# Results of a moist shallow-water model on locally refined Spherical Voronoi grids

## *The Andes Problem*

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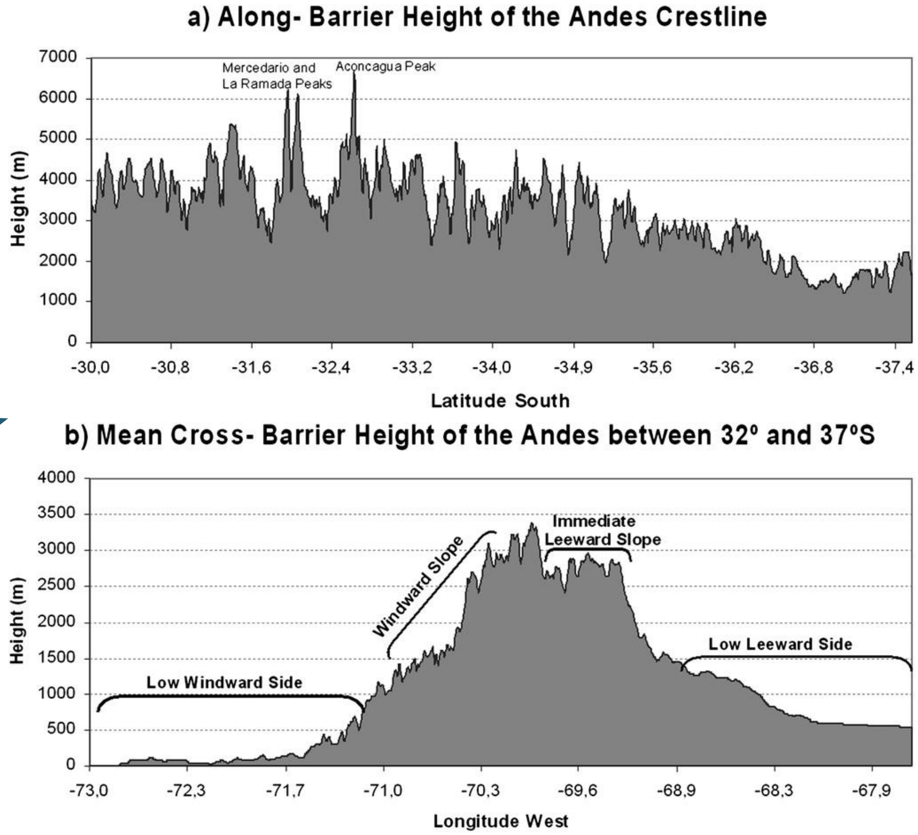
This work was funded by FAPESP, being presented by Pedro Peixoto at the SIAM CSE conference (01/03/2021 - 05/03/2021).

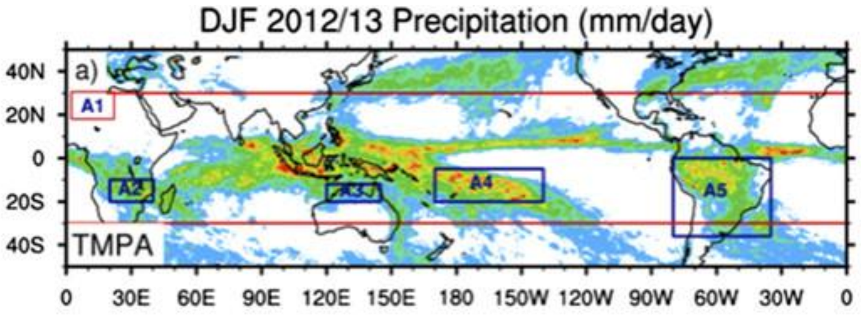
SIAM-SCE - March 2021

# The Andes Range

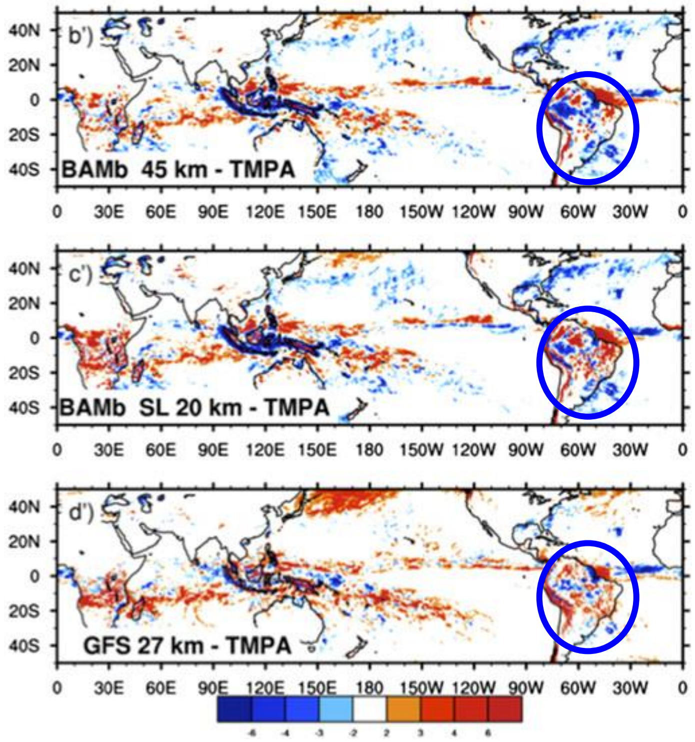




Viale and Nuñez, Journal of Hydrometeorology 12, 4

# The Andes Problem for Weather and Climate




TRMM Multisatellite Precipitation Analysis (TMPA)




