Chaotic composition operators on L^p-spaces

Benito Pires (USP - Ribeirão Preto)

Continuous linear operators acting on Banach spaces of infinite dimension may present chaotic behaviour. One of the simplest examples of chaotic linear operator is the Rolewicz operator :

$$(x_0, x_1, x_2, \ldots) \in \ell_p \mapsto (2x_1, 2x_2, \ldots) \in \ell_p.$$

Another class of chaotic linear operator is that consisting of backward weighted shifts

$$(x_0, x_1, x_2, \ldots) \in \ell_p \mapsto (\omega_1 x_1, \omega_2 x_2, \ldots) \in \ell_p,$$

where $(\omega_1, \omega_2, \ldots)$ is a bounded sequence of positive weights ω_i satisfying

$$\sum_{i \in \mathbb{N}} \frac{1}{(\omega_1 \omega_2 \cdots \omega_i)^p} < \infty.$$

In this talk we are going to discuss a recent result obtained by the speaker in a joint work with Udayan Darji. More precisely, we provide conditions on a σ-finite measure space $(X, \mathcal{B}, \mu)$ and a self-map f of X so that the composition operator

$$\varphi \in L^p(\mu) \to \varphi \circ f \in L^p(\mu)$$

is chaotic. Composition operators include backward weighted shifts and non-singular odometers as a particular case.