On the base problem for the polynomial identities of matrix algebras

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Since 1950, the Amitsur-Levitzki Theorem [1]–[7] is known: the minimal polynomial identity for the algebra $M_n(F)$ of $n \times n$ matrices over a field $F$ of characteristic 0 is the standard polynomial

$$st_{2n} = \sum_{\sigma \in S_{2n}} \text{sgn}(\sigma) x_{\sigma(1)} x_{\sigma(2)} \ldots x_{\sigma(2n)},$$

where $S_k$ is the group of permutations on the elements \{1, 2, \ldots, k\} and \(\text{sgn}(\sigma) = \begin{cases} 1, & \text{if } \sigma \text{ is even;} \\ -1, & \text{if } \sigma \text{ is odd.} \end{cases}\)

In 1973, Ju. P. Razmyslov [8] provided the 9 polynomials forming a base for identities of $M_2(F)$. V. S. Drensky [9] reduced the Razmyslov’s base up to 2 polynomials. However, the problem on finding a base for polynomial identities of $M_n(F)$ $(n \geq 3)$ is still open and there is no solution in sight, even for $n = 3$. Actually, we only know [10, 11] that all the identities of $M_3(F)$ of degree $d \leq 8$ follow from $st_6$.

In this talk, we consider an embedding of $M_n(F)$ into a $\mathbb{Z}_n$-graded associative ring $A^{(n)}$ obtained as a quotient ring $A^{(n)} = R[x, \varphi]/(x^n - 1)$ for a ring $R[x, \varphi]$ of (associative but not commutative) skew-polynomials on one variable $x$ over a free associative commutative ring $R$ with an automorphism $\varphi : R \rightarrow R$. We present some results on the connections between the polynomial identities of $M_n(F)$ and $A^{(n)}$. We also formulate some problems on the identities of $A^{(n)}$ related naturally with $M_n(F)$.

References


*Supported by the Capes–PPgMAE program of the Federal University of Rio Grande do Norte (UFRN)
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