- **Wet-dry biases:**
  - Over the Andes
  - Amazon
  - South-East of Brazil

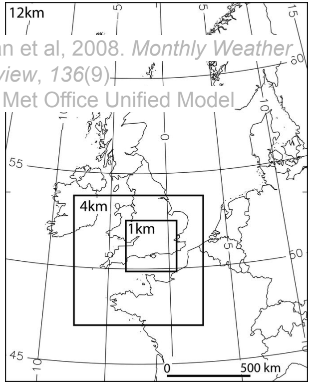

Figueroa, S. N., Bonatti, J. P., Kubota, P. Y., Grell, G. A., Morrison, H., Barros, S. R., ... & Panetta, J. (2016). The Brazilian global atmospheric model (BAM): performance for tropical rainfall forecasting and sensitivity to convective scheme and horizontal resolution. *Weather and Forecasting*, *31*(5), 1547-1572.

# Locally refined grids

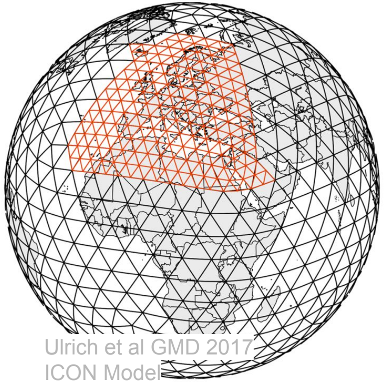
## HPC Scalability Bottleneck

- Move away from:
  - Globally coupled **spectral model** -> scalability problems at high resolution
  - **Latitude-longitude** -> the “pole problem”

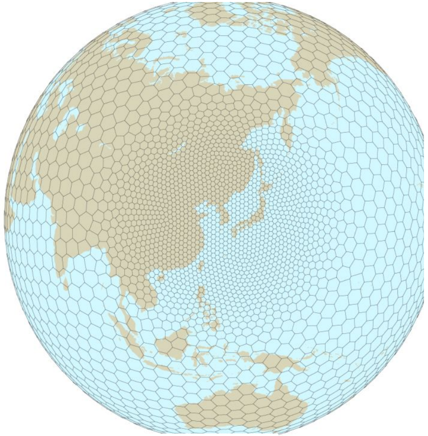
- Move towards:
  - Quasi-uniform grids (icosahedral, cubed-sphere, Voronoi, Ying-Yang, ...)
  - Finite-volume and finite/spectral-element methods



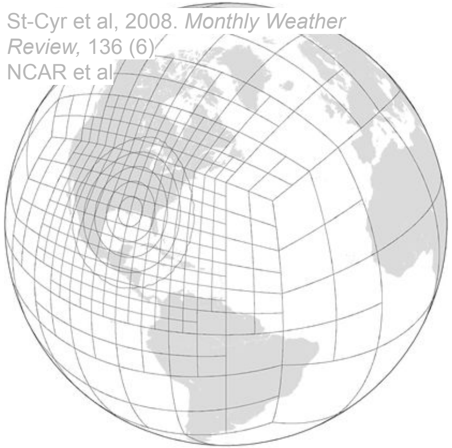

Lean et al, 2008. *Monthly Weather Review*, *136*(9)
UK Met Office Unified Model



Ulrich et al GMD 2017
ICON Model



MPAS Training Workshop 2012



St-Cyr et al, 2008. *Monthly Weather Review*, 136 (6)
NCAR et al

# Goals

Investigate a simplified numerical model using **locally refined grids** capturing the Andes Range

Focus on:

- Model of Prediction Across Scales (MPAS)
- Voronoi grids (flexibility)
- Local refinement over South America featuring the Andes Range
- Low order finite volume (TRiSK)
- Shallow Water Equations **with Moist**


https://www.alpineguides.co.nz/aconcagua-expedition

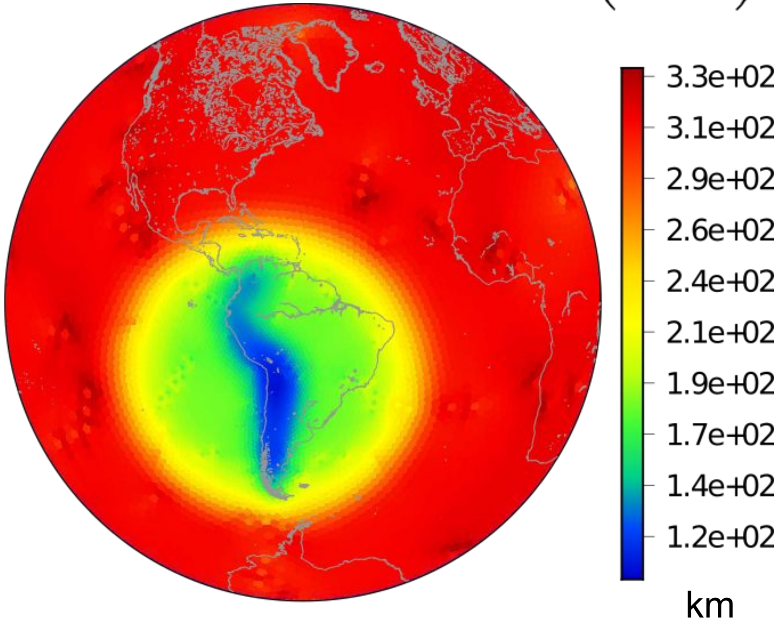
What issues one encounters solely due to grid changes?

