1. Introduction

“The Probabilistic Method has now become one of the most important and indispensable tools for the combinatorialist. There have been several hundred papers written which employ probabilistic ideas and some wonderful monographs. . . . Over the past two decades, the explosion of research material, along with the wide array of very impressive results, has caused an additional impact of the probabilistic method: some of the techniques involved are subtle, one needs to know how to use those tools, more than simply understand the theoretical underpinnings.”

Noga Alon and Zoltan Furedi, published in 1989, on a result on legitimate colorings of projective planes [1] in which this method is applied via the Local Lov´asz Lemma for a particular combinatorial problem (instead of proving the theorem). The idea is to construct a randomized procedure that solves a particular combinatorial problem (instead of proving the result). The method, which became popular after several contributions of Paul Erdős, can be summarized as follows: to prove the existence of an object (a graph with certain properties, for example) one builds a probability space in which our object has positive probability. So, even though we don’t know how to build it, we know it exists. See Noga Alon and Zoltan Furedi, published in 1989, on a result on legitimate colorings of projective planes [1] in which this method is applied via the Local Lovász Lemma.

3. Results

Definition 1: \( P = \{1, \ldots, n\} \) is a 1-coloring of \((P, L_R, R, \delta_L, \alpha, \beta, R_L, \delta_R, \gamma, \delta_R, \gamma)\).

Definition 2: The type of a 1-coloring is the vector \( t_\alpha = (0, \delta_P(\alpha_{L})\alpha_{R}, \beta_{L}, \gamma_{R}, \delta_{R}, \gamma)\).

Definition 3: \( D \) is a 1-coloring of \((P, L_R, R, \delta_L, \alpha, \beta, R_L, \delta_R, \gamma, \delta_R, \gamma)\) if, for some \( \chi \), all the lines of \((P, L_R, R, \delta_L, \alpha, \beta, R_L, \delta_R, \gamma, \delta_R, \gamma)\) have different types, this coloring is called legitimate.

Definition 4: \( \chi = \chi(P, L_R, R, \delta_L, \alpha, \beta, R_L, \delta_R, \gamma, \delta_R, \gamma) \) is the minimum number of colors necessary in order to exist a legitimate \( \chi \)-coloring of \((P, L_R, R, \delta_L, \alpha, \beta, R_L, \delta_R, \gamma, \delta_R, \gamma)\).

In [1] it is shown that every projective plane of order greater than 3 \( 3^{3m} \) can be legitimate colored with 8 colors. In our research we concluded that \( L_R \) \( \geq \delta_L \) belong to more than \( \delta_L \) points colored. In this case, it is impossible to legitimate color the plane. It was discovered lately (and somewhat unexpectedly), by Grytczuk, Kozik and Mirek in [8], that one can obtain better combinatorial results by a direct application of the entropy compression method rather than appealing to the LLL. The idea is considered a randomized procedure that solves a particular combinatorial problem (instead of proving the theorem). For a particular combinatorial problem (instead of proving the result). The method, which became popular after several contributions of Paul Erdős, can be summarized as follows: to prove the existence of an object (a graph with certain properties, for example) one builds a probability space in which our object has positive probability. So, even though we don’t know how to build it, we know it exists. See Noga Alon and Zoltan Furedi, published in 1989, on a result on legitimate colorings of projective planes [1] in which this method is applied via the Local Lovász Lemma. The method, which became popular after several contributions of Paul Erdős, can be summarized as follows: to prove the existence of an object (a graph with certain properties, for example) one builds a probability space in which our object has positive probability. So, even though we don’t know how to build it, we know it exists.

4. Conclusion

In this paper we have shown a sketch of our proof. First we need some results used in [6]. We now need to compute \( |R| \) as the sequence of words on the alphabet \( L_R \) \( \geq \delta_L \) belong to more than \( \delta_L \) points colored. In this case, it is impossible to legitimate color the plane. It was discovered lately (and somewhat unexpectedly), by Grytczuk, Kozik and Mirek in [8], that one can obtain better combinatorial results by a direct application of the entropy compression method rather than appealing to the LLL. The idea is considered a randomized procedure that solves a particular combinatorial problem (instead of proving the theorem). For a particular combinatorial problem (instead of proving the result). The method, which became popular after several contributions of Paul Erdős, can be summarized as follows: to prove the existence of an object (a graph with certain properties, for example) one builds a probability space in which our object has positive probability. So, even though we don’t know how to build it, we know it exists.

References


