MAT 2110 - Cálculo Diferencial e Integral I para Química

Segunda Prova - 30 de junho de 2008

Nome : ____

Número USP : ______________________________________

Assinatura : _______________________________________

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Total	

Questão 1: (1,5 pt) Dada $f(x) = \int_0^{x^2} \operatorname{sen}(t^2)\, dt$, calcule $f'((\pi/2)^{1/4})$.

$f'(x) = (\operatorname{sen} x^4) \cdot 2x$

$f'\left(\left(\frac{\pi}{2}\right)^{1/4}\right) = \left(\operatorname{sen} \frac{\pi}{2}\right) \cdot 2 \cdot \left(\frac{\pi}{2}\right)^{1/4} = 2 \cdot \left(\frac{\pi}{2}\right)^{1/4}$

Questão 2: (2 pts) Mostre que $\left|\frac{1}{e} - \frac{1}{2}\right| < \frac{1}{6}$.

Sugestão: Use o polinômio de Taylor de ordem 2 de $f(x) = e^{-x}$ em torno de $x = 0$.

$f(x) = e^{-x} = f''(x) \quad \therefore \quad f(0) = f''(0) = 1$

$f'(x) = -e^{-x} = f'''(x) \quad \therefore \quad f'(0) = -1$

$$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 = 1 - x + \frac{x^2}{2}$$

T. Taylor $\Rightarrow \exists \; c \in (0,1)$ tal que $f(1) = P_2(1) + \frac{f'''(c)}{3!}$

Isto é
$$\frac{1}{e} = 1 - 1 + \frac{1}{2} - \frac{e^{-c}}{6}$$

Logo $\left|\frac{1}{e} - \frac{1}{2}\right| = \frac{e^{-c}}{6} < \frac{1}{6}$ (pois $c > 0$)

Questão 3: (2 pts) Calcule $\displaystyle\int_1^2 \frac{\ln x}{x^2}\,dx$.

$$\int \underbrace{\ln x}_{u} \underbrace{\frac{1}{x^2}\,dx}_{dv} = -\ln x \cdot \frac{1}{x} + \int \frac{1}{x} \cdot \frac{1}{x}\,dx$$

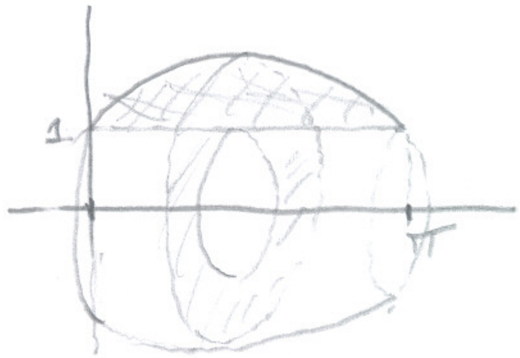
$$du = \frac{1}{x}\,dx \qquad v = -\frac{1}{x}.$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C.$$

$$\int_1^2 \frac{\ln x}{x}\,dx = -\frac{\ln 2}{2} - \frac{1}{2} + \ln 1 + 1 =$$

$$= \frac{1}{2} - \frac{\ln 2}{2}.$$

Questão 4: (2,5 pts) Calcule o volume do sólido que se obtém girando em torno do eixo x a região $\{(x,y);\, 0 \le x \le \pi, 1 \le y \le 1 + \operatorname{sen} x\}$.



$$A(x) = \pi\,(1+\operatorname{sen} x)^2 - \pi \cdot 1^2 \,, \quad 0 \le x \le \pi$$

$$A(x) = 2\pi \operatorname{sen} x + \pi \operatorname{sen}^2 x \,, \quad 0 \le x \le \pi$$

$$V = 2\pi \int_0^\pi \operatorname{sen} x \, dx + \pi \int_0^\pi \operatorname{sen}^2 x \, dx = (*)$$

$$\int_0^\pi \operatorname{sen} x \, dx = -\cos x \Big|_0^\pi = 2$$

$$\int_0^\pi \operatorname{sen}^2 x \, dx = \int_0^\pi \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx = \frac{\pi}{2}$$

$$(*) = 4\pi + \frac{\pi^2}{2}$$

Questão 5: (2 pts) Calcule o comprimento da curva $y = x^2/2$, $0 \leq x \leq 1$.

Sugestão: Use sem demonstrar que $\int \sec^3\theta \, d\theta = \frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|) + C$.

$$L = \int_0^1 \sqrt{1 + f'(x)^2}\, dx \qquad f(x) = \frac{x^2}{2}$$

$$f'(x) = x$$

$$L = \int_0^1 \sqrt{1+x^2}\, dx = \circledast$$

$$\boxed{\begin{array}{ll} x = tg\,\theta & 0 \leq \theta \leq \frac{\pi}{4} \Rightarrow 0 \leq x \leq 1 \\ dx = \sec^2\theta \, d\theta & \\ \sqrt{1+x^2} = \sec\theta & \end{array}}$$

$$\circledast = \int_0^{\pi/4} \sec^3\theta \, d\theta =$$

$$= \frac{1}{2}\left(\sec\theta \, tg\,\theta + \ln|\sec\theta + tg\,\theta|\right)\Big|_0^{\pi/4}$$

$$= \frac{1}{2}\left[\sqrt{2} + \ln(1+\sqrt{2})\right]$$