# Locally Refined Grid Over the Andes

Iterative Centroidal Voronoi Method
(Lloyd's method)

Density function

$$\rho(x) = \frac{1}{\gamma^4} + \left(1 - \frac{1}{\gamma^4}\right)(\lambda s(x) + (1-\lambda) b(x))$$




$$\gamma = 3$$

$$\lambda = 0.6$$

$$\frac{h_i}{h_j} \approx \left(\frac{\rho(x_j)}{\rho(x_i)}\right)^{\frac{1}{4}}$$

Smoothed ETOPO data b(x) (Jacobi iterations) and Smoothed Continent Region s(x) (moving average)
Important for regularity : Ensures all triangles contain their circumcenters!!!

# Shallow Water Model and FV scheme

- Model for Prediction Across Scales (MPAS)
  - Horizontal discretization: TRiSK [1]
  - Mimetic properties (conservation, geometric)
  - Sensitive to grid distortions (grid-imprinting)[2]

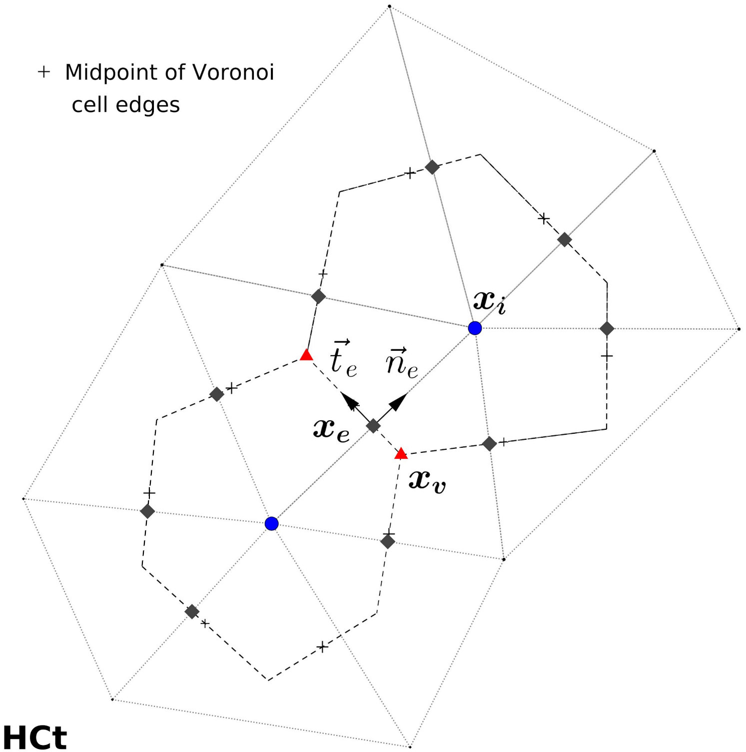
$$\frac{\partial h}{\partial t} + \nabla \cdot (h\vec{u}) = 0,$$

$$\frac{\partial \vec{u}}{\partial t} + qh\vec{u}^{\perp} = -g\nabla(h+b) - \nabla K,$$

