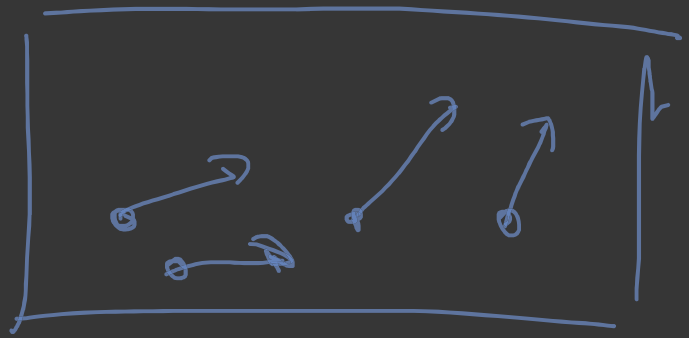
Campo vetorial



Defi Seja $D \subset \mathbb{R}^2 (\mathbb{R}^3)$

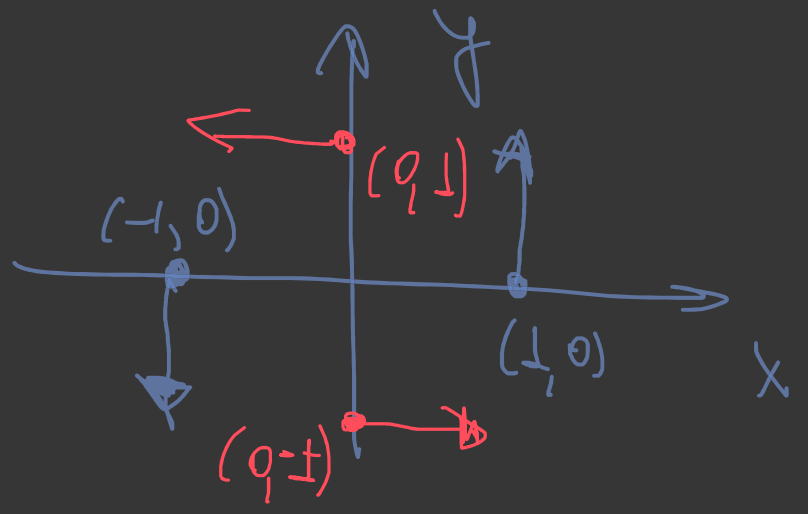
Um **campo vetorial** sobre $\mathbb{R}^2 (\mathbb{R}^3)$ é uma função que transforme todo ponto em D em vetor (no plano ou no espaço) : $\vec{F} : D \longrightarrow V^2 (V^3)$, ou seja

$\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$

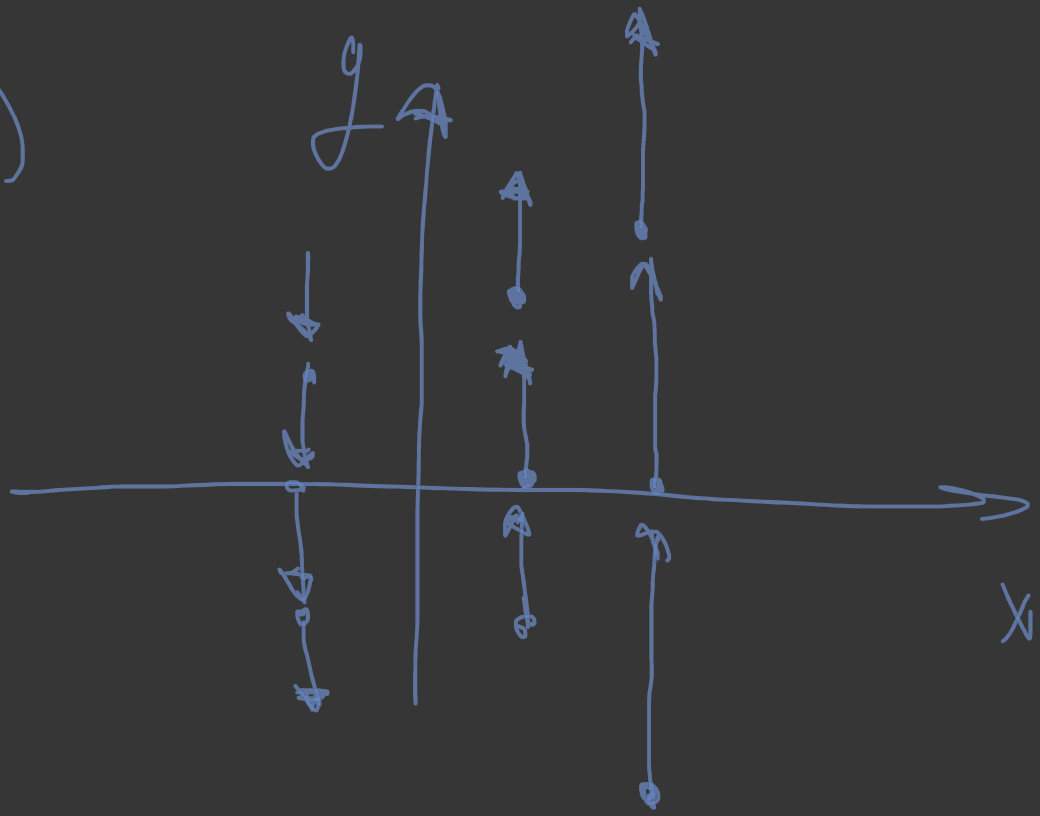
$$\left(\vec{F}(x,y,z) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}\right)$$

