

MAT 144 - IOUSP - Cálculo I - Período Diurno

9ª Lista de Exercícios - 1º semestre de 2010

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1. Calcule:

a) $\int_0^1 (3x + 1)^4 \, dx$

b) $\int_{-3}^4 \sqrt[3]{5 - x} \, dx$

c) $\int_1^2 \frac{2}{(3x - 2)^3} \, dx$

d) $\int_0^1 xe^{x^2} \, dx$

e) $\int_{-1}^0 x\sqrt{x+1} \, dx$

f) $\int_0^{\pi/3} \cos 2x \, dx$

g) $\int_0^1 \frac{x^2}{1+x^3} \, dx$

h) $\int_0^1 \frac{x^2}{(1+x^3)^2} \, dx$

i) $\int_{-1}^0 x^2\sqrt{1+x^3} \, dx$

j) $\int_0^1 \frac{x}{(x+1)^5} \, dx$

k) $\int_{-1}^0 x(x+1)^{100} \, dx$

l) $\int_1^2 x^2(x-2)^{10} \, dx$

2. Calcule $\int_{-\pi}^{\pi} \frac{\sin x}{x^4 + x^2 + 1} \, dx$.

3. Calcule:

a) $\int_0^1 x\sqrt{x^2 + 3} \, dx$

b) $\int_1^2 x(x^2 - 1)^5 \, dx$

c) $\int_{-1}^0 x^2 e^{x^3} \, dx$

d) $\int_1^2 \frac{3s}{1+s^2} \, ds$

e) $\int_0^1 \frac{1}{1+4s} \, ds$

f) $\int_0^3 \frac{x}{\sqrt{x+1}} \, dx$

g) $\int_0^1 \frac{s}{\sqrt{s^2 + 1}} \, ds$

h) $\int_0^3 \frac{x^2}{\sqrt{x+1}} \, dx$

i) $\int_0^1 \frac{x^2}{(x+1)^2} \, dx$

j) $\int_{-1}^{+1} x^3(x^2 + 3)^{10} \, dx$

k) $\int_0^{\sqrt{3}} x^3\sqrt{x^2 + 1} \, dx$

l) $\int_0^{\pi/3} \sin x \cos^2 x \, dx$

m) $\int_0^{\pi/6} \cos x \sin^5 x \, dx$

n) $\int_{\pi/3}^{\pi/2} \sin^3 x \, dx$

4. Calcule:

$$\text{a)} \int \sqrt[5]{x^2} dx$$

$$\text{b)} \int \frac{1}{x^3} dx$$

$$\text{c)} \int \frac{x+x^2}{x^2} dx$$

$$\text{d)} \int \left(x^2 + \frac{3}{x} \right) dx$$

$$\text{e)} \int (e^{2x} + e^{-x}) dx$$

$$\text{f)} \int \left(e^{4x} + \frac{1}{x^2} \right) dx$$

$$\text{g)} \int \frac{x^5 + x + 1}{x^2} dx$$

$$\text{h)} \int e^{\sqrt{2x}} dx$$

5. Calcule:

$$\text{a)} \int_0^1 e^{2x} dx$$

$$\text{b)} \int_{-1}^{+1} e^{-x} dx$$

$$\text{c)} \int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx$$

$$\text{d)} \int_1^2 \frac{x^3 + 1}{x} dx$$

6. Calcule:

$$\text{a)} \int_0^{\pi/3} (\sin 3x + \cos 3x) dx$$

$$\text{b)} \int_{-\pi/2}^{\pi/2} \cos\left(\frac{x}{2}\right) dx$$

$$7. \text{ a)} \text{ Verifique: } \sin^2 x = \frac{1 - \cos 2x}{2} \quad \text{e} \quad \cos^2 x = \frac{1 + \cos 2x}{2};$$

$$\text{b)} \text{ Calcule } \int \sin^2 x dx.$$

8. Calcule:

$$\text{a)} \int \cos^2 2x dx$$

$$\text{b)} \int \cos^2 5x dx$$

$$\text{c)} \int \sin^2 3x dx$$

$$\text{d)} \int \cos^2\left(\frac{x}{2}\right) dx$$

$$\text{e)} \int \cos^4 x dx$$

$$\text{f)} \int \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right)^2 dx$$

$$\text{g)} \int (\sin x - \cos x)^2 dx$$

$$\text{h)} \int (5 + \sin 3x)^2 dx$$

9. Calcule:

a) $\int (x + \sec^2 3x) dx$

b) $\int (1 + \sec x)^2 dx$

10. a) Determine α, β tais que $\sin 6x \cos x = \frac{1}{2}(\sin \alpha x + \sin \beta x)$;

b) Calcule $\int \sin 6x \cos x dx$;

c) Calcule $\int \sin(mx) \cos(nx) dx$; $m, n \in \mathbb{N}$.

11. a) Determine α, β tais que $\sin 3x \sin 2x = \frac{-1}{2}(\cos \alpha x - \cos \beta x)$;

b) Calcule $\int \sin 3x \sin 2x dx$;

c) Calcule $\int \sin(mx) \sin(nx) dx$; $m, n \in \mathbb{N}$.

12. a) Calcule $\int \cos 5x \cos 2x dx$;

b) Calcule $\int \cos(mx) \cos(nx) dx$; $m, n \in \mathbb{N}$.

