EXERCÍCIOS: SOLUÇÕES

I.1. (A)

Payback period

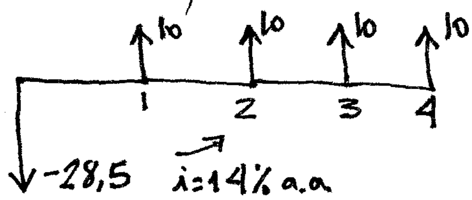
$$p.b.p._M = \frac{28{,}5}{\left(\frac{10+10+10+10}{4}\right)} = 2{,}85 \text{ anos}$$

$$p.b.p._N = \frac{27}{\left(\frac{11+10+9+8}{4}\right)} = 2{,}84 \text{ anos}$$

O payback period dos projetos é similar.

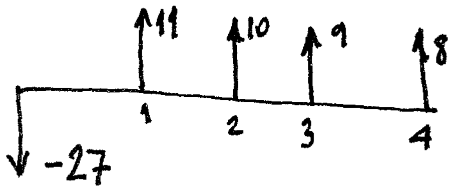
(B) Valor Presente Líquido (VPL)

Projeto M



$$VPL_M = -28{,}5 + \frac{10}{1{,}14} + \frac{10}{(1{,}14)^2} + \frac{10}{(1{,}14)^3} + \frac{10}{(1{,}14)^4} = 0{,}637 \text{ (em milhares)}$$

Projeto N



$$VPL_N = -27 + \frac{11}{1{,}14} + \frac{10}{(1{,}14)^2} + \frac{9}{(1{,}14)^3} + \frac{8}{(1{,}14)^4} = 1{,}155 \text{ (em milhares)}$$

(C) Taxa Interna de Retorno (TIR)

$TIR_M = 15{,}09\%$ a.a.

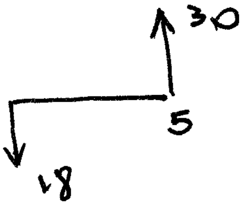
$TIR_N = 16{,}19\%$ a.a.

(D) Os payback periods são totalmente equivalentes. No entanto, se levarmos a estrutura temporal dos pagamentos em conta notaremos que o VPL do projeto N é superior ao do projeto M. Da mesma forma, como seria esperado, a TIR do Proj. N também é superior à do Proj. M. Nossa conclusão é que o projeto N é financeiramente mais interessante que o Proj. M.

I.2.

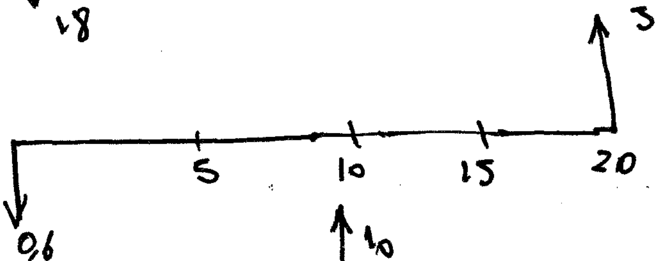
Sua TMA = 10% a.a.

A



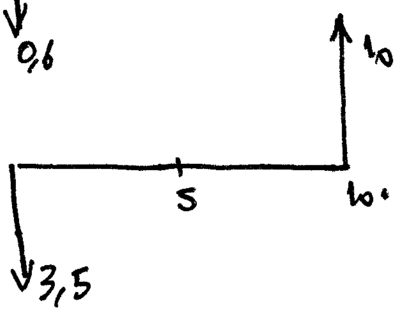
$$VPL_A = -18 + \frac{30}{(1,1)^5} = 0,628$$