Discrete Finite Volume Operators

$$\frac{\partial h_i}{\partial t} = -[\nabla \cdot (h\mathbf{u})]_i,$$

$$\frac{\partial u_e}{\partial t} = -[qh\mathbf{u}^{\perp}]_e - [\nabla B]_e,$$



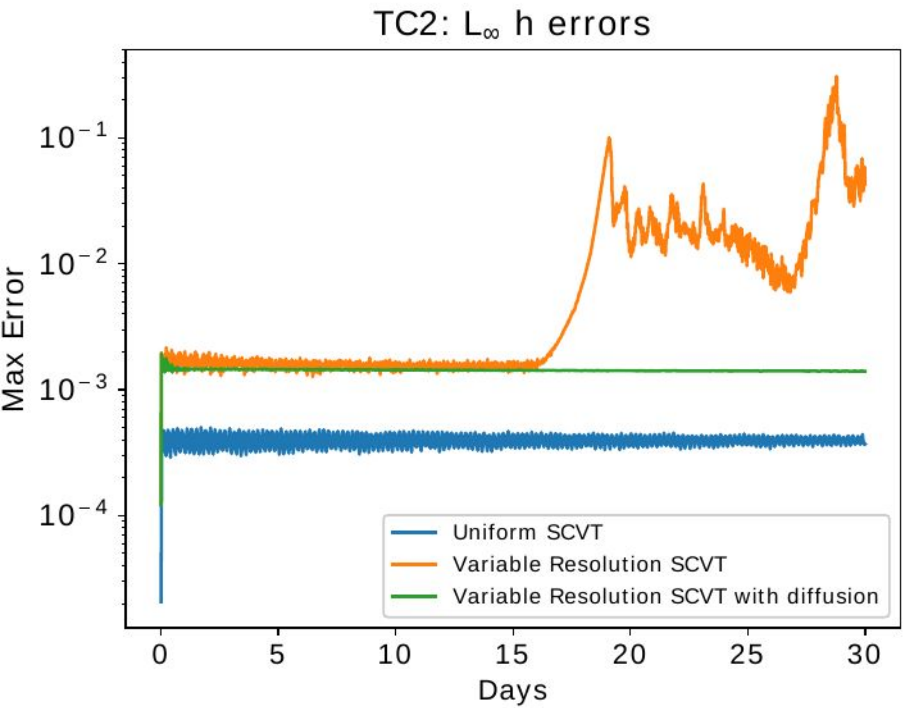

**HCt**

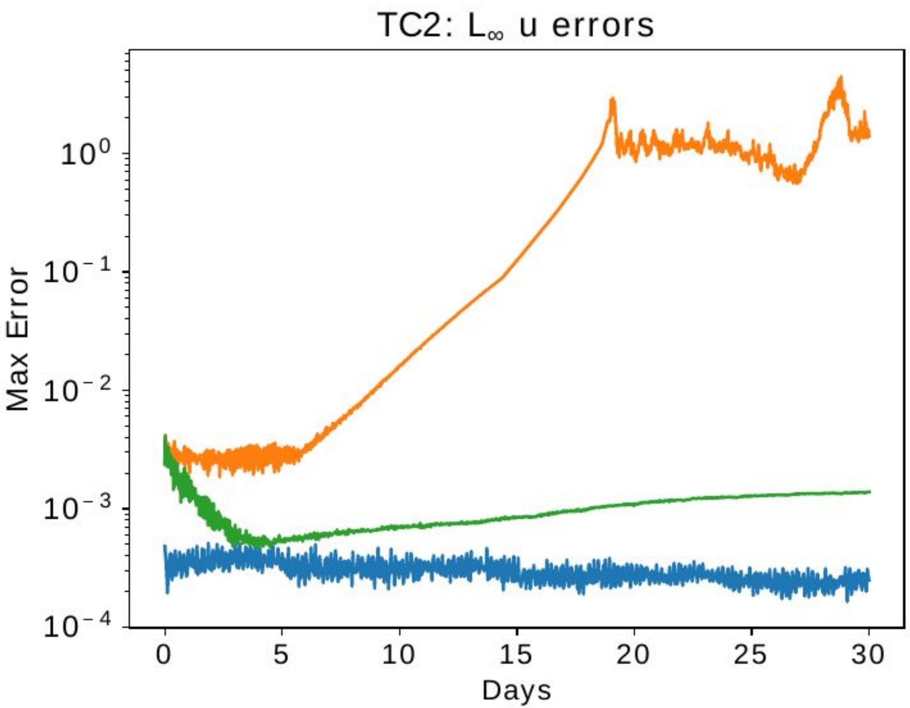
[1] Thuburn, Ringler, Skamarock, Klemp (2009). Journal of Computational Physics, 228(22), 8321-8335.

[2] Peixoto (2016). Journal of Computational Physics, 310, 127-160.

# Stability

Geostrophically balanced flow - Test Case 2 of Williamson et al (1992)



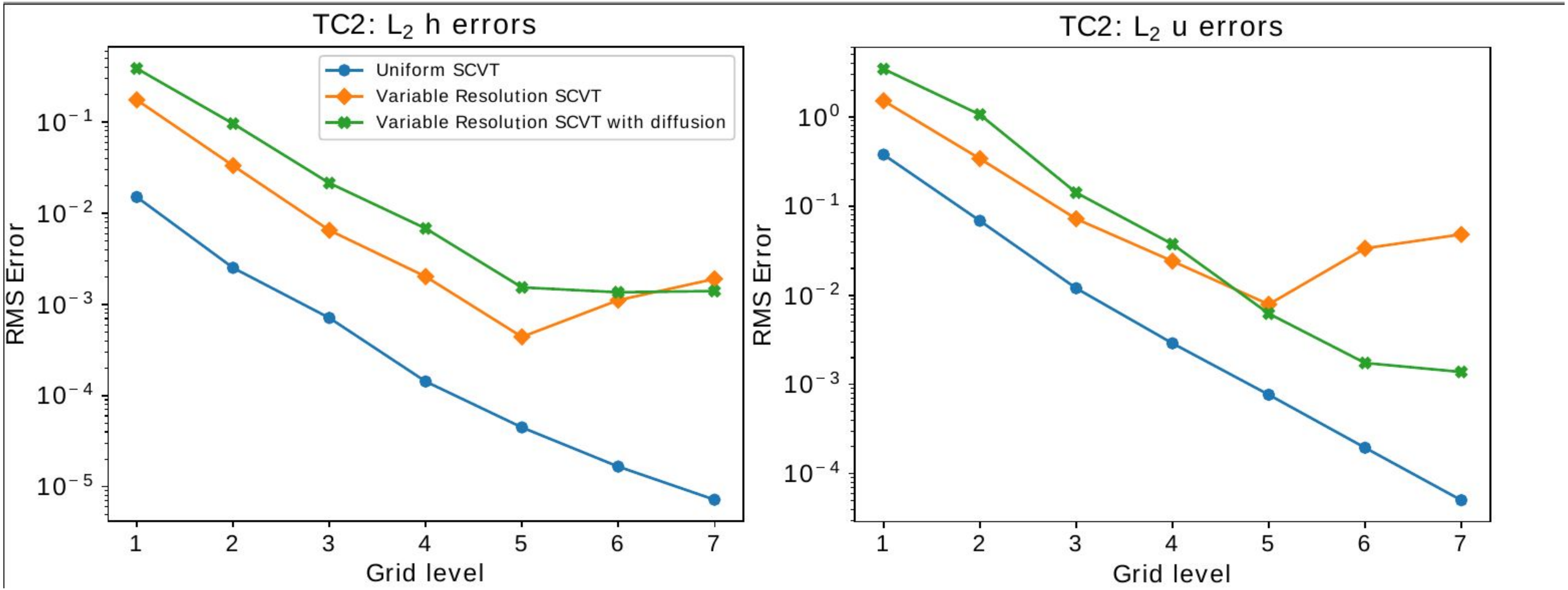




163842 nodes in all grids

Diffusion coefficient 8x10³ m²/s (approx what is used in T85 spectral models)

