

UNIVERSIDADE FEDERAL DO CEARÁ  
X WORKSHOP DE ANÁLISE GEOMÉTRICA  
Fortaleza, 07 a 09 de junho, 2017  
Títulos e resumos das conferências

QUARTA-FEIRA - 07/06

**Keti Tenenblat - UNB** (*keti@mat.unb.br*)

**Ricci Almost Solitons on Riemannian Warped Products**

Joint work with Valter Borges.

We characterize Ricci almost solitons on semi-Riemannian warped products, considering the potential function to depend on the fiber or not. We show that the fiber is necessarily an Einstein manifold. As a consequence of our characterization, we prove that when the potential function depends on the fiber, if the gradient of the warping function does not act by translations then the base and the warped product are also Einstein manifolds. Moreover, we show the existence of conformal vector fields on the base, the fiber and on the warped product. Assuming completeness of the warped product we provide a classification of such manifolds. When the potential function depends on the fiber and the gradient of the warping function is an improper vector field, we show that the base is a Brinkmann space and the fiber is Ricci flat. We use the characterization also to prove that the potential function of a complete Ricci soliton depends only on the base.

**Vicente Miquel Molina - Espanha** (*Vicente.F.Miquel@uv.es*)

**Evolution by mean curvature flow of Lagrangian spherical surfaces in complex Euclidean plane**

joint work with Ildefonso Castro and Ana M. Lerma.

We describe the evolution under the mean curvature flow of Lagrangian spherical surfaces in the complex Euclidean plane  $\mathbb{C}^2$ . In particular, for embedded surfaces, we answer the question addressed by A. Neves in 2011 about finding out a condition on a starting Lagrangian torus in  $\mathbb{C}^2$  such that the corresponding mean curvature flow becomes extinct at finite time and converges after rescaling to the Clifford torus. On the other hand, we also provide examples of Lagrangian surfaces with self-intersection which develop Type II singularities under the mean curvature flow.

**Gregório Silva Neto - UFAL** (*gregoriosilvaneto@gmail.com*)

**Isoperimetric inequalities and monotonicity formulas for submanifolds in warped products manifolds**

In this talk we first present some new linear isoperimetric inequalities for submanifolds in the de Sitter-Schwarzschild and Reissner-Nordstrom manifolds. Moreover, the equality

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is attained. Next, we show some monotonicity formulas for submanifolds with bounded mean curvature vector in warped product manifolds and, as consequences, we give lower bound estimates for the volume of these submanifolds in terms of the warping function.

**Ezequiel Barbosa - UFMG** (*zikebarbosa@gmail.com*)

**Resultados de gap para área de subvariedades mínimas free-boundary na bola unitária Euclidiana**

Nessa palestra, vamos discutir resultados de gap envolvendo a área de  $k$ -subvariedades mínimas free-boundary na bola unitária Euclidiana  $B^n$ . Mais precisamente, vamos mostrar que existe um  $\varepsilon(k, n)$  tal que a única  $k$ -subvariedade mínima free-boundary  $\Sigma$  na bola Euclidiana  $B^n$  com área  $\varepsilon(n)$ -próxima da área do disco flat é o próprio disco flat. Também vamos discutir resultados de gap para a energia de Willmore para superfícies CMC's free-boundary na bola unitária Euclidiana  $B^3$ . Este é um trabalho em conjunto com C. Viana.

**Fábio Reis dos Santos - UFCG** (*fabiogauss23@gmail.com*)

**On the complete locally conformally at Riemannian manifolds**

In this talk we will talk briefly about complete locally conformally flat manifolds  $(M^n, g)$  with constant scalar curvature. In this setting, we apply the Omori-Yau maximum principle in order to show that such a manifold  $(M^n, g)$  must be either a manifold of constant sectional curvature or universal cover with the pull-back metric isometric to  $\mathbb{R} \times \mathbb{S}^{n-1}(\kappa)$ ,  $\mathbb{S}^1 \times \mathbb{S}^{n-1}(\kappa)$  or  $\mathbb{H}^1 \times \mathbb{S}^{n-1}(\kappa)$  for some  $\kappa > 0$ . Moreover, we also study the case in which these manifolds are parabolic.

