Cálculo de limite com o método dos infinitésimos

Calculemos o limite:

$$\lim_{x\to 0} \frac{\sin(x^3) - \tan^3 x}{(1-e^x)(1-\cos x)^2}$$

Temos:

$$\sin x = x - \tfrac{1}{6}x^3 + o(x^4) \quad \text{para } x \to 0,$$

daí:

$$\sin(x^3) = x^3 - \tfrac{1}{6}x^9 + o(x^{12}) = x^3 + o(x^8) \quad \text{para } x \to 0.$$

O Polinômio de Taylor de ordem 4 da tangente, centrado em 0, é:

$$\tan(x) = x + \tfrac{1}{3}x^3 + o\left(x^4\right) \quad \text{para } x \to 0,$$

segue:

$$\tan^3 x = \left[x + \frac{1}{3}x^3 + o\left(x^4\right)\right]^3 = x^3 + x^5 + o(x^6) \quad \text{para } x \to 0,$$

e portanto:

$$\sin(x^3) - \tan^3 x = -x^5 + o(x^6) \quad \text{para } x \to 0.$$

Analogamente:

$$1 - e^x = -x + o(x) \quad \text{para } x \to 0,$$

e:

$$1 - \cos x = \tfrac{1}{2}x^2 + o(x^3) \quad \text{para } x \to 0,$$

$$(1-\cos x)^2 = \left(\tfrac{1}{2}x^2 + o(x^3)\right) = \tfrac{1}{4}x^4 + o(x^4) \quad \text{para } x \to 0,$$

$$(1-e^x)(1-\cos x)^2 = -\tfrac{1}{4}x^5 + o(x^5) \quad \text{para } x \to 0.$$

Finalmente:

$$\lim_{x\to 0} \frac{\sin(x^3) - \tan^3 x}{(1-e^x)(1-\cos x)^2} = \lim_{x\to 0} \frac{-x^5 + o(x^6)}{-\frac{1}{4}x^5 + o(x^5)} = 4.$$