Ex 1 $\vec{F}(x,y) = -y\vec{i} + x\vec{j} = (-y, x)$

- $\vec{F}(1,0) = (0,1)$
- $\vec{F}(-1,0) = (0,-1)$
- $\vec{F}(0,1) = (-1,0)$
- $\vec{F}(0,-1) = (1,0)$

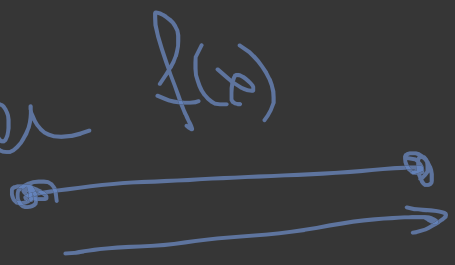


Ex 2 $\vec{F}(x,y) = (0,b)$



Integral de linha do campo vetorial

Lembre: $W = \int_a^b f(x)\,dx$ é o trabalho da força f que move uma partícula de a para b ao longo do eixo x

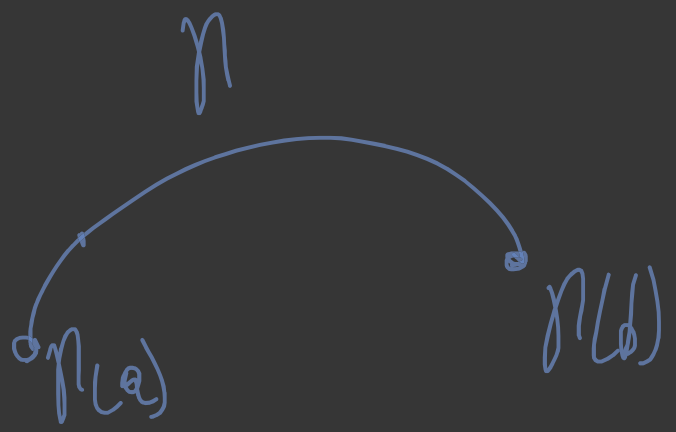


• Seja $\vec{F}$ força constante que move um objeto de X_1 para X_2 ao longo do segmento $\overline{X_1X_2}$ $\Rightarrow$

$$W = \vec{F} \cdot \overrightarrow{X_1X_2}$$

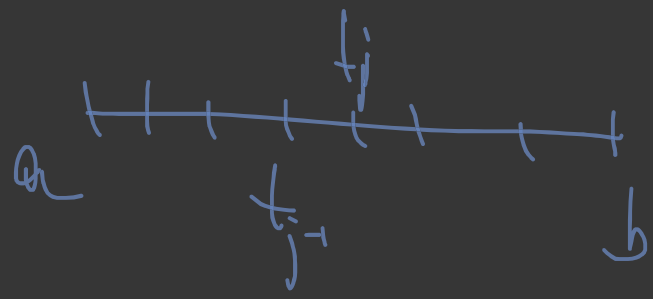
• Seja $\vec{F} : D \to V^2(V^3)$ um campo vet-l e $\gamma : [a,b] \to D$ uma curva. Queremos calcular o trabalho realizado por $\vec{F}$ no