# Convergence

Geostrophically balanced flow - Test Case 2 of Williamson et al (1992)




Lacks convergence in Max Norm

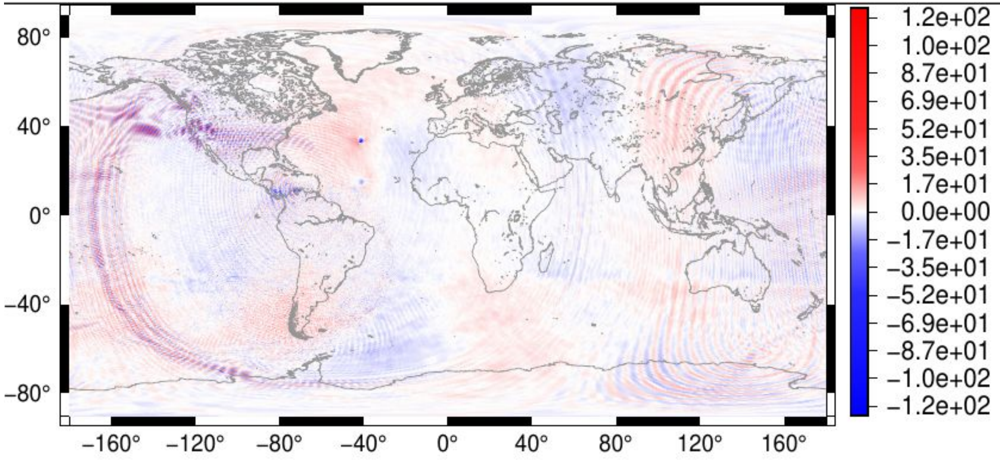
Diffusion coefficient $8 \times 10^3$ m²/s (approx what is used in T85 spectral models)

# Grid imprinting

Geostrophically balanced flow - Test Case 2 of Williamson et al (1992)

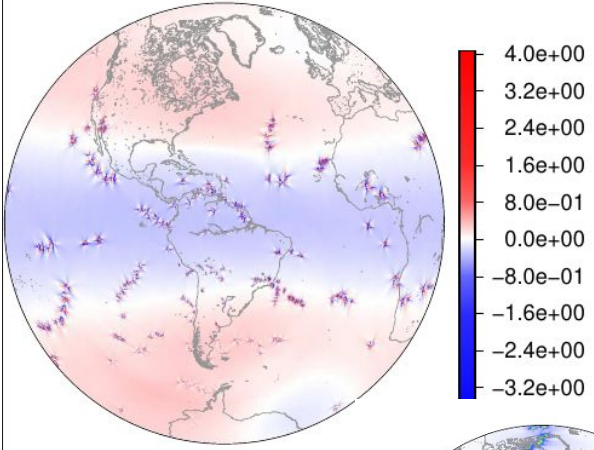
Locally refined (Andes) grid with 163842 nodes

(But no topography!!!)

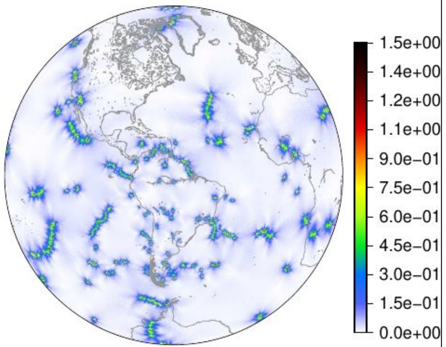



Error at day 30 (depth) - No diffusion

Error at day 30 (depth) - with diffusion




Alignment index

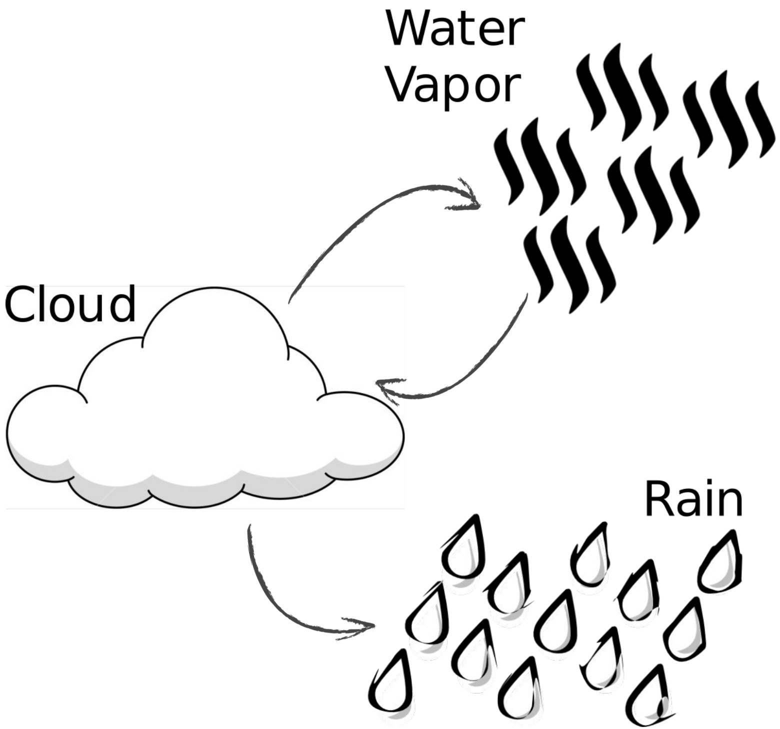



Peixoto (2016). Journal of Computational Physics, 310, 127-160.

