Current trends for global atmosphere dynamical core development

Traditional and modern approaches

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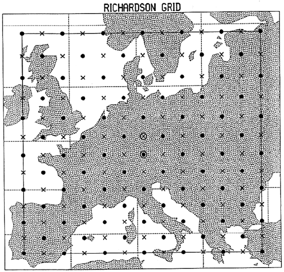
Summary

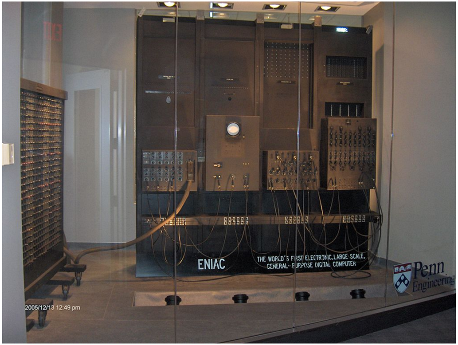
Beginning

- Lewis Fry Richardson (1922) *Weather Prediction by Numerical Process*
    - Primitive equations
    - Finite differences (staggered E-grid)
    - Regional: Europe
    - 2 years of (hand) calculation
    - Problems with initial data
    - See Lynch (1999) review *Richardson's marvellous forecast*



Early days

- 1950-1960 - Beginning of regular computer aided forecasting
    - Computers, ENIAC
    - More/better surveillance data
    - Primitive equations
    - Finite differences



Governing equations - Dynamics

Compressible Euler equations for atmosphere (ideal gas) in vector form:

$$\begin{aligned}\frac{D\mathbf{u}}{Dt} &= -2\,\mathbf{\Omega}\times\mathbf{u} - \frac{1}{\rho}\nabla p + \mathbf{g} + \mathbf{F}_r \;\; \text{(Momentum)}\\ \frac{D\rho}{Dt} &= -\rho\nabla\cdot\mathbf{u} \;\; \text{(Continuity)}\\ c_v\frac{DT}{Dt} &= -\frac{p}{\rho}\nabla\cdot\mathbf{u}\,\text{(Thermodynamics)}\end{aligned}$$

- $\mathbf{u} = (u, v, w)$: wind velocity
- p: pressure
- ρ: density
- T: temperature
- $D/Dt = \partial/\partial t + \mathbf{u}\cdot\nabla$: Material derivative

Governing equations

- Compressible Euler
  - Incompressible
  - Hydrostatic vs Non-hydrostatic
  - Shallow atmosphere vs Deep atmosphere
- Primitive equations: hydrostatic and shallow atmosphere
  - Quasigeostrophic equations
  - Shallow water equations
    - Barotropic vorticity equations
  - Passive transport equation

Summary

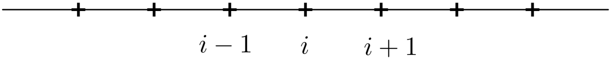
Basic Finite Difference

Example: Transport equation 1D :

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$$

Finite differences: Change partial derivatives with finite deviations

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_{i-1}}{\Delta x}$$



Main concerns: Accuracy and stability.

Global

Latitude and Longitude Grids
with Finite Differences

- Traditional Eulerian
  - Stability usually requires $\Delta t \propto \Delta x$
  - Pole requires Δt very small
- Semi-Lagrangian semi-implicit
  - Allows large Δt
  - Solve a very large linear system at each time-step

EndGame - UK MetOffice

- Even Newer Dynamics for General Atmospheric Modelling of the Environment - Met. Office
- Global latitude - longitude grid
- Differences on C-Grid (with some Finite Volume)
- Semi-implicit Semi-Lagrangian
- Two-time level scheme - iterations for correction
- Non Hydrostatic / Deep Atmosphere
- Terrain Following (Height based) Vertical Coordinate
- Operational (17 km resolution from 07/2014 - time-step 450 s)

Problems...

