On Brauer configurations induced by finite groups

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Throughout this presentation

- K will denote a field.
- Quivers $\mathcal{Q} = (\mathcal{Q}_0, \mathcal{Q}_1, s, t)$ will be finite, where s the start of an arrow and t the end of an arrow.
- Composition of arrows will be from left to right.
- Usually, letters G, H and L denote groups, subgroups.
- If H is a subgroup of the group G, we denote this by $H \leqslant G$. The order of H is denoted by $|H|$.
- We also denote by $|\mathcal{C}|$ the cardinal number of a set $\mathcal{C}$.
- If G is a group and $x \in G$, we denote by $|x|$ the order of x in G.

Brauer configuration algebras were introduced in 2017 by Edward L. Green and Sibylle Schroll in

- Green, Edward L.; Schroll, Sibylle. *Brauer Configuration Algebras: A Generalization of Brauer Graph Algebras*, Bull. Sci. math., **141** (2017), 539-572.

Initially, these algebras were introduced as a generalization of Brauer graph algebras, as well as a particular class of multiserial symmetric algebras.

To define a Brauer configuration algebra we need to define first the combinatorial object called *Brauer configuration*. Let $\Gamma = (\Gamma_0, \Gamma_1)$ be a tuple such that

- Γ_0 is a finite set of elements that we call *vertices* of Γ;
- Γ_1 is a finite collection of labeled finite multisets of vertices. We call each element of Γ_1 a *polygon*.

Example

Let $\Gamma = (\Gamma_0, \Gamma_1)$ be the tuple given by the following data:

- $\Gamma_0 = \{1, 2, 3, 4\}$;
- $\Gamma_1 = \{V_1, V_2, V_3, V_4\}$ where

$$\begin{aligned} V_1 &= \{1, 2\} \\ V_2 &= \{1, 2\} \\ V_3 &= \{1, 1, 3, 3\} \\ V_4 &= \{3, 4\} \end{aligned}$$

Consider the tuple $\Gamma = (\Gamma_0, \Gamma_1)$. If α is a vertex in Γ_0, we define the following values

- $\mathrm{occ}(\alpha, V)$, *occurrences of* α *in* V, is the number of times that α occurs as a vertex in V.
- The *valence of* α is the nonnegative integer

$$\mathrm{val}(\alpha) := \sum_{V \in \Gamma_1} \mathrm{occ}(\alpha, V).$$

So, by the example presented previously if $\Gamma = (\Gamma_0, \Gamma_1)$ where $\Gamma_0 = \{1, 2, 3, 4\}$ and $\Gamma_1 = \{V_1, V_2, V_3, V_4\}$, then we have

$$\begin{aligned}
\mathrm{occ}(1, V_1) = \mathrm{occ}(1, V_2) = 1, &\quad \mathrm{occ}(1, V_3) = 2,\\
\mathrm{occ}(2, V_1) = \mathrm{occ}(2, V_2) = 1, &\quad \mathrm{occ}(3, V_3) = 2,\\
\mathrm{occ}(1, V_4) = \mathrm{occ}(3, V_1) = 0, &\quad \mathrm{occ}(3, V_4) = 1.
\end{aligned}$$

Thus, in particular, we see that

$$\mathrm{val}(1) = 4$$

Consider the tuple $\Gamma = (\Gamma_0, \Gamma_1)$. We need two more data to define what exactly a Brauer configuration is.

One of these data is a function $\mu : \Gamma_0 \to \mathbb{Z}_{>0}$ that we will call a *multiplicity function*.

The other data is what we call an orientation $\mathfrak{o}$ of Γ. An *orientation* $\mathfrak{o}$ *of* Γ is a choice, for each vertex $\alpha \in \Gamma_0$, of a cyclic ordering of the polygons in which α occurs as a vertex, including repetitions.

Example

Let $\Gamma = (\Gamma_0, \Gamma_1)$ be the tuple considered previously where $\Gamma_0 = \{1, 2, 3, 4\}$ and $\Gamma_1 = \{V_1, V_2, V_3, V_4\}$. We have the following list of polygons in each vertex

- at vertex 1: V_1, V_2, V_3, V_3;
- at vertex 2: V_1, V_2;
- at vertex 3: V_3, V_3, V_4;
- at vertex 4: V_4.

