Online Network Leasing Problems

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Online Steiner Network Leasing

We also propose a generalization of problem PP in which there is a varying demand for multiple copies of the same resource. This problem is a special case of the 2D Parking Permit (PP2D) problem proposed by Hu et al. [5], but we give an offline approximation algorithm and a randomized \(M/K\)-competitive algorithm for both stochastic-time algorithms.

The input of this problem, which we call the Multi Parking Permit (MPP) problem, consists of:

- \(K\) permit types, with lengths \(l_1, \ldots, l_K\) and costs \(c_1, \ldots, c_K\);
- a sequence of demands \(d_1, \ldots, d_T\) in \(Z_+\).

Every time a demand \(d_t\) arrives, we may buy some permits (maybe multiple permits of the same type and starting time), obtaining a multiset \(S = \{t | \sum_{i \leq t} d_i < \ldots \leq t < \ldots \leq T\}\) which must have at least \(r_t\) active permits in instant \(t\). The objective is to minimize the total cost of the permits.

Our algorithm consists in a reduction to problem PP. This reduction is based on the Interval Model proposed by Meyerson: if we sort the permits by length, then \(d_t\) must be divisible by \(d_{t-1}\), and permits of type \(K\) may only begin at times that are multiple of \(d_{t-1}\). If we suppose the Interval Model, then the guarantee factor may become a ratio of \(O(1/2)\).

Problem MPP may be reduced to \(O(1/2)^{-1}\) instances of problem PP, where \(O(1/2)^{-1}\) is the maximum demand, by putting as much requests in the lowest-indexed instances as possible. This reduction is pseudopolynomial, as we may circumvent the problem by naming \(|\{t | t \leq T\}\) instances and leasing at most \(O(1/2)^{-1}\) per the guarantee ratio. Thus, we obtain a constant-factor approximation algorithm for the offline setting, and for the online setting, a deterministic \(M/K\)-competitive algorithm and a randomized \(M/K\)-competitive algorithm.

In the Online Steiner Network Leasing (OSNL) problem, given a graph \(G = (V, E)\) with edge costs and \(K\) lease types with uniform costs on the edges, at every instant \(t\) we receive a pair \((s_t, e_t)\) of vertices and a demand \(r_t\). Then, we must connect this pair with \(r_t\) edge-disjoint paths of leased edges, note that we may have multiple leases for the same edge. This problem may be solved by approximating the underlying metric by an HST [4].

The problem reduces to problem MPP on each edge of the HST. We obtain a \((O(1/K)\sqrt{E})\)-competitive algorithm.

Similarly, the leasing variant of the Online Buy-at-Bulk Network Design problem may be solved by using tree metric approximations and by solving problem PP2D on each edge of the HST, obtaining a \((O(1/K)\sqrt{E})\)-competitive algorithm.

Conclusions

- We propose a leasing variant of the Online Connected Facility Location problem and give an algorithm which is \((O(1/\gamma)\sqrt{n})\)-competitive for the special case in which the scaling factor \(M = 1\). Our algorithm is based on [10] and [9].

- We propose a multi-demand variant of the Parking Permit problem, and we give:
- a strictly polynomial-time algorithm with constant approximation ratio;
- a deterministic \(M/K\)-competitive algorithm based on [8];
- a randomized \(M/K\)-competitive algorithm based on [8].

- Our techniques may be used to turn the algorithms in [5] into strictly polynomial-time algorithms.

- We also point that the Online Buy-at-Bulk Network Leasing problem has a \((O(1/K)\sqrt{E})\)-competitive algorithm.

Forthcoming Research

We intend to solve leasing variants of other network design problems. For problem OFCLeN, we wish to prove to the same competitive factor for arbitrary \(M\), maybe by modifying the algorithm so as to take into account core edges when deciding whether to lease a facility.

References


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