B



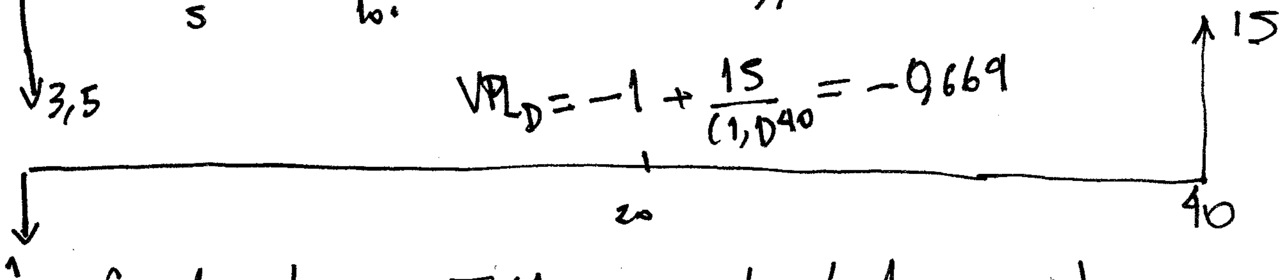
$$VPL_B = -0,6 + \frac{3}{(1,1)^{20}} = -0,154$$

C



$$VPL_C = -3,5 + \frac{10}{(1,1)^{10}} = 0,355$$

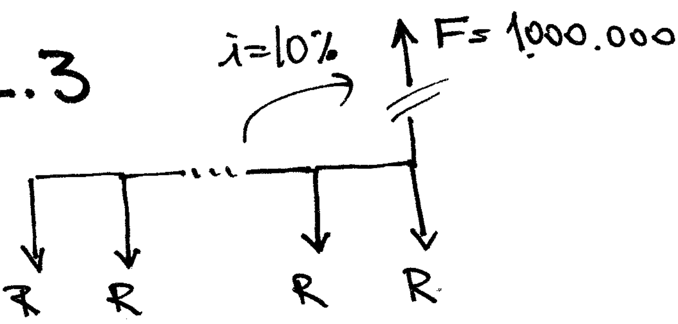
D



$$VPL_D = -1 + \frac{15}{(1,1)^{40}} = -0,669$$

Considerando nossa TMA, o investimento A apresenta o maior VPL e deve ser recomendado.

I.3



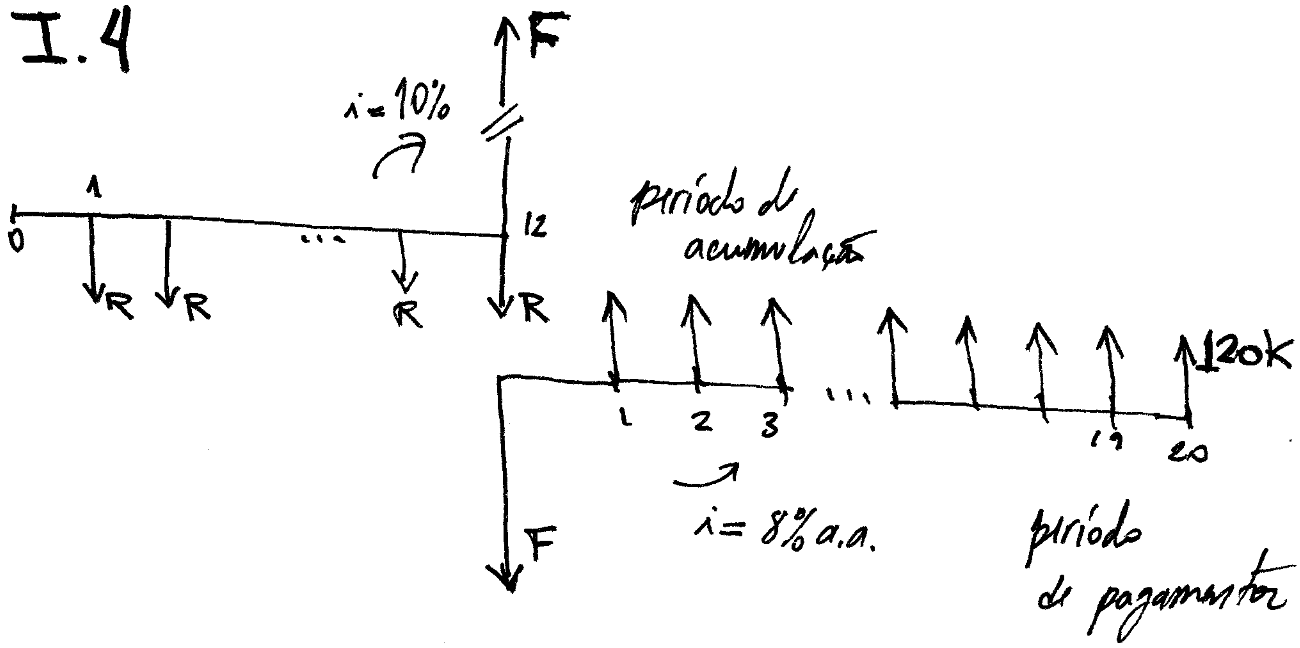
$$S_n = 1 + r + r^2 + \dots + r^n$$
$$-rS_n = -r - r^2 - r^3 - \dots - r^{n+1}$$
$$S_n(1-r) = 1 - r^{n+1}$$
$$\boxed{S_n = \frac{1-r^{n+1}}{1-r}}$$