Example

Now, an orientation for Γ can be chosen by

$$\begin{aligned}
\text{at vertex } 1 &: \quad V_1 < V_2 < V_3 < V_3, \\
\text{at vertex } 2 &: \quad V_1 < V_2, \\
\text{at vertex } 3 &: \quad V_3 < V_4 < V_3, \\
\text{at vertex } 4 &: \quad V_4.
\end{aligned}$$

Example

With the additional condition that in each vertex the last polygon is less than the first one, i.e,

$$\begin{aligned} \text{at vertex } 1 &: V_3 < V_1, \\ \text{at vertex } 2 &: V_2 < V_1, \\ \text{at vertex } 3 &: V_3 < V_3, \\ \text{at vertex } 4 &: V_4 < V_4. \end{aligned}$$

Example

Thus, we may represent an orientation $\mathfrak{o}$ of Γ as giving by

$$\begin{aligned} 1 &: V_1 < V_2 < V_3 < V_3, \\ 2 &: V_1 < V_2, \\ 3 &: V_3 < V_4 < V_3, \\ 4 &: V_4. \end{aligned}$$

Definition (Brauer Configuration)

A *Brauer configuration* is a tuple $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ where

- Γ_0 is a set of vertices and Γ_1 a set of polygons,
- μ is a multiplicity function,
- and $\mathfrak{o}$ is an orientation for Γ,

such that the following conditions hold

C1. Every vertex in Γ_0 is a vertex in at least one polygon in Γ_1.

C2. Every polygon in Γ_1 has at least two vertices.

C3. Every polygon in Γ_1 has at least one vertex α such that $\mathrm{val}(\alpha)\mu(\alpha) > 1$.

Definition (Truncated vertex)

Let $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ be a Brauer configuration. We say that a vertex $\alpha \in \Gamma_0$ is *truncated* if $\mathrm{val}(\alpha) = \mu(\alpha) = 1$. A vertex of the configuration that is not truncated is called a *nontruncated vertex*.

From this definition we have the following set of vertices

$$\mathscr{T}_\Gamma := \{ \alpha \in \Gamma_0 \,|\, \mathrm{val}(\alpha) = \mu(\alpha) = 1 \}.$$

This set is just the collection of all truncated vertices of Γ.

Let α be a nontruncated vertex of the Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$, and consider the sequence of polygons of α induced by the orientation $\mathfrak{o}$:

$$\alpha : V_{i_1} < V_{i_2} < \cdots < V_{i_{\mathrm{val}(\alpha)}}.$$

We call this cyclically ordered sequence a *successor sequence at* α.

We also say that $V_{i_{j+1}}$ is the *successor of* V_{i_j} *at* α, for each $1 \leq j \leq \mathrm{val}(\alpha)$, where $V_{i_{\mathrm{val}(\alpha)+1}} = V_{i_1}$

Let $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ be a Brauer configuration. Now, let us see that Γ induces a quiver $\mathcal{Q}_\Gamma$ using the combinatorial data inside of it.

In order to this, we need to define what are going to be the vertices and the arrows of $\mathcal{Q}_\Gamma$.

Induced quiver

Given a Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ we define the induced quiver $\mathcal{Q}_\Gamma$ as follows.

- The vertex set $\{ v_1, \dots, v_m \}$ of $\mathcal{Q}_\Gamma$ is in bijection with the set of polygons $\{ V_1, \dots, V_m \}$ in Γ_1, noting that there is exactly one vertex in $\mathcal{Q}_\Gamma$ for every polygon in Γ_1. We call v_i (resp. V_i) the vertex (resp. polygon) *associated* to V_i (resp. v_i).

Induced quiver

- In order to define the arrows in $\mathcal{Q}_\Gamma$, we use the successor sequences. For each $\alpha \in \Gamma_0 \setminus \mathscr{T}_\Gamma$, and each successor V' of V at α, there is an arrow from v to v' in $\mathcal{Q}_\Gamma$, where v and v' are the vertices in $\mathcal{Q}_\Gamma$ associated to the polygons V and V' in Γ_1, respectively

When it is clear from the context we denote the quiver $\mathcal{Q}$ instead of $\mathcal{Q}_\Gamma$.

