Lista 4
MAT 2351 / 2013

1) b) Calcule aproximadamente $\sqrt{1{,}01} + \sqrt[3]{7{,}9}$

Seja $f(x,y) = \sqrt{x} + \sqrt[3]{y}$ com $x_0 = 1$ e $y_0 = 8$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{2\sqrt{x}} \quad e \quad \frac{\partial f}{\partial y}(x,y) = \frac{1}{3\sqrt[3]{y^2}}$$

Plano tangente ao gráfico de f no ponto $(x_0, y_0, f(x_0,y_0))$:

$$z = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\cdot(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)\cdot(y-y_0)$$

$$z = \sqrt{x_0} + \sqrt[3]{y_0} + \frac{1}{2\sqrt{x_0}}(x-x_0) + \frac{1}{3\sqrt[3]{y_0^2}}(y-y_0)$$

$$z = \sqrt{1} + \sqrt[3]{8} + \frac{1}{2\cdot\sqrt{1}}\cdot(x-1) + \frac{1}{3\sqrt[3]{8^2}}(y-8)$$

$$z = 1 + 2 + \frac{x}{2} - \frac{1}{2} + \frac{y}{12} - \frac{8}{12}$$

$$z = \frac{x}{2} + \frac{y}{12} + \frac{11}{6}$$

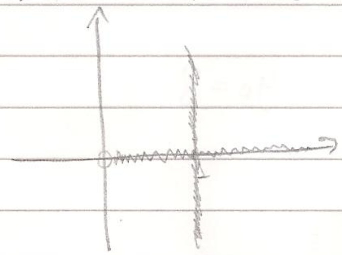
$$\sqrt{1{,}01} + \sqrt[3]{7{,}9} = f(1{,}01;\, 7{,}9) \approx \frac{1{,}01}{2} + \frac{7{,}9}{12} + \frac{11}{6} = \frac{101}{200} + \frac{79}{120} + \frac{11}{6}$$

$$= \frac{303 + 395 + 1100}{600} = \frac{1798}{600} = \frac{899}{300} = 2{,}99666\ldots$$

② Seja $f(x,y) = x^y$ com domínio $D = \{(x,y) \in \mathbb{R}^2 \mid x > 0\}$

a) Determine a curva de nível 1 de f dentro de D.

$x^y = 1 \Rightarrow x = 1$ ou $y = 0$



b) Calcule aproximadamente $(1{,}01)^{2{,}03}$

Sejam $x_0 = 1$ e $y_0 = 2$

$\frac{\partial f}{\partial x}(x,y) = y \cdot x^{y-1}$ e $\frac{\partial f}{\partial y}(x,y) = \ln x \cdot x^y \Rightarrow \nabla f(1,2) = (2,0)$

Plano tangente: $z = f(1,2) + \nabla f(1,2) \cdot (x-1, y-2)$

$z = 1 + 2 \cdot (x-1)$

$(1{,}01)^{2{,}03} = f(1{,}01; 2{,}03) \approx 1 + 2 \cdot 0{,}01 = 1{,}02$

③ a) Sendo $g(t) = f(3t, 2t^2-1)$ calcule $g'(0)$ dado que $f_x(0,-1) = \frac{1}{3}$

$g'(t) = \nabla f(3t, 2t^2-1) \cdot (3, 4t) \Rightarrow g'(0) = \nabla f(0,-1) \cdot (3,0) = 3 \cdot f_x(0,-1) = 3 \cdot \frac{1}{3} = 1$

b) Suponha que $f(3t, t^3) = \text{arctg}\, t$, $\forall t \in \mathbb{R}$. Admitindo que $f_y(3,1) = 2$, calcule $f_x(3,1)$ e determine o plano tangente ao gráfico de f no ponto $(3, 1, f(3,1))$

$g(t) = \text{arctg}\, t \Rightarrow f(3,1) = g(1) = \pi/4$

$g'(t) = \nabla f(3t, t^3) \cdot (3, 3t^2) \Rightarrow \frac{1}{1+t^2} = 3 \cdot f_x(3t, t^3) + 3t^2 \cdot f_y(3t, t^3) \Rightarrow$

$\frac{1}{1+1^2} = 3 f_x(3,1) + 3 f_y(3,1) \Rightarrow f_x(3,1) = \frac{\frac{1}{2} - 6}{3} = -\frac{11}{6}$

$\pi(3,1)$ $z = \frac{\pi}{4} - \frac{11}{6}(x-3) + 2 \cdot (y-1)$ ou $z = -\frac{11x}{6} + 2y + \frac{7}{2} + \frac{\pi}{4}$

5) Calcule os limites abaixo, se existirem, ou mostre que o limite não existe.

d) $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x+y)^2}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} f(x,y)$

$\gamma_1(t)=(t,0)$ e $\gamma_2(t)=(t,t)$

$\lim\limits_{t\to0} f(\gamma_1(t)) = \lim\limits_{t\to0} \dfrac{(t+0)^2}{t^2+0^2} = 1 \neq \lim\limits_{t\to0} f(\gamma_2(t)) = \lim\limits_{t\to0} \dfrac{(2t)^2}{t^2+t^2} = 2$

$\therefore \nexists \lim\limits_{(x,y)\to(0,0)} \dfrac{(x+y)^2}{x^2+y^2}$

f) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+xy^2}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \dfrac{x\cdot(x^2+y^2)}{(x^2+y^2)} = \lim\limits_{(x,y)\to(0,0)} x = 0$

i) $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2y}{x^4+y^2} = \lim\limits_{(x,y)\to(0,0)} f(x,y)$

$\gamma_1(t)=(t,0)$ e $\gamma_2(t)=(t,t^2)$

$\lim\limits_{t\to0} f(\gamma_1(t)) = \lim\limits_{t\to0} \dfrac{2\cdot t^2\cdot 0}{t^4+0} = 0 \neq \lim\limits_{t\to0} f(\gamma_2(t)) = \lim\limits_{t\to0} \dfrac{2t^2\cdot t^2}{t^4+(t^2)^2} = \lim\limits_{t\to0} 1 = 1$

$\therefore \nexists \lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2y}{x^4+y^2}$

j) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \underbrace{x^3}_{\to 0} \cdot \underbrace{\dfrac{y^2}{x^2+y^2}}_{\text{limitada em } [0,1]} = 0$

k) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} \cdot \left(\dfrac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}\right) = \lim\limits_{(x,y)\to(0,0)} \dfrac{\cancel{x^2+y^2}}{\cancel{x^2+y^2}} \cdot \left(\sqrt{x^2+y^2+1}+1\right) = 2$

l) $\lim\limits_{(x,y)\to(2,0)} \dfrac{xy-2y}{x^2+y^2-4x+4} = \lim\limits_{(x,y)\to(2,0)} \dfrac{y\cdot(x-2)}{(x-2)^2+y^2} = \lim\limits_{(x,y)\to(2,0)} f(x,y)$

$\gamma_1(t)=(2,t)$ e $\gamma_2(t)=(t+2,t)$

$\lim\limits_{t\to0} f(\gamma_1(t)) = \lim\limits_{t\to0} \dfrac{t\cdot(2-2)}{(2-2)^2+t^2} = 0 \neq \lim\limits_{t\to0} f(\gamma_2(t)) = \lim\limits_{t\to0} \dfrac{t\,(t+2-2)}{(t+2-2)^2+t^2} = \lim\limits_{t\to0} \dfrac{t^2}{2t^2} = \dfrac{1}{2}$

$\therefore \nexists \lim\limits_{(x,y)\to(2,0)} \dfrac{xy-2y}{x^2+y^2-4x+4}$