# Moist Shallow Water Model

$$\frac{\partial \mathbf{u}}{\partial t} + qh\mathbf{u}^{\perp} + \nabla B = S_u,$$
$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0,$$
$$\frac{\partial h\theta}{\partial t} + \nabla \cdot (h\theta\mathbf{u}) = hS_\theta,$$
$$\frac{\partial hq^k}{\partial t} + \nabla \cdot (hq^k\mathbf{u}) = hS_q^k$$

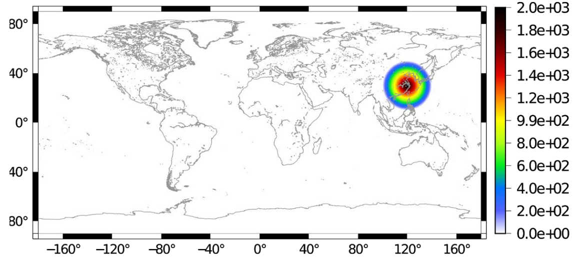
$\theta$ = temperature
$q^1$ = water vapor state
$q^2$ = cloud state
$q^3$ = rain state.



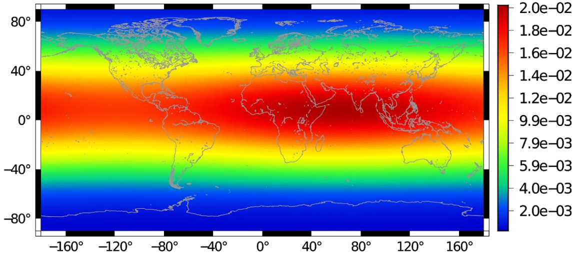

Zerroukat, M. and Allen, T. (2015). Journal of Computational Physics, 290

# Flow Over a Mountain (not the Andes)

Only the grid is refined to follow the Andes Range, but the mountain used here is a circle “bump”

