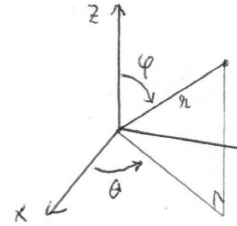
Mat 2352 : dia 17/10 (e ~~20/10~~)

Integrais triplas em coordenadas esféricas:

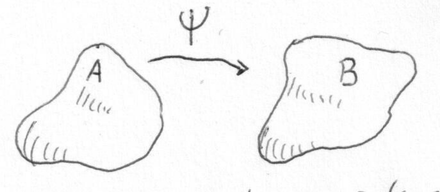


$z = r\cos\varphi$

$x = r \operatorname{sen}\varphi \cos\theta$

$y = r \operatorname{sen}\varphi \operatorname{sen}\theta$

$r^2 = x^2+y^2+z^2$



$$\iiint_B F(x,y,z)\,dx\,dy\,dz = \iiint_A F(x(r,\theta,\varphi),y(r,\theta,\varphi),z(r,\theta,\varphi))\,r^2\operatorname{sen}\varphi\,dr\,d\theta\,d\varphi$$

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ex: $\displaystyle\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\,dx\,dy\,dz = \int_{\varphi=0}^{\pi}\int_{\theta=0}^{2\pi}\int_{r=0}^{1} e^{(r^2)^{3/2}}\, r^2 \operatorname{sen}\varphi\, dr\,d\theta\,d\varphi = \cdots = \frac{4}{3}\pi(e-1).$

$B = \{x^2+y^2+z^2 \leq 1\}$

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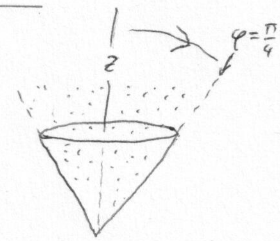
ex: "Cone de sorvete" $B = \{(x,y,z) / z \geq \sqrt{x^2+y^2} \text{ e } x^2+y^2+z^2-z \leq 0\}$. Calcule o volume.

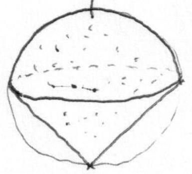


ⓐ Desenho:

- cone $z \geq \sqrt{x^2+y^2}$ pode ser descrito em coordenadas esféricas como: $\{0 \leq \varphi \leq \frac{\pi}{4}\}$.
- "completamento do quadrado":

$x^2+y^2+z^2-z=0 \iff x^2+y^2+(z-\frac{1}{2})^2 = \frac{1}{4}$ é a esfera $S\left((0,0,\frac{1}{2}), \text{raio} = \frac{1}{2}\right)$, também $\underbrace{x^2+y^2+z^2}_{= r^2} \leq \underbrace{z}_{= r\cos\varphi} \iff 0 \leq r \leq \cos\varphi$.

Consequência:

sólido B = 

, volume (B) $= \displaystyle\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\varphi} 1.\, r^2 \operatorname{sen}\varphi\, dr\,d\varphi\,d\theta.$

$$= \int_0^{2\pi}\int_0^{\pi/4} \operatorname{sen}\varphi \left[\frac{r^3}{3}\right]_0^{\cos\varphi} d\varphi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{\operatorname{sen}\varphi}{3}(\cos\varphi)^3\, d\varphi\,d\theta$$

$$= \frac{2\pi}{3}\left[\frac{(\cos\varphi)^4}{-4}\right]_0^{\pi/4} = \frac{2\pi}{3}\left(\frac{-1}{4}\right)\left(\frac{1}{2^2}-1\right) = \frac{\pi}{8}.$$

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exo: calcule o jacobiano de $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\operatorname{sen}\varphi\cos\theta \\ r\operatorname{sen}\varphi\operatorname{sen}\theta \\ r\cos\varphi \end{pmatrix}$ e $\left(\text{Resp: } \dfrac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2 \operatorname{sen}\varphi\right)$.

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exo: calcule $\displaystyle\iiint_B x\,dx\,dy\,dz$ onde $B = \{(x,y,z) / x^2+y^2+z^2 \leq 4, x \geq 0\}$.

Resp: B pode ser descrita como $\{-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2},\ 0 \leq \varphi \leq \pi,\ 0 \leq r \leq 2\}$.

então $\displaystyle\iiint_B x\,dx\,dy\,dz = \int_0^{\pi}\int_{-\frac{\pi}{2}}^{\pi/2}\int_0^2 r\operatorname{sen}\varphi\cos\theta \,.\, r^2\operatorname{sen}\varphi\, dr\,d\theta\,d\varphi = \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos t\,dt\right)\cdot\left(\int_0^{\pi}\operatorname{sen}^2\varphi\,d\varphi\right)\cdot\left(\int_0^2 r^3\,dr\right)$

$= 4\pi.$