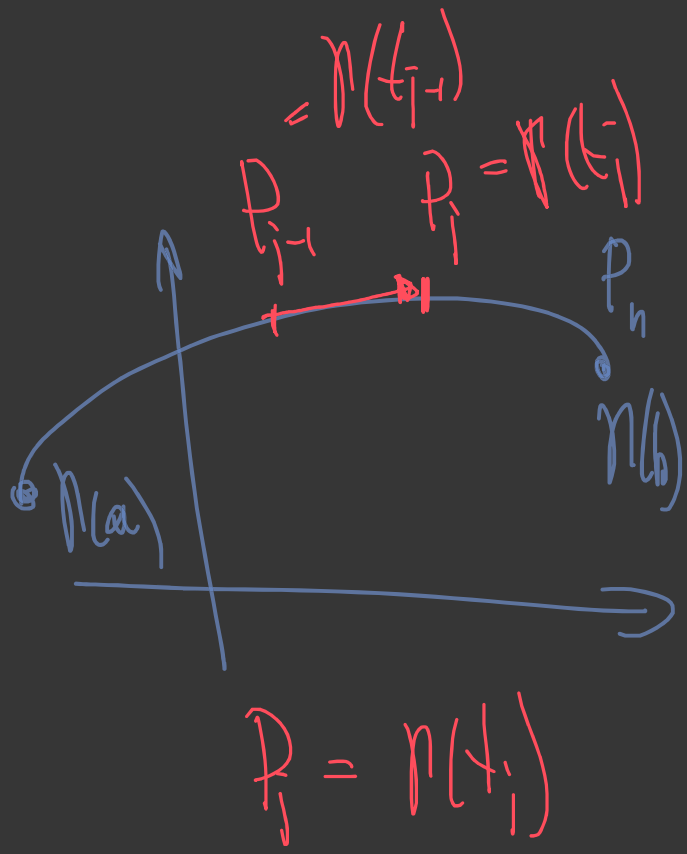
processo de movimento de uma partícula de $\gamma(a)$ para $\gamma(b)$



1) Partição de $[a,b]$

$a = t_0 < t_1 < \dots < t_n = b$





2) $W \approx \sum_{i=1}^{n} \vec{F}(P_{i-1}) \cdot \overrightarrow{P_{i-1}P_i} \Rightarrow$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \vec{F}(P_{i-1}) \cdot \overrightarrow{P_{i-1}P_i}$$

3) $\overrightarrow{P_{i-1}P_i} = (x(t_i) - x(t_{i-1}), y(t_i) - y(t_{i-1}))$ = T. do Valor médio

$= (x'(\tilde{t}_i)\Delta t_i, y'(\tilde{\tilde{t}}_i)\Delta t_i) \approx (x'(t_{i-1}), y'(t_{i-1}))\Delta t_i =$

$= \vec{\gamma}'(t_{i-1})\Delta t_i \quad , \quad \tilde{t}_i, \tilde{\tilde{t}}_i \in [t_{i-1}, t_i]$