$$F = R + R(1,1) + R(1,1)^2 + \dots + R(1,1)^{14}$$
$$= R\Big[1 + \underbrace{1,1 + (1,1)^2 + \dots + (1,1)^{14}}_{PG}\Big]$$
$$= R\left[\frac{1-(1,1)^{15}}{1-1,1}\right]$$

$$R = \frac{1000\,000}{31,47} = 31772,00$$

I.4

(A)



(B)

$$F = \frac{120}{1{,}08} + \frac{120}{(1{,}08)^2} + \cdots + \frac{120}{(1{,}08)^{20}}$$

$$= \frac{120}{1{,}08}\left[\underbrace{1 + \frac{1}{1{,}08} + \cdots + \frac{1}{(1{,}08)^{19}}}_{P.G.}\right] \qquad S_n = \frac{1 - r^{n+1}}{1 - r}$$

$$= \frac{120}{1{,}08}\left[\frac{1 - \left(\frac{1}{1{,}08}\right)^{20}}{1 - \frac{1}{1{,}08}}\right] = \frac{120}{1{,}08} \times 10{,}6036 = 1.178{,}2\ K$$

$$\boxed{F = R\$\ 1.178.200{,}00}$$

(C)

$$F = R + R(1{,}1) + \cdots + R(1{,}1)^{11}$$

$$= R\left[\underbrace{1 + (1{,}1) + (1{,}1)^2 + \cdots (1{,}1)^{11}}_{P.G.}\right]$$

$$= R\left[\frac{1 - 1{,}1^{12}}{-0{,}1}\right] = R \times 21{,}384$$

$$R = \frac{1.178.200{,}00}{21{,}384} = 55{,}097\ K$$

$$\boxed{R = R\$\ 55.097{,}00}$$

(D)

$$F_{perp} = \frac{120}{0{,}08} = 1500\ K$$

$$\boxed{F_{perp} = R\$\ 1.500.000{,}00}$$

I.5

Melhor caso

$$F = P(1+i)^n$$
$$= 40(1+0,03)^5$$
$$= 46,37\,k$$

$$\boxed{F_{melhor} = R\$\,46.370,00}$$

Pior caso

$$F = P(1+i)^n$$
$$= 40(1+0,05)^5$$
$$= 51,05\,k$$

$$\boxed{F_{pior} = R\$\,51,050,00}$$

Inflação de 8% a.a.

$$F = 40(1+0,08)^5$$
$$= 58,77\,k$$

$$\boxed{F_{8\%} = R\$\,58.779,00}$$

II.1.