Example

Let $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ be the Brauer configuration defined by

- $\Gamma_0 = \{1, 2, 3, 4\}$
- $\Gamma_1 = \{V_1, V_2, V_3, V_4\}$
- $\mu(3) = \mu(4) = 1$ and $\mu(1) = \mu(2) = 2$
- And the orientation $\mathfrak{o}$ is given by the following successor sequences

$$\begin{aligned} 1 &: \quad V_1 < V_2 < V_3 < V_3; \\ 2 &: \quad V_1 < V_2; \\ 3 &: \quad V_3 < V_4 < V_3. \end{aligned}$$

Example

If $\mathcal{Q}$ is the quiver induced by Γ we have that the successor sequence at vertex 1 induces in $\mathcal{Q}$ the sequence of arrows

$$1 : v_1 \xrightarrow{a_1^{(1)}} v_2 \xrightarrow{a_2^{(1)}} v_3 \xrightarrow{a_3^{(1)}} v_3 \xrightarrow{a_4^{(1)}} v_1;$$

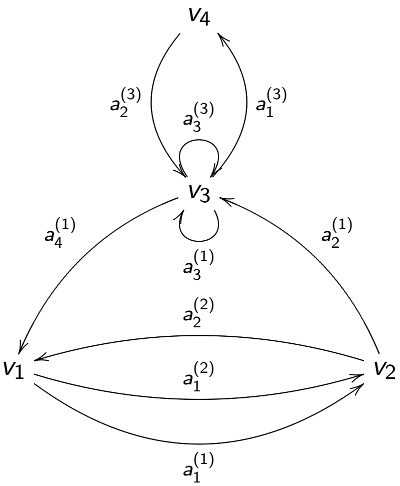
at the vertex 2 and the vertex 3 we have in $\mathcal{Q}$, respectively, the sequences

$$2 : v_1 \xrightarrow{a_1^{(2)}} v_2 \xrightarrow{a_2^{(2)}} v_1,$$

$$3 : v_3 \xrightarrow{a_1^{(3)}} v_4 \xrightarrow{a_2^{(3)}} v_3 \xrightarrow{a_3^{(3)}} v_3.$$

So, the quiver $\mathcal{Q}$ induced by these sequences of arrows looks as follow

Example



Special cycles

Let Γ be a Brauer configuration with induced quiver $\mathcal{Q}$. If $\alpha \in \Gamma_0$ with $\mathrm{val}(\alpha) = t > 1$ and successor sequence $\alpha : V_{i_1} < \cdots < V_{i_t}$ then we have in $\mathcal{Q}$ the sequence of arrows

$$v_{i_1} \xrightarrow{a^{(\alpha)}_{j_1}} v_{i_2} \xrightarrow{a^{(\alpha)}_{j_2}} \cdots \xrightarrow{a^{(\alpha)}_{j_{t-1}}} v_{i_t} \xrightarrow{a^{(\alpha)}_{j_t}} v_{i_1}.$$

Let $C_l = a^{(\alpha)}_{j_l} a^{(\alpha)}_{j_{l+1}} \cdots a^{(\alpha)}_{j_t} a^{(\alpha)}_{j_1} \cdots a^{(\alpha)}_{j_{l-1}}$ be the oriented cycle in $\mathcal{Q}$, for every $1 \leq l \leq t$.

We call any of these cycles a *special* α*-cycle*.

Brauer relations

Let $K\mathcal{Q}_\Gamma$ be the path algebra induced by the quiver $\mathcal{Q}_\Gamma$. Now we define three types of relations in $K\mathcal{Q}_\Gamma$.

Relations of type I

For each polygon $V = \{\alpha_1, \ldots, \alpha_m\}$ in Γ_1 and each pair of nontruncated vertices α_i and α_j in V we define a *relation of type I* as any expression in $K\mathcal{Q}_\Gamma$ of the form

$$C^{\mu(\alpha_i)} - C'^{\mu(\alpha_j)} \text{ or } C'^{\mu(\alpha_j)} - C^{\mu(\alpha_i)},$$

where C is a special α_i-cycle and C' a special α_j-cycle.

Brauer relations

Relation of type II

For a nontruncated vertex α we define a *relation of type II* as an expression in KQ_Γ of the form

$$C^{\mu(\alpha)}a,$$

where C is a special α-cycle and a is the first arrow in C.

Relation of type III

These relations are defined as quadratic monomial expressions of the form ab in KQ_Γ where ab is not a subpath of any special cycle.