Weather forecasting needs to be done within a short time windows (1 or 2 hours wall clock time)

Massively Distributed Memory Parallel Machines

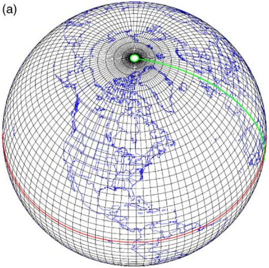
- Pole communicates with many other computer nodes
- A lot of global communication required for the solution of the global linear system
- Limited scalability on large supercomputers (cannot do the forecast within the time window)

GEM

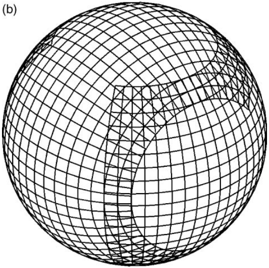
- Global Environmental Multiscale Model - Environment Canada / CMC
- Staggered finite differences
- Global latitude - longitude grid
- Grid Stretching for variable resolution
- Semi-implicit Semi-Lagrangian
- Non Hydrostatic / Shallow Atmosphere
- Terrain Following Vertical Coordinate

- GEM Yin-Yang with 2 way coupling.

(a)



(b)



Summary

Spectral methods

Emerged around 1960-1970.

- Derivatives are calculated in spectral space
- Fourier Transforms

$$\phi(x) = \sum_k \hat{\phi}_k e^{2\pi ikx}$$

- Derivatives ($\frac{\partial \phi}{\partial x}$):
  - Given a vector of values of $\phi = [\phi_i]$
  - Calculate Fast Fourier Transform FFT to obtain $\hat{\phi} = [\hat{\phi}_k]$
  - Calculate derivatives (in spectral space, simply multiply by $2\pi ik$)
  - Return to physical space with Inverse FFT
- 1970s: Viability for Atmosphere shown by Eliasen et al (1970) & Orszag (1970)

Example

1D transport with constant speed (c) and periodic boundaries:

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial x} = 0$$

Substituting the Fourier transform $\phi(t,x) = \sum_k \hat{\phi}_k(t)e^{2\pi ikx}$ into the transport equation, results in

$$\sum_k \frac{\partial \hat{\phi}_k(t)}{\partial t} e^{2\pi ikx} + c\sum_k \hat{\phi}_k(t)\frac{\partial e^{2\pi ikx}}{\partial x} = 0$$

Using that $\frac{\partial e^{2\pi ikx}}{\partial x} = 2\pi ike^{2\pi ikx}$ we have

$$\sum_k \left(\frac{\partial \hat{\phi}_k(t)}{\partial t} + 2\pi ikc\hat{\phi}_k(t)\right) e^{2\pi ikx} = 0$$

Example

1D transport with constant speed (c):

1. FFT ϕ at initial time
2. Solve $\frac{\partial \hat{\phi}_k(t)}{\partial t} + 2\pi ikc\hat{\phi}_k(t) = 0$ for every k with your favourite time-stepping scheme to obtain $\hat{\phi}(t)$ for future times
3. IFFT $\hat{\phi}(t)$ to obtain $\phi(t)$

Very accurate!

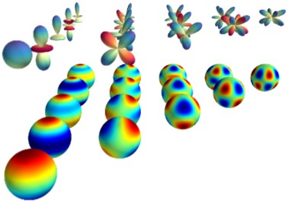
Spectral methods

What about doing this on the sphere?

- Spherical harmonics: Fourier expansion for each latitude circle, Legendre polinomials on meridians

$$\Upsilon_n^m(\lambda, \theta) = e^{-im\lambda} P_n^m(\sin\theta)$$

$$P_n^m(\mu) = \frac{1}{\sqrt{2}} \frac{(1-\mu^2)^{|m|/2}}{2^n n!} \frac{d^{n+|m|}(1-\mu^2)}{d\mu^{n+|m|}}.$$



Spectral methods

- Spherical harmonics with Fast Fourier Transform and “Fast” Legendre tramsforms
- Pseudo-spectral method
- Avoids the requirement of special treatment at the poles
- Semi-implicit is easier in spectral space
- With also Semi-Langrangian : allows large Δt!
- Very accurate!
- Used in most operational Weather Forecasting models and in many Climate models (BAM, IFS, ...).

