Eigenvalues and eigenvectors for nonlinear problems with Fredholm operators

Nonlinear spectral theory is a research field of increasing interest, which finds application to properties of the structure of the solution set of differential equations. In this context a nontrivial question consists in studying nonlinear perturbations of linear problems and investigating the so-called "persistence" of eigenvalues and eigenvectors.

More precisely, let G and H denote two real Hilbert spaces. By a "perturbed eigenvalue problem" we mean a system of the following type:

$$\begin{cases} Lx + sN(x) = \lambda Cx \\ x \in S, \end{cases} \tag{1}$$

where s, λ are real parameters, $L, C\colon G \to H$ are bounded linear operators with C compact and L Fredholm of index zero, while S denotes the unit sphere of G, and $N\colon S \to H$ is a nonlinear map. We call *solution* of (1) a triple $(s, \lambda, x) \in \mathbb{R} \times \mathbb{R} \times S$ satisfying the above system. The element $x \in S$ is then said a *unit eigenvector* corresponding to the *eigenpair* (s, λ) of (1), and the set of solutions of (1) will be denoted by $\Sigma \subseteq \mathbb{R} \times \mathbb{R} \times S$.

In this seminar we present some recent results obtained in collaboration with A. Calamai, M. Furi and M.P. Pera, concerning the topological properties of the set Σ as well as continuation and bifurcation of the solutions.

Some of the presented results need, to be proven, the use of a recent and advanced topological degree theory, which extends the Leray-Schauder degree.