Brauer configuration algebra

Definition (Brauer configuration algebra)

Let K be a field and Γ a Brauer configuration. The *Brauer configuration algebra* Λ_Γ *associated to* Γ is defined to be $K\mathcal{Q}_\Gamma/I_\Gamma$, where $\mathcal{Q}_\Gamma$ is the quiver associated to Γ and I_Γ is the ideal in $K\mathcal{Q}_\Gamma$ generated by the set of relations of type I, II and III.

When is clear of the context we write $\Lambda = K\mathcal{Q}/I$ instead of $\Lambda_\Gamma = K\mathcal{Q}_\Gamma/I_\Gamma$.

Example

Let $\Gamma' = (\Gamma'_0, \Gamma'_1, \mu', \mathfrak{o}')$ be the Brauer configuration given by

$$\Gamma'_0 = \{1\}, \Gamma'_1 = \{V\}$$

where $V = \{1, 1\}$, $\mu' \equiv 1$ and the orientation $\mathfrak{o}'$ is defined by the successor sequence

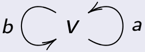
$$1 : V < V$$

Let $\mathcal{Q}'$ be the induced quiver by Γ' with sequence of arrows

$$1 : v \xrightarrow{a} v \xrightarrow{b} v$$

Example

Then $\mathcal{Q}'$ is



We observe that there are only two special cycles: ab and ba.

- Relations of type one: $ab - ba$.
- Relations of type two: aba and bab.
- Relations of type three: a^2 and b^2.

Example

If $\Lambda_{\Gamma'}$ is the induced Brauer configuration algebra, then by the relation of type one we can see that this algebra is commutative. It is not difficult to prove that the Brauer configuration algebra of this example is isomorphic to the algebra

$$K[x_1, x_2]/\langle x_1^2, x_2^2 \rangle.$$

Let Λ be a basic finite dimensional associative K-algebra with a complete set $\{e_1, \ldots, e_n\}$ of primitive orthogonal idempotents. Let $\mathcal{C}_\Lambda = (c_{i,j})$ be the $n \times n$ matrix with entries in $\mathbb{Z}_{\geq 0}$ where

$$c_{i,j} = \dim_K e_j \Lambda e_i.$$

The matrix $\mathcal{C}_\Lambda$ is called the *Cartan matrix* of Λ.

Given a Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ and V a polygon of Γ, we denote by $\overline{V}$ the subset of Γ_0 defined by

$$\overline{V} := \{\alpha \in \Gamma_0 \,|\, \alpha \text{ occurs in } V\}.$$

Theorem (Sierra, 2018)

Let $\Lambda = KQ/I$ be the Brauer configuration algebra associated to $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$. If $\mathcal{C}_\Lambda = (c_{v,w})_{V,W\in\Gamma_1}$ is the Cartan matrix of Λ then

$$c_{v,w} = \begin{cases} 2 + \sum\limits_{\alpha\in\overline{V}} \mathrm{occ}(\alpha, V)\left(\mathrm{occ}(\alpha, V)\mu(\alpha) - 1\right), & V = W; \\ \sum\limits_{\alpha\in\overline{V}\cap\overline{W}} \mu(\alpha)\mathrm{occ}(\alpha, V)\mathrm{occ}(\alpha, W), & V \neq W. \end{cases}$$

Let Λ be the Brauer configuration algebra associated to the Brauer configuration Λ, and let v be the vertex in $\mathcal{Q}$ associated to the polygon V in Γ. By considering the class of all **uniserial modules** contained in the indecomposable projective module $v\Lambda$, we can compute the module length of $v\Lambda$, i.e, the integer value

$$\ell(v\Lambda).$$

Theorem (Sierra, 2019)

Let $\Lambda = K\mathcal{Q}/I$ be a Brauer configuration algebra associated to $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$, and let V be a polygon of Γ. If v is the vertex in $\mathcal{Q}$ associated to V then

$$\ell(v\Lambda) = 2 + \sum_{\alpha \in \overline{V}} \mathrm{occ}(\alpha, V)(\mathrm{val}(\alpha)\mu(\alpha) - 1).$$

As we know $\ell(v\Lambda) = \dim_K v\Lambda$, hence we also obtain the formula

$$\dim_K v\Lambda = 2 + \sum_{\alpha \in \overline{V}} \mathrm{occ}(\alpha, V)(\mathrm{val}(\alpha)\mu(\alpha) - 1).$$

Let Λ be Brauer configuration algebra associated to $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$, and let $\mathcal{C}_\Lambda$ its Cartan matrix. Any Brauer configuration algebra is symmetric, hence the matrix $\mathcal{C}_\Lambda$ is symmetric.