**Eraldo Lima Jr - UFPB** (*eraldojrps21@gmail.com*)

**Stability and parabolicity of hypersurfaces in Lorentzian spacetimes**

In this presentation, we deal with parabolic stable spacelike hypersurfaces with constant mean curvature immersed in a Lorentzian product. In this setting, we use some cut-off functions directly related to the parabolicity. We obtained rigidity properties for stable hypersurfaces, such as constant angle with the vertical direction timelike parallel direction  $\partial_t$ , maximality and Calabi-Bernstein results in this process we also observed that such hypersurfaces must have zero Gauss-Kronecker curvature. We also attained to the case of surfaces where we can consider classic results as well obtain more refined theorems than in the general case.

**Patrícia Klaser - UFRGS** (*patriciak@mat.ufrgs.br*)

**Bounded  $\lambda$ -harmonic functions in domains of the hyperbolic space with asymptotic boundary with fractional dimension**

The existence and nonexistence of  $\lambda$  harmonic functions in unbounded domains of  $\mathbb{H}^n$  are investigated. We define Hausdorff dimension on the asymptotic boundary of  $\mathbb{H}^n$  and relate

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the dimension of the asymptotic boundary of an unbounded domain to the existence of bounded  $\lambda$ -harmonic functions in the domain. This is a joint work with L. Bonorino from UFRGS.

**QUINTA-FEIRA - 08/06**

**Marcos Dacjzer - IMPA** (*marcos@impa.br*)

**Entire unbounded constant mean curvature Killing graphs**

We provided conditions for an entire constant mean curvature Killing graph lying inside a possible unbounded region to be necessarily a slice. This is joint work with Jorge H. Lira.

**Paulo Alexandre Sousa - UFPI** (*paulosousa@ufpi.edu.br*)

**Convexity and Some Geometric Properties**

In this talk we will prove that the conservativity of the geodesic flow on a Riemannian manifold with infinite volume is an obstruction to the existence of convex functions and that Riemannian manifolds that admit monotone vector fields that satisfy a weaker condition than strictly monotone has infinite volume. These results generalize, in a certain sense, the result proved by Yau.

**Esko Heinonen - Finlândia** (*esko.heinonen@helsinki.fi*)

**Existence and non-existence results for minimal graphic and  $p$ -harmonic functions**

In the Euclidean space, by the celebrated result due to Bombieri, De Giorgi, and Miranda, all positive entire solutions of the minimal graph equation are constant. It turns out that on Riemannian manifolds similar results can be obtained for solutions with at most linear growth if the manifold has only one end and asymptotically non-negative sectional curvature. In this talk I will discuss about recent results concerning the existence and non-existence of entire minimal graphic and  $p$ -harmonic functions. Talk is based on joint work with Jean-Baptiste Casteras and Ilkka Holopainen.

**Feliciano Vitório - UFAL** (*feliciano@pos.mat.ufal.br*)

**Gap theorems for free boundary hypersurfaces**

In this talk, we prove gap theorems for free boundary hypersurface  $M^n \looparrowright \mathbb{B}^{n+1} \subset \mathbb{R}^{n+1}$ ,  $n \geq 3$ , in terms of traceless second fundamental form. This is a joint work with M. Cavalcante and A. Mendes.

**Benedito Leandro Neto - UFG** (*bleandroneto@ufg.br*)

**Critical point metrics of the total scalar curvature functional**

Nesta apresentação vamos tratar da conjectura CPE (Critical Point Equation). Num primeiro momento, vamos estabelecer as bases da teoria e apresentar a conjectura. Depois

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vamos discutir alguns resultados já conhecidos sobre a conjectura CPE. Por fim, daremos uma condição necessária e suficiente sobre a norma do gradiente da função potencial para que a métrica CPE seja Einstein.

**Antonio Wilson Cunha - UFPI** (*wilsoncunha@ufpi.edu.br*)

**On the first strong stability eigenvalue of closed submanifolds in the unit sphere**

We extend a sharp upper bound of the first strong stability eigenvalue recently due to Chen and Cheng for the context of closed submanifold  $M^n$  immersed with nonzero parallel mean curvature vector field in the Euclidean unit sphere  $\mathbb{S}^{n+p}$  and, as a consequence of this estimate, we obtain new characterizations for certain Clifford torus.

**SEXTA-FEIRA 09/06**

**Paolo Piccione - USP** (*piccione.p@gmail.com.br*)

**T.B.A.**

**Álvaro Krüger Ramos - (akramos7h@gmail.com)**

**Properly immersed surfaces of constant mean curvature  $|H| < 1$  in hyperbolic 3-manifolds of finite volume**

In this talk, we give a necessary and sufficient condition for an immersed  $H$ -surface of finite topology to be properly immersed in a hyperbolic 3-manifold of finite volume. We also give an asymptotic description of such surfaces and prove area and second fundamental form bounds.