IFS Model

- Integrated Forecasting System - ECMWF
- Global Spectral Model - Triangular Truncation
- Gaussian Reduced (Linear) Grid
- Semi-implicit Semi-Lagrangian
- Two-time level scheme
- Transposition Approach for Parallelization
- Developed Fast Legendre Transforms

Introduction – A history

- Resolution increases of the deterministic 10-day medium-range Integrated Forecast System (IFS) over ~28 years at ECMWF:
  - 1983: T 63 (~316km)
  - 1987: T 106 (~188km)
  - 1991: T 213 (~95km)
  - 1998: T_L319 (~63km)
  - 2000: T_L511 (~39km)
  - 2006: T_L799 (~25km)
  - 2010: T_L1279 (~16km)
  - 2015: T_L2047 (~10km) Hydrostatic, parametrized convection
  - 2020-???: (~1-10km) Non-hydrostatic, explicit deep convection, different cloud-microphysics and turbulence parametrization, substantially different dynamics-physics interaction…

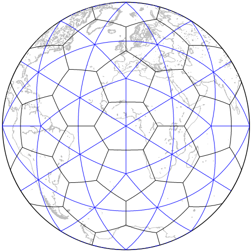
Problems...

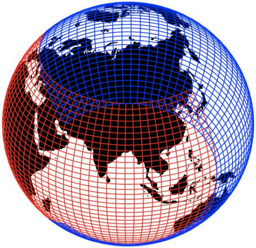
- Most of the computational time is spent solving the Spherical Harmonics transform (Legendre + Fourier).
- This part implies in a global communication, which reduces its scalability
- We might not be able to fit the necessary time windows for very high resolution models.

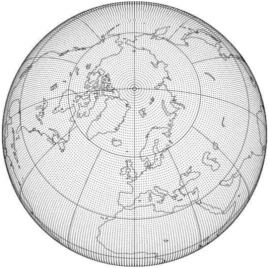
Summary

Search for alternatives - more isotropic grids

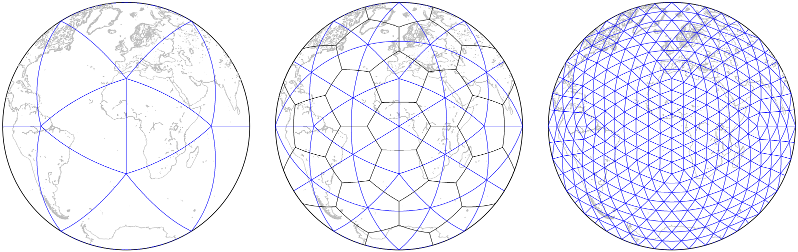
- Icosahedral (triangular / hexagonal)
- Cubed Sphere
- Yin-Yang Grids
- Reduced Gaussian grid







Icosahedral grids



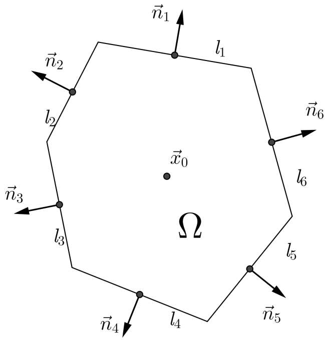
- May be used as triangular or Hexagonal/Pentagonal grid
- May be optimized (Spring Dynamics, Centroidal Voronoi, HR95)
- May be locally refined (Hierarchically or with Centroidal optimizations)
- But are not perfectly isotropic ...

Example

Horizontal continuity equation (Shallow water model)

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\vec{u}) = 0$$

- h is the fluid depth
- $\vec{u} = (u, v)$ is the fluid horizontal velocity

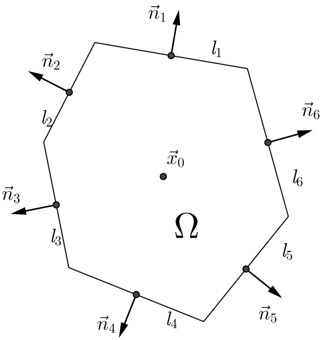


Divergence theorem

$$\nabla \cdot (h\vec{u})|_{\vec{x}_0} \approx \frac{1}{|\Omega|}\int_\Omega \nabla \cdot (h\vec{u})\, d\Omega$$

$$(DivThm) = \frac{1}{|\Omega|}\int_{\partial\Omega} h\vec{u}\cdot\vec{n}\, d\partial\Omega$$

$$\approx \frac{1}{|\Omega|}\sum_{i=1}^{n} h_i\vec{u}_i \cdot \vec{n}_i\, l_i.$$



$$\frac{\partial h}{\partial t} = -\frac{1}{|\Omega|}\sum_{i=1}^{n} h_i\vec{u}_i \cdot \vec{n}_i\, l_i$$

Interpolations required to obtain h_i and $\vec{u}_i$ depending on the staggering (A,C,...)