Let $\mathcal{Q}$ be the quiver induced by Γ and let $v \in \mathcal{Q}$. It is a known property that the sum of all the entries of the v-th row of $\mathcal{C}_\Lambda$ coincides with the vector dimension of $v\Lambda$.

Theorem (Sierra, 2019)

Let $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ be a Brauer configuration. Then

$$\sum_{\alpha \in \overline{V}} \operatorname{occ}(\alpha, V)\operatorname{val}(\alpha)\mu(\alpha) = \sum_{W \in \Gamma_1} \left(\sum_{\alpha \in \overline{V} \cap \overline{W}} \mu(\alpha)\operatorname{occ}(\alpha, V)\operatorname{occ}(\alpha, W) \right),$$

for each polygon V in Γ.

Subgroup-ccurrence of an element in a group

Definition

Let G be a group. For $x \in G$, the *subgroup-occurrence of* x *in* G is defined as the value

$$\mathrm{occ}_G(x) := |\{H \leqslant G \,|\, x \in H\}|,$$

that is, $\mathrm{occ}_G(x)$ is the cardinal number of the set of all subgroups in G where x belongs.

If e is the identity element of G, then $\mathrm{occ}_G(e)$ coincides with the number of subgroups contained in G.

Example

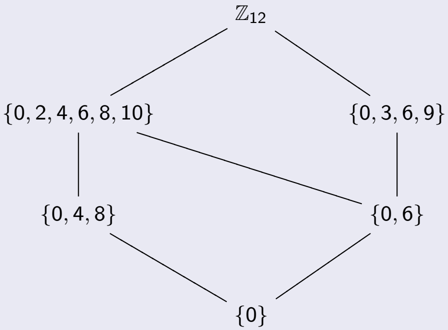
Let us consider the additive group $\mathbb{Z}$ of integers. Using the fact that $\mathbb{Z}$ is a principal ideal domain, we can prove that $\mathrm{occ}_{\mathbb{Z}}(m) < \infty$, for any $m \neq 0$. In fact, we have the values

$$\mathrm{occ}_{\mathbb{Z}}(m) = \begin{cases} \tau(|m|), & m \neq 0; \\ \infty, & m = 0, \end{cases}$$

where τ is the number of divisors function and $|m|$ is the absolute value of m.

Example

The cyclic group $\mathbb{Z}_{12}$ has exactly six subgroups



x	0	1	2	3	4	5	6	7	8	9	10	11
$\mathrm{occ}_{\mathbb{Z}_{12}}(x)$	6	1	2	2	3	1	4	1	3	2	2	1

Proposition

Let G be a group with identity element e. Then

1. $G = \{e\} \iff \mathrm{occ}_G(e) = 1$.
2. G is a cyclic group $\iff \exists x \in G; \mathrm{occ}_G(x) = 1$.
3. $\mathrm{occ}_G(x) = \mathrm{occ}_G(x^{-1})$, for all $x \in G$.
4. If the order of G is finite then
   1. $\mathrm{occ}_G(e) = 2 \iff |G|$ is a prime number.
   2. $\mathrm{occ}_G(x) = \mathrm{occ}_G(e) \iff x = e$.

Proposition

Let G be a finite group with identity element e, and such that $|G| > 1$ is not a prime number. Let $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ be the configuration defined by

1. $\Gamma_0 = G$.
2. $\Gamma_1 = \{H \mid H \leqslant G \text{ and } H \neq \{e\}\}$.
3. $\mu : G \to \mathbb{Z}_{>0}$ is a function such that $\mu(e) = 1$.
4. $\mathfrak{o}$ is a chosen fixed orientation over the objects in Γ_1.

Then Γ is a Brauer configuration.

Let G be a finite group with identity element e, and such that $|G| > 1$ is not a prime number. Let $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$ be the induced Brauer configuration as in the previous proposition. If $\mathrm{val} : \Gamma_0 \to \mathbb{Z}_{>0}$ is the valence function associated to the Brauer configuration Γ, we can easily prove that

$$\mathrm{val}(x) = \begin{cases} \mathrm{occ}_G(x), & x \neq e; \\ \mathrm{occ}_G(x) - 1, & x = e. \end{cases}$$

Theorem (Sierra, 2020)

Let G be a finite group with identity element e and let $\mu : G \to \mathbb{Z}_{>0}$ be a function such that $\mu(e) = 1$. If $H \leqslant G$ then

$$\sum_{x\in H} \mu(x)\mathrm{occ}_G(x) = \sum_{L\leqslant G}\left(\sum_{x\in H\cap L} \mu(x)\right).$$

For n a positive integer let $G = \mathbb{Z}_n$ be the additive group of the integers module n. We have the following corollary.

