On the representatives $k$-fold coloring polytope

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Abstract

A $k$-fold $x$-coloring of a graph $G$ is an assignment of (at least) $k$ distinct colors from the set $\{1, 2, \ldots, x\}$ to each vertex such that any two adjacent vertices are assigned disjoint sets of colors. The $k$-th chromatic number of $G$, denoted by $\chi_k(G)$, is the smallest $x$ such that $G$ admits a $k$-fold $x$-coloring. We present an ILP formulation to determine $\chi_k(G)$ and study the facial structure of the corresponding polytope $P_k(G)$. We show facets that $P_{k+1}(G)$ inherits from $P_k(G)$. We also relate $P_k(G)$ to $P_1(G \circ K_k)$, where $G \circ K_k$ is the lexicographic product of $G$ by a clique with $k$ vertices. In both cases, we can obtain facet-defining inequalities from most of those known for the 1-fold coloring polytope. In addition, we present a class of facet-defining inequalities based on strongly $\chi_k$-critical webs, which extend and generalize known corresponding results for 1-fold coloring. We introduce this criticality concept and characterize the webs having such a property.

Keywords: $(k$-fold$)$ graph coloring, facet, web graph, critical graph, lexicographic product.

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