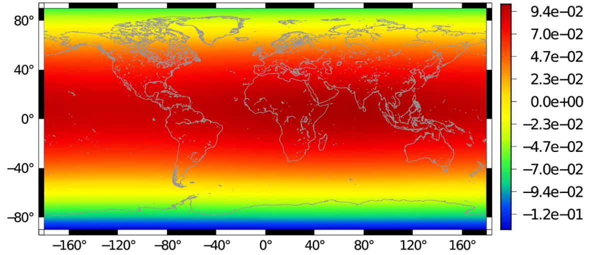



Topography

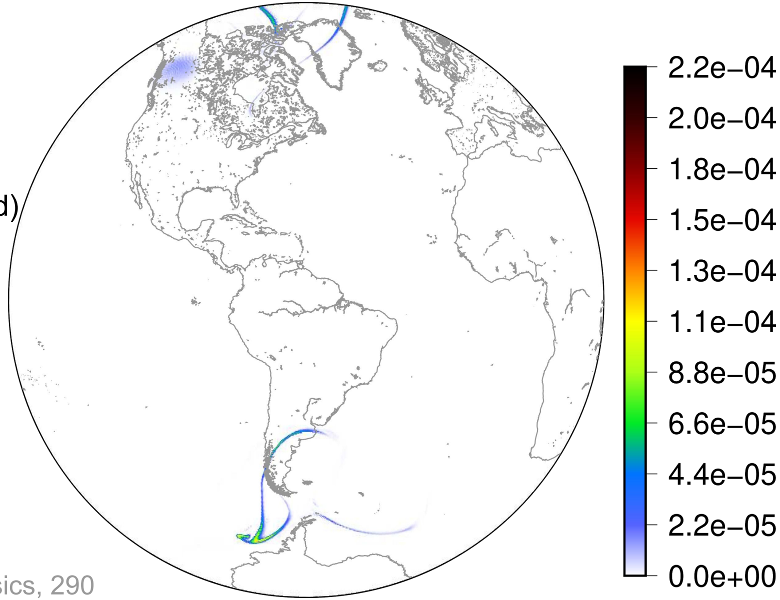



Initial Temperature

Initial vapor



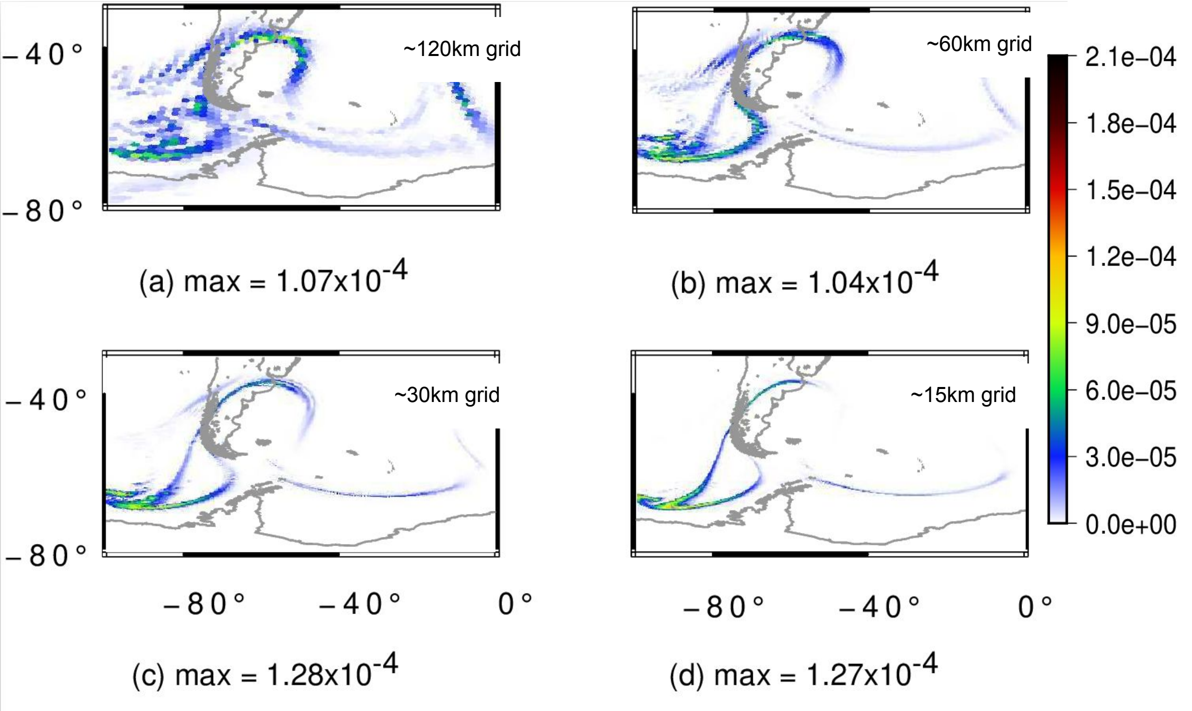

Rain after 30 days (UR 15km SCVT grid)