Corollary

Let n be a positive integer. If k is a positive divisor of n then

$$\sum_{x \in \langle \frac{n}{k} \rangle} \mathrm{occ}_{\mathbb{Z}_n}(x) = \sum_{d \mid n} \gcd(k, d),$$

$$\sum_{x \in \langle \frac{n}{k} \rangle} |x| \mathrm{occ}_{\mathbb{Z}_n}(x) = \sum_{d \mid n} \left(\sum_{t \mid \gcd(k,d)} \phi(t) t \right),$$

where ϕ is the Euler phi-function.

There are two formulas to compute the vector dimension of a Brauer configuration algebra.

Green, Schroll (2017)

Let Λ be a Brauer configuration algebra associated to the Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$. Then

$$\dim_K \Lambda = 2|\Gamma_1| + \sum_{\alpha \in \Gamma_0} \mathrm{val}(\alpha)(\mu(\alpha)\mathrm{val}(\alpha) - 1).$$

Sierra (2019)

Let Λ be a Brauer configuration algebra associated to the Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$. Then

$$\dim_K \Lambda = 2|\Gamma_1| + \sum_{V \in \Gamma_1} \left(\sum_{\alpha \in \overline{V}} \mathrm{occ}(\alpha, V)(\mu(\alpha)\mathrm{val}(\alpha) - 1) \right).$$

Once again, considering the Brauer configuration induced by G when $|G|$ is not a prime number, and considering separately the case when $|G|$ is a prime number, and applying the previous formulae, we can demonstrate the following result.

Theorem (Sierra, 2020)

Let G be a finite group with identity element e and let $\mu : G \to \mathbb{Z}_{>0}$ be a function such that $\mu(e) = 1$. Then

$$\begin{aligned}\sum_{x\in G}\mu(x)\mathrm{occ}_G(x)^2 &= \sum_{H\leqslant G}\left(\sum_{x\in H}\mu(x)\mathrm{occ}_G(x)\right)\\ &= \sum_{H\leqslant G}\left(\sum_{L\leqslant G}\left(\sum_{x\in H\cap L}\mu(x)\right)\right).\end{aligned}$$

Corollary

Let n be a positive integer. Then

$$\sum_{x\in\mathbb{Z}_n} \mathrm{occ}_{\mathbb{Z}_n}(x)^2 = \sum_{k\,|\,n}\left(\sum_{d\,|\,n} \gcd(k,d)\right),$$

$$\sum_{x\in\mathbb{Z}_n} |x|\mathrm{occ}_{\mathbb{Z}_n}(x)^2 = \sum_{k\,|\,n}\left(\sum_{d\,|\,n}\left(\sum_{t\,|\,\gcd(k,d)} \phi(t)t\right)\right),$$

where ϕ is the Euler phi-function.

A final construction

Given a pair (G, μ) where G is a nontrivial finite group with identity element e, and $\mu : G \to \mathbb{Z}_{>0}$ is a function such that

$$\mu(e) = 1,$$

we associate a finite collection $\mathscr{G}_{(G,\mu)}$ that can be formed by either Brauer configurations induced by G and μ, or only by one configuration which is not a Brauer configuration.

Now, for each $\Gamma \in \mathscr{G}_{(G,\mu)}$ a basic finite dimensional algebra Λ_Γ is associated such that:

- if Γ is a Brauer configuration, then Λ_Γ is the Brauer configuration algebra induced by Γ;
- if Γ is not a Brauer configuration, then $\Lambda_\Gamma = K[x]/(x^2)$.

According to this association we have the following result.

Theorem

Let Λ_Γ be the basic finite dimensional algebra associated to the configuration $\Gamma \in \mathscr{G}_{(G,\mu)}$. If $\mu \equiv 1$, then

$$\dim_K Z(\Lambda_\Gamma) = \text{occ}_G(e).$$

That is, if the function μ is constant and equal to 1, the vector dimension of $Z(\Lambda_\Gamma)$ coincides with the number of subgroups contained in G.

Thanks for watching!

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