Notas sobre o capítulo 4 de Diestel

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Alguns comentários sobre o capítulo 4 (*Planar graphs*) do livro de Diestel.

4.5 Algebraic planarity criteria

Não sei por que Diestel não menciona a conjectura da cobertura dupla (das arestas) por ciclos logo depois de definir um subconjunto simples do espaço de ciclos, no início da seção 4.5 (ou talvez na seção 1.9). (Mas Diestel menciona a conjectura no exercício 6.13, p.144.) Eis as definições e fatos relevantes.

A *cyle double cover* (CDC[1]) of a graph G is a list of cyles of G that uses each edge exactly twice[2]. A cycle may appear twice in the list; this is necessary because if G is itself a cyle then the only CDC consists of G taken twice. Thus, in Diestel's terms, a CDC is essentially the same as a simple subset of the edge space $\mathcal{E}(G)$ of G. The CDC conjecture claims that CDC

Every graph without bridges has a CDC.

This conjecture belongs to the graph theory folklore; it was first stated in print by Szekeres in 1973 and then by Seymour in 1977. I think it remains open.

For a contrast, consider the following "cut double cover" fact (Proposition 1.9.3 on page 21 in Diestel): *Every multigraph without a loop has a list of cuts that uses each edge exactly twice.* (Take all cuts of the form $E(v)$, where v is a vertex.)

MacLane's theorem (Theorem 4.5.1 in Diestel) can be restated as follow: *A graph is planar if and only if its cycle space can be generated by a* CDC.

[1] not to be confused with the Center for Desease Control

[2] I guess the definition could be changed to "uses each edge at least once but at most twice"

Exercises

1. Suppose $C_1, \ldots, C_k$ is a set of cycles of a (possibly nonplanar) graph G such that each edge of G belongs to at least one and at most two of the cycles. Show that there exists a collection of cycles such that each edge belongs to exactly two of them.

2. Write a CDC of K^5. Show that it does not generate the cycle space of the graph (e.g., show that some cycle cannot be written as the symmetric difference of members of the CDC).

3. Repeat the exercise for $K_{3,3}$.