx = 2^{3/2} + \frac{3}{2}\left[\sqrt{2} + \ln(\sqrt{2}+1)\right]$$

$$\therefore\ \int_0^{\pi/4} \sec^5 x\,dx = \frac{1}{\sqrt{2}} + \frac{3}{8}\left[\sqrt{2} + \ln(\sqrt{2}+1)\right]$$