13. Calcule:

a) $\int \sqrt{3x - 2} dx$

b) $\int \frac{dx}{3x - 2}$

c) $\int \frac{1}{(3x - 2)^2} dx$

d) $\int x \sin x^2 dx$

e) $\int x e^{x^2} dx$

f) $\int x^2 e^{x^3} dx$

g) $\int x^3 \cos x^4 dx$

h) $\int \cos^3 x \sin x dx$

i) $\int \frac{3x}{5 + 6x^2} dx$

j) $\int \frac{x}{(1 + 4x^2)^2} dx$

k) $\int x \sqrt{1 + 3x^2} dx$

l) $\int e^x \sqrt{1 + e^x} dx$

14. Calcule:

a) $\int_0^1 xe^{-x^2} dx$

b) $\int_0^{\pi/3} \sin^4 x \cos x dx$

c) $\int_1^2 \frac{x}{1+3x^2} dx$

d) $\int_0^1 \frac{x}{\sqrt{1+x^2}} dx$

e) $\int_0^1 \frac{x^3}{\sqrt{1+x^2}} dx$

f) $\int_{-\frac{3}{2}}^{-1} (2x+3)^{100} dx$

g) $\int_0^{\sqrt{\pi}} x \sin(3x^2) dx$

h) $\int_2^3 \frac{1}{(x-1)^3} dx$

i) $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} dx$

j) $\int_0^1 \frac{x}{1+x^4} dx$

15. Calcule:

a) $\int \sin^2 x \cos x dx$

b) $\int \sin^2 x \cos^3 x dx$

c) $\int \cos^3 x \sin^3 x dx$

d) $\int \sin x \sqrt{\cos x} dx$

e) $\int \sin 2x \sqrt{1+\cos^2 x} dx$

f) $\int \sin 2x \sqrt{5+\sin^2 x} dx$

g) $\int \sin^3 x dx$

h) $\int \cos^5 x dx$

i) $\int \tan^3 x \sec^2 x dx$

j) $\int \tan x \sec^2 x dx$

k) $\int \tan x \sec^3 x dx$

l) $\int \tan^3 x \sec^4 x dx$

m) $\int \sin x \sqrt{3+\cos x} dx$

n) $\int \sin x \sec^2 x dx$

o) $\int \sin x \sec^3 x dx$

p) $\int \sin^2 x \cos^2 x dx$

q) $\int \tan^3 x \cos x dx$

r) $\int \frac{\sec^2 x}{3+2\tan x} dx$

16. Calcule:

a) $\int \frac{2x+3}{x+1} dx$

b) $\int \frac{x^2}{x+1} dx$

17. Suponha α, β, m e n constantes, $\alpha \neq \beta$. Mostre que existem constantes A e B tais que $\frac{mx+n}{(x-\alpha)(x-\beta)} = \frac{A}{x-\alpha} + \frac{B}{x-\beta}$.

18. Calcule:

a) $\int \frac{1}{(x+1)(x-1)} dx$

b) $\int \frac{2x+3}{x(x-2)} dx$

c) $\int \frac{x}{x^2-4} dx$

d) $\int \frac{1}{x^2-4} dx$

e) $\int \frac{5x+3}{x^2-3x+2} dx$

f) $\int \frac{x+1}{x^2-x-2} dx$

g) $\int \frac{2}{x^2-5x+6} dx$

h) $\int \frac{x-3}{x^2+3x+2} dx$

19. Calcule:

a) $\int \frac{1}{2+5x^2} dx$

b) $\int \frac{3x+2}{1+x^2} dx$

c) $\int \frac{1}{1+(x+1)^2} dx$

d) $\int \frac{1}{x^2+2x+2} dx$

e) $\int \frac{1}{x^2+4x+8} dx$

f) $\int \frac{1}{x^2+x+1} dx$

20. Sejam $\alpha \neq 0$ e β constantes. Verifique:

a) $\int \frac{1}{x^2-\alpha^2} dx = \frac{1}{2\alpha} \ln \left| \frac{x-\alpha}{x+\alpha} \right| + k$

b) $\int \frac{1}{\alpha^2+(x+\beta)^2} dx = \frac{1}{\alpha} \arctg \left(\frac{x+\beta}{\alpha} \right) + k.$

21. Calcule:

a) $\int \frac{x^3}{(16+x^4)^3} dx$

b) $\int \frac{x^3}{16+x^4} dx$

c) $\int \frac{x}{16+x^4} dx$

d) $\int \operatorname{tg} 2x dx$

e) $\int \frac{1}{x \ln x} dx$

f) $\int \frac{1}{x(\ln x)^2} dx$ ^{ll}

g) $\int \operatorname{tg}^2 x dx$

h) $\int \frac{1}{\sqrt{1-x^2}} dx$

$$\text{i) } \int \frac{x}{\sqrt{1-4x^2}} dx$$

$$\text{j) } \int \frac{2x+3}{\sqrt{1-4x^2}} dx$$

$$\text{k) } \int \frac{2}{4-9x^2} dx$$

$$\text{l) } \int \frac{x}{\sqrt{1-x^4}} dx$$

$$\text{m) } \int \frac{e^x}{\sqrt{1-e^{2x}}} dx$$

$$\text{n) } \int \frac{e^x}{\sqrt{1-e^x}} dx$$

$$\text{o) } \int \frac{1}{x\sqrt{1-(\ln x)^2}} dx$$

$$\text{p) } \int \frac{2}{\sqrt{1-(x+1)^2}} dx$$

$$\text{q) } \int \frac{e^x}{1+e^{2x}} dx$$

$$\text{r) } \int \frac{e^x}{\sqrt{1+3e^x}} dx$$

$$\text{s) } \int \frac{1}{x} \cos(\ln x) dx$$

$$\text{n) } \int \frac{x^3}{1+x^8} dx$$

22. Esboce os gráficos de:

$$\text{a) } f(x) = \frac{x^2}{x^2 - x - 2}$$

$$\text{b) } f(x) = \frac{x^2 - x - 1}{x^2}$$