Problems...

- Can we get all the nice properties obtained in finite difference models and also scalability?

Desired:

1. Accurate
2. Stable
3. Conservative (mass, energy, PV, axial-angular momentum)
4. Mimetic Properties (spurious modes)

And also:

- Scalable on supercomputers
- Arbitrary spherical grids

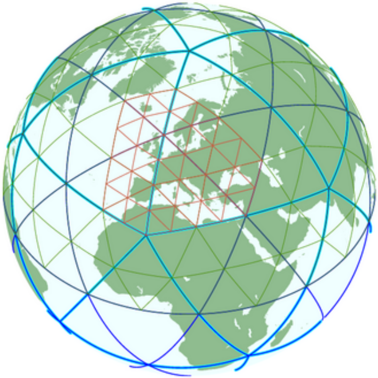
Is it possible?

Let's see some models with Finite Volume or Finite Differences on quasi uniform grids...

ICON

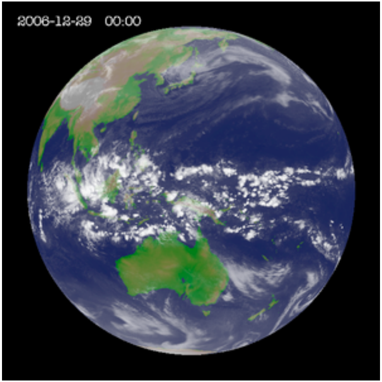
- Icosahedral non-hydrostatic
- MPI-M and DWD
- Triangular C grid
- Conservation of mass
- Highly scalable
- Hierarchically local refinement
- Spring dynamics optimization

- ICON-IAP (University of Rostock): Uses Hexagons



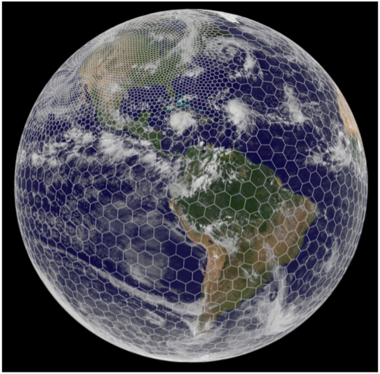
NICAM

- Nonhydrostatic ICosahedral Atmospheric Model
- RIKEN, JAMSTEC, University of Tokyo
- Hexagonal/pentagonal A grid
- Spring dynamics
- Highly scalable (3.5km, 15s)
- Operational
- JCP 2008 paper: Global cloud resolving simulations
- `https://earthsystemcog.org/projects/dcmip-2012/nicam`



MPAS

- Model for Prediction Across Scales
- NCAR and Los Alamos Nat Lab
- Spherical Centriodal Voronoi Tesselations (Smooth local refinement)
- Voronoi C grid (Hexagonal/Pentagonal)
- Fully mimetic
- Highly scalable
- Non-hydrostatic
- MWR 2012 paper: Multiscale Nonhydrostatic Atmospheric Model
- `http://mpas-dev.github.io/`

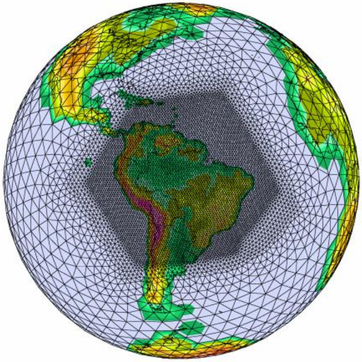


FV3

- Finite Volume Cube ([3])
- Geophysical Fluid Dynamics Laboratory-NOAA
- Shallow Atmosphere (plans for deep)
- Gnomonic Cubed - non orthogonal
- Finite Volume
- D-grid, with C-grid winds used to compute fluxes
- Vertical mass based Lagrangian
- Refinement: stretching and two-way nested grid

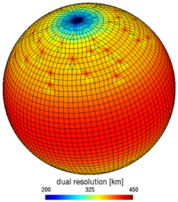
OLAM

- Ocean Land Atmosphere Model
- University of Miami / Colorado State University
- Non-hydrostatic / Deep Atmosphere
- Triangular / Hexagonal grids (possible refinements)
- Vertical Coordinate / Cut Cells
- Operational - US Environmental Protection Agency
- Split / Explicit time-stepping

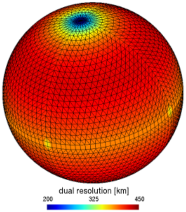


IFS-FV

Finite Volume schemes from Computational Fluid Dynamics models.