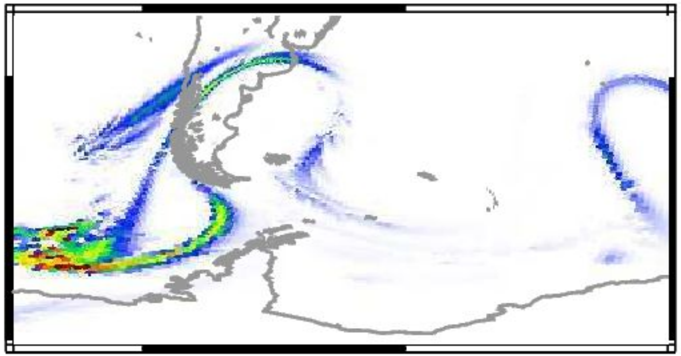

Zerroukat, M. and Allen, T. (2015). Journal of Computational Physics, 290

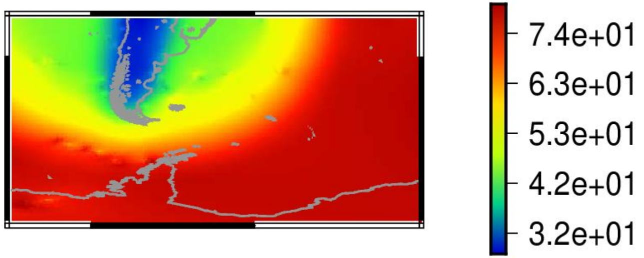
# Precipitation (Rain) - 30 days

Uniform grids



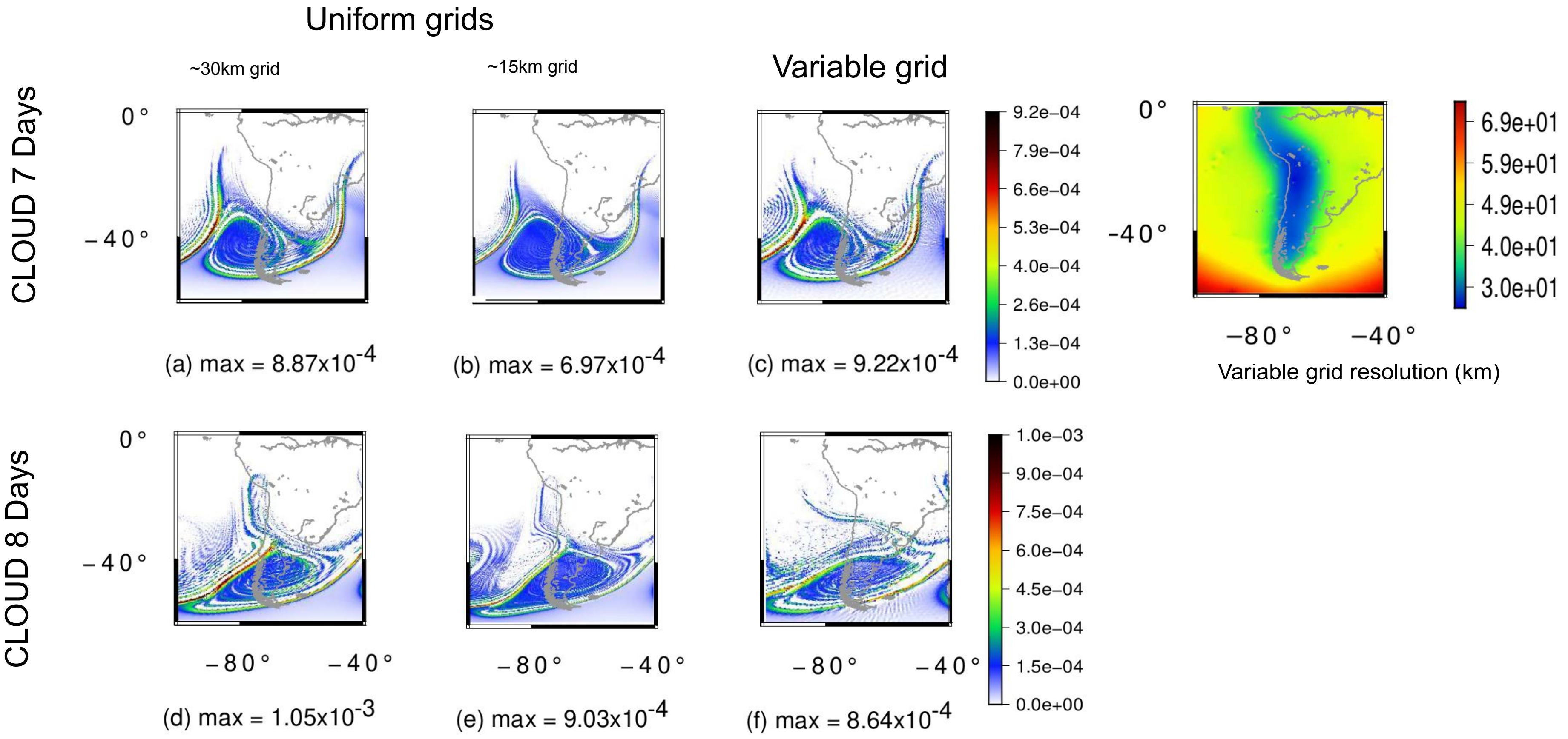

Variable grid






Variable grid resolution (km)

# Unstable Barotropic Jet (no topography)



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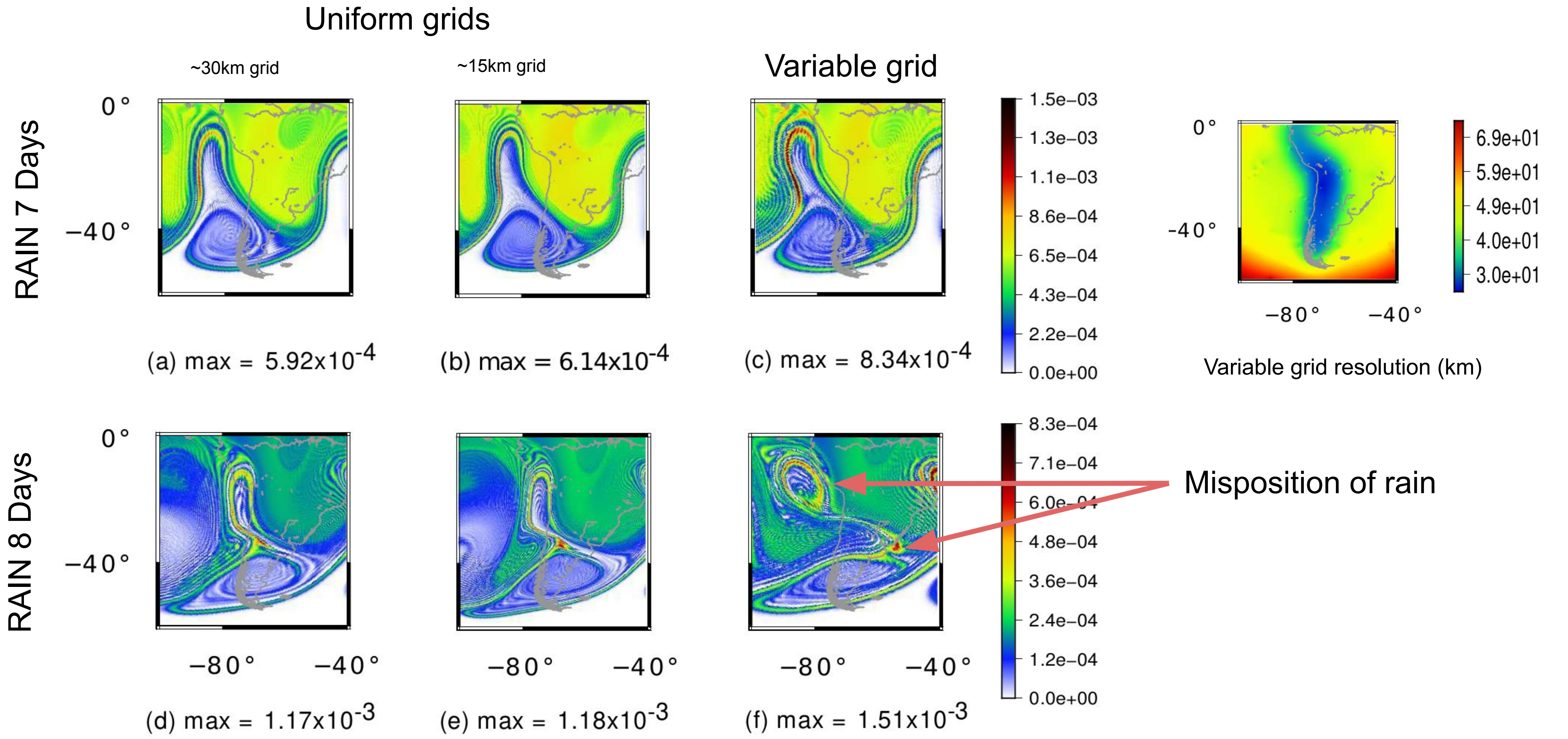

# Take away message

We are not focusing on the usual analysis of benefits of local refinement with narrow timeframes, but ...

... rather on the grid effects of usual global simulations on mid-range weather forecast windows.

- Be careful with grid quality
- Spurious numerical waves -> requires small amount of diffusion
- Spurious cloud/rain patterns due solely to grid influence

Possible gains of local refinement may be weakened due to added grid irregularity issues

# Acknowledgements


- Grants from FAPESP, CNPq, CAPES
- More at http://pedrosp.ime.usp.br


Thanks!


Contact for collaboration/advisory: ppeixoto@usp.br