(A) Maximizar $Z = 30x_1 + 20x_2 + 15x_3$

máquina A $\quad 9x_1 + 3x_2 + 5x_3 \leqslant 500$

máquina B $\quad 5x_1 + 4x_2 \leqslant 350$

máquina C $\quad 3x_1 + 2x_3 \leqslant 150$

$x_3 \leqslant 20$

$x_1, x_2, x_3 \geqslant 0$

x_1	x_2	x_3	s_1	s_2	s_3	s_4		razão
9	3	5	1	0	0	0	500	55,56
5	4	0	0	1	0	0	350	70
3	0	2	0	0	1	0	150	50
0	0	1	0	0	0	1	20	—
-30	-20	-15	0	0	0	0	0	

0	3	-1	1	0	-3	0	50	16,67
0	4	-3,33	0	1	-1,667	0	100	25
1	0	0,667	0	0	0,333	0	50	—
0	0	1	0	0	0	1	20	—
0	-20	5	0	0	10	0	1500	

0	1	-0,333	0,333	0	-1	0	16,67	—
0	0	-2	-1,333	1	2,333	0	33,33	14,29
1	0	0,667	0	0	0,333	0	50	150,15
0	0	1	0	0	0	1	20	—
0	0	-1,667	6,667	0	-10	0	1833	

0	1	-1,19	-0,238	0,4286	0	0	30,95	—
0	0	-0,857	-0,5714	0,4286	1	0	14,29	—
1	0	0,952	0,1905	-0,1429	0	0	45,24	47,52
0	0	1	0	0	0	1	20	20
0	0	-10,24	0,9524	4,286	0	0	1976	

$$
\begin{array}{ccccccc|c}
\boxed{x_1} & x_2 & x_3 & s_1 & s_2 & \cancel{s_3}\ s_3 & s_4 & \\
\hline
0 & 1 & 0 & -0,2381 & 0,4286 & 0 & 1,19 & 54,76 \\
0 & 0 & 0 & -0,5714 & 0,4286 & 1 & 0,8571 & 31,43 \\
\boxed{1} & 0 & 0 & 0,1905 & -0,1429 & 0 & -0,9524 & \boxed{26,19} \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 20 \\
\hline
0 & 0 & 0 & 0,9524 & 4,286 & 0 & 10,24 & 2181
\end{array}
$$