**Rondinelle Marcolino Batista - UFPI** (*rmarcolino@ufpi.edu.br*)

**Critical metrics of the volume functional on compact three-manifolds with boundary**

In the last decades very much attention has been given to characterizing critical metrics of the Riemannian functionals, as for instance, the total scalar curvature functional and the volume functional. Critical metrics of the volume functional constrained to the space of metrics of constant scalar curvature on a given compact manifold with boundary are called Miao-Tam critical metrics. In this talk, we provide an estimate to the area of the boundary of Miao-Tam critical metrics on compact three-manifolds. In addition, we obtain a Bochner type formula which enables us to show that a Miao-Tam critical metric on a compact three-manifold with positive scalar curvature must be isometric to a geodesic ball in  $\mathbb{S}^3$ . This is a joint work with R. Diogenes, M. Ranieri and E. Ribeiro Jr.

**Frederico Girão - UFC** (*fred@mat.ufc.br*)

**Geometric Inequalities for Hypersurfaces in Space Forms**

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We will present several geometric inequalities for hypersurfaces in space forms. Each of these inequalities is obtained by defining a related quantity and showing that this quantity is monotone along some geometric flow (mainly the inverse mean curvature flow). One of our results gives a partial answer to a conjecture of Ge, Wang and Wu concerning the weighted Alexandrov-Fenchel inequalities for hypersurfaces in hyperbolic space.

**Rodrigo Avalós - UFPB** (*rdravalos@gmail.com*)

**On the proof of the Thin Sandwich Conjecture in arbitrary dimensions** The Cauchy problem for general relativity (GR) consists in finding a solution of the Einstein equations in a 4-dimensional Lorentzian manifold, which satisfies some prescribed initial conditions on a 3-dimensional Riemannian hypersurface. It is known that GR has a well-posed Cauchy problem for initial data satisfying some constraint equations. This implies that we cannot arbitrarily give the initial data set for the Cauchy problem, motivating the study of this set of constraint equations so as to determine under what conditions it has a solution, and what part of this data can in fact be given arbitrarily on the initial manifold. It is customary to regard this system as a system of partial differential equations for a Riemannian metric and for some  $(0, 2)$  symmetric tensor field defined on this Riemannian hypersurface, which, in the end, will play the role of the extrinsic curvature. It is known that, under some hypotheses on the topology of the space-like manifold, we can specify a conformal metric to the physical one and the trace of the second fundamental form, and then get a well-posed system for the remaining undetermined quantities. Another way to look at this problem was proposed by John A. Wheeler. His idea was to consider space-time as a curve in what he called Superspace. Given an  $n$ -dimensional manifold  $M$ , the Superspace  $S(M)$  related to it, is the space of geometries that can be defined on  $M$ . That is, each point in  $S(M)$  is a pair  $(M, g)$  with  $g$  a Riemannian metric defined on  $M$ . In this context, Wheeler proposed the Thin Sandwich Problem, where the idea is to give as initial data a Riemannian metric  $g$  and a tangent vector  $\dot{g}$  to  $(M, g)$ , and then study whether we can solve the constraint equations for these initial data. If we can, this would mean that there is a unique curve in Superspace satisfying these initial conditions and compatible with the Einstein equations. In this talk, we will discuss how, under certain geometric conditions, Wheeler's conjecture can be shown to be true, not only for GR, but also for higher-dimensional theories of gravity. The main result to be presented is that, on any compact  $n$ -dimensional manifold,  $n \geq 3$ , there is an open set in the space of solutions of the constraint equations where the thin sandwich problem is well-posed.

**Flávio França Cruz - URCA** (*flavio.franca@urca.br*)

**On the existence of radial graphs with constant scalar curvature**

In this talk I will discuss about the problem of finding hypersurfaces of constant curvature and prescribed boundary in the Euclidean space, using the theory of fully nonlinear elliptic equations. The main result says that if the given data admits a suitable radial graph as a

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subsolution, then we can find a radial graph with constant curvature and that realizes the prescribed boundary. As an application we prove that if  $\Omega \subset \mathbb{S}^n$  is a mean convex domain whose closure is contained in an open hemisphere of  $\mathbb{S}^n$  then, for  $0 < R < n(n-1)$ , there exists a radial graph of constant scalar curvature  $R$  and boundary  $\Omega$ .