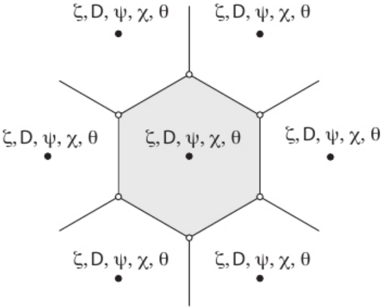
"classical" reduced Gaussian grid (N24)



reduced octahedral Gaussian grid (N24)

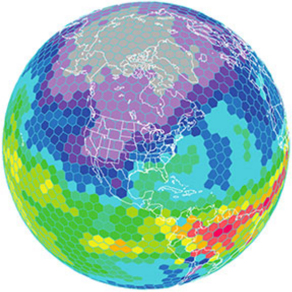
UZIM

- Unified Z-grid Icosahedral Model
- Colorado State University, Fort Collins
- Non-hydrostatic
- Heikes and Randall (1995) grid optimization
- Vorticity-Divergence Z-grid (Randall (1994))
- Less computational modes
- Multigrid solver



NIM

- Non-hydrostatic Icosahedral Model
- NOAA/ESRL
- GPU and MIC(Intel)
- Icosahedral - optimized - hexag/pentag
- Unstaggered finite-volume (A-grid)
- Local coodinate system - Flow following
- Time: RK4
- HEVI
- Vertical : Height based
- Shallow Atmosphere



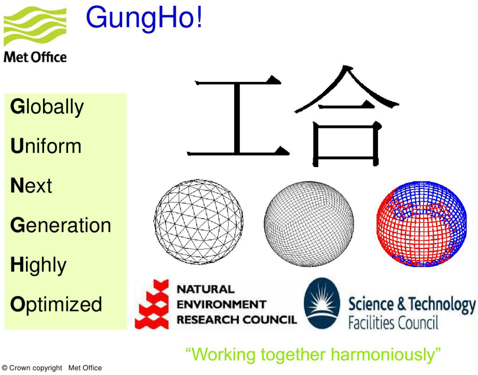
Summary

Finite elements

- Traditional Finite element
- Spectral Elements
- Discontinuous Galerkin
- Mixed finite elements

(More complicated to give simple examples...)

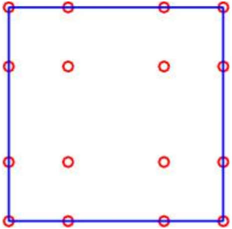
Gung Ho project- UK MetOffice



- Mixed Finite Elements - Fully mimetic - Cubed Sphere grid
- Finite Volumes advection
- Challenges: Quadrature - Mass Matrix inversion - Solver - ....

CAM-SE Model

- Community Atmosphere Model - Spectral Element - NCAR
- Operational - Hydrostatic - Shallow Atmosphere
- Continuous Galerkin Formulation - Cubic Polynomials
- Gauss-Lobato Quadrature
- Runge-Kutta time integration
- Hybrid Vertical coordinate (terrain following)
- Hyperviscosity
- Highly Scalable parallelism
- Hydrostatic
- Developing non-hydrostatic



NUMA

- NRL (Navy)
- Element-based Galerkin methods (continuous or discontinuous high-order)
- Mesoscale (limited-area) or global model
- Grid: Any rectangular based (cubed sphere)
- Multiple methods (modular): IMEX, RK, ...
- Highly scalable

numa
Non-hydrostatic Unified Model of the Atmosphere

Other models

- DYNAMICO: France LMD-Z part of IPSL-CM Earth System Model - Mimetic Finite Volume (still hydrostatic - going non-hydrostatic) - Focused in Climate.
- KIAPS: Korea Spectral Element
- Apologies for those that I forgot...

Summary

Conclusion

"All models are wrong but some are useful"

— George Box

Thank you!
Questions?