$x_1^* = 26,19$

$x_2^* = 54,76$ $\qquad Z^* = 2181$

$x_3^* = 20$

(c) Maximize 30 x1 + 20 x2 + 15 x3

Subject To

9 x1 + 3x2 + 5x3 <= 500

5x1 + 4x2 <= 350

3x1 + 2 x3 <= 150

x3 <= 20

Bounds

x1 >= 0

x2 >= 0

x3 >= 0

End

Z* = 2180, 95

x1* = 26,19

x2* = 54,76

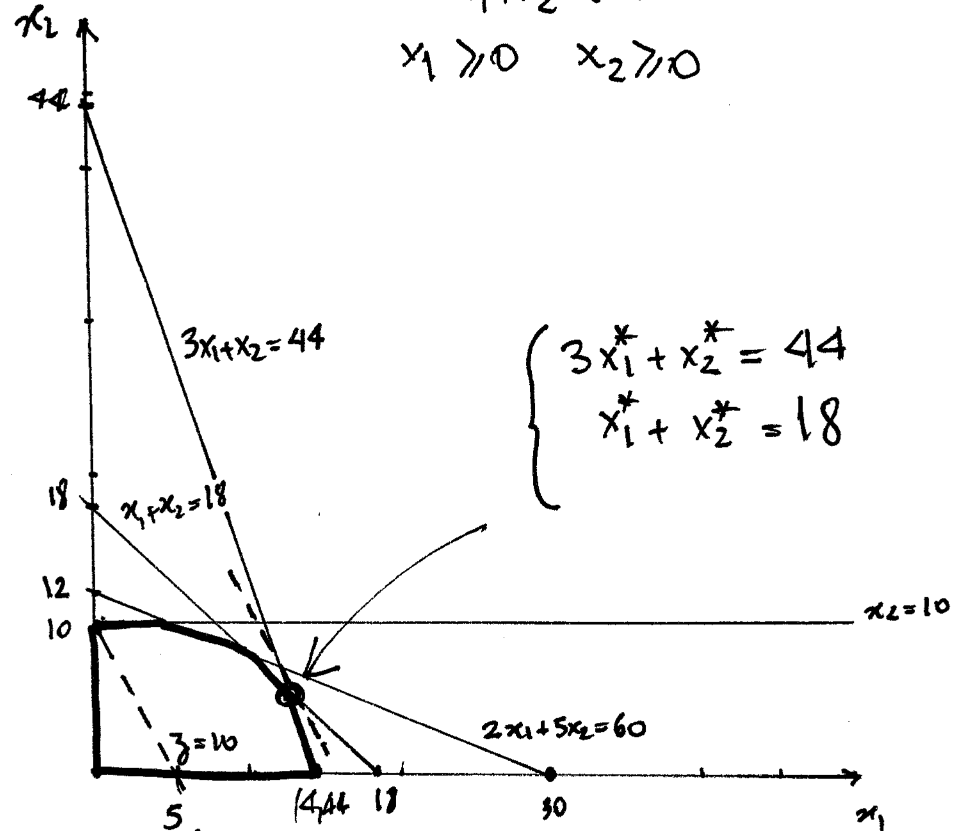
x3* = 20

II.2.

$$\max\ Z = 2x_1 + x_2$$

sujeito a

$$x_2 \leq 10$$
$$2x_1 + 5x_2 \leq 60$$
$$x_1 + x_2 \leq 18$$
$$3x_1 + x_2 \leq 44$$
$$x_1 \geq 0 \quad x_2 \geq 0$$



$$\begin{cases} 3x_1^* + x_2^* = 44 \\ x_1^* + x_2^* = 18 \end{cases}$$

$$x_1^* = \frac{\begin{vmatrix} 44 & 1 \\ 18 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{44-18}{3-1} = 13$$

$$z^* = 31$$

$$x_2^* = \frac{\begin{vmatrix} 3 & 44 \\ 1 & 18 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{10}{2} = 5$$

II. 3.

x_1	x_2	s_1	s_2	s_3	s_4		газб
2	4	1	0	0	0	22	11
-1	4	0	1	0	0	10	—
2	-1	0	0	1	0	7	3,5
1	-3	0	0	0	1	1	1
-3	-2	0	0	0	0	0	

0	10	1	0	0	-2	20	2
0	1	0	1	0	1	11	11
0	5	0	0	1	-2	5	1
1	-3	0	0	0	1	1	—
0	-11	0	0	0	3	3	

0	0	1	0	-2	2	10	5
0	0	0	1	-0,2	1,4	10	7,14
0	1	0	0	0,2	-0,4	1	—
1	0	0	0	0,6	-0,2	4	—
0	0	0	0	2,2	-1,4	14	

0	0	0,5	0	-1	1	5
0	0	-0,7	1	1,2	0	3
0	1	0,2	0	-0,2	0	3
1	0	0,1	0	0,4	0	5
0	0	0,7	0	0,8	0	21

$x_1^* = 5 \quad x_2^* = 3 \quad z^* = 21$

II.4. Minimizar $Z = 7x_1 + 6x_2 + 5x_3$

(A) sujeito a

$$9x_1 + 2x_2 + 4x_3 \geq 20$$
$$3x_1 + 8x_2 + 6x_3 \geq 18$$
$$x_1 + 2x_2 + 6x_3 \geq 15$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$$

(B) Minimizar $Z = 7x_1 + 6x_2 + 5x_3$

sujeito a
$$9x_1 + 2x_2 + 4x_3 - e_1 = 20$$
$$3x_1 + 8x_2 + 6x_3 - e_2 = 18$$
$$x_1 + 2x_2 + 6x_3 - e_3 = 15$$

$$x_1, x_2, x_3 \geq 0 \quad e_1, e_2, e_3 \geq 0$$

(C)
```
Minimize 7 x1 + 6 x2 + 5 x3
Subject To
    9 x1 + 2 x2 + 4 x3 >= 20
    3 x1 + 8 x2 + 6 x3 >= 18
    x1 + 2 x2 + 6 x3 >= 15
Bounds
x1 >= 0
x2 >= 0
x3 >= 0
End
```

rodando glpsol obtemos:

$$x_1^* = 1,1429$$
$$x_2^* = 0$$
$$x_3^* = 2,429$$
$$Z^* = 20,14$$

9