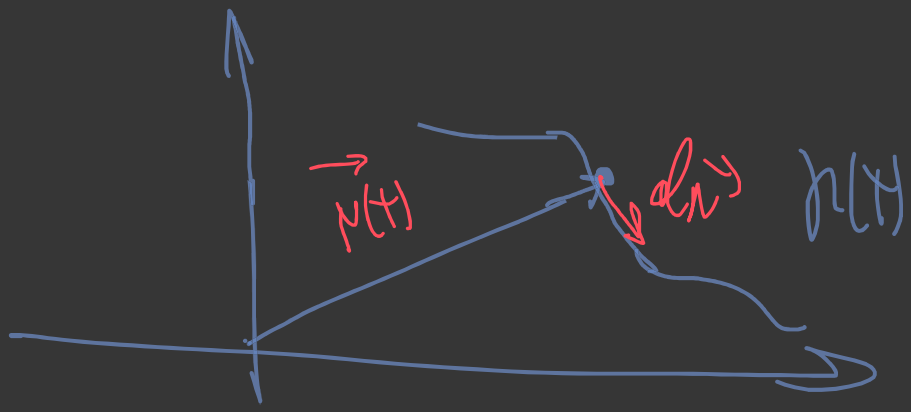
Finalmente, $\sum_{i=1}^{n} \vec{F}(P_{i-1})\overrightarrow{P_{i-1}P_i} \approx \sum_{i=1}^{n} F(\gamma(t_{i-1})) \cdot \underbrace{\gamma'(t_{i-1})}_{(x'(t_{i-1}),\, y'(t_{i-1}))} \Delta t_i$

Conclusão $W = \lim_{n\to\infty} \sum_{i=1}^{n} \vec{F}(\gamma(t_{i-1})) \cdot \gamma'(t_{i-1}) \Delta t_i =$

$$= \int_a^b \vec{F}(\gamma(t)) \cdot \gamma'(t)\, dt$$

Usaremos notação para $\gamma(t) = (x(t), y(t))$:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$$



Seja $\vec{F}: D \to V^2$ um campo vet-l e

$\gamma(t) = (x(t), y(t)) : D \to \mathbb{R}^2$ uma curva $\Rightarrow$

a integral de linha de $\vec{F}$ sobre γ

é $\int_\gamma \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\gamma(t)) \cdot \gamma'(t)\, dt$

Ex 2 Ache $\int_\gamma \vec{F} \cdot d\vec{r}$ sendo

1) $\vec{F}(x,y) = x\vec{i} + y\vec{j}$ e $\gamma(t) = (t, t^2)$, $t \in [-1, 1]$

$x(t)\ \ y(t)$

Temos $\int_\gamma \vec{F} \cdot d\vec{r} = \int_{-1}^{1} \vec{F}(\gamma(t)) \cdot \gamma'(t)\, dt$ e

$\vec{F}(\gamma(t)) = (t, t^2)$, $\gamma'(t) = (1, 2t)$ e

$\vec{F}(\gamma(t)) \cdot \gamma'(t) = t + 2t^3 \Rightarrow$

$$\int_\gamma \vec{F} \cdot d\vec{r} = \int_{-1}^{1} \left(t + 2t^3\right) dt = \left(\frac{t^2}{2} + \frac{t^4}{2}\right)\Big|_{-1}^{1} =$$

$$= 0$$

2) $\vec{F}(x,y,z) = x^2\vec{i} + y^2\vec{j} + z^2\vec{k} = (x^2, y^2, z^2)$,

$\gamma(t) = (\underbrace{2\cos t}_{x(t)}, \underbrace{3\operatorname{sen} t}_{y(t)}, \underbrace{t}_{z(t)})$, $t \in [0, 2\pi]$

Temos $\cdot\ \vec{F}(\gamma(t)) = \vec{F}(x(t), y(t), z(t)) =$

$= (4\cos^2 t, 9\operatorname{sen}^2 t, t^2)$

$\cdot\ \gamma'(t) = (-2\operatorname{sen} t, 3\cos t, 1)$ $\Rightarrow$

$\cdot\ \vec{F}(\gamma(t)) \cdot \gamma'(t) = (4\cos^2 t, 9\operatorname{sen}^2 t, t^2) \cdot (-2\operatorname{sen} t, 3\cos t, 1)$

$$= -8\cos^2 t \operatorname{sen} t + 2t \operatorname{sen}^2 t \cos t + t^2 \Rightarrow$$

$$\int_\gamma \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\alpha(t))\cdot \alpha'(t)\,dt = \int_0^{2\pi} \left(-8\cos^2 t \operatorname{sen} t + 2t \operatorname{sen}^2 t \cos t + t^2\right) dt$$

(*)

Obs $\int \cos^2 t \operatorname{sen} t\, dt = \left| \begin{array}{l} \cos t = z \\ dz = -\operatorname{sen} t\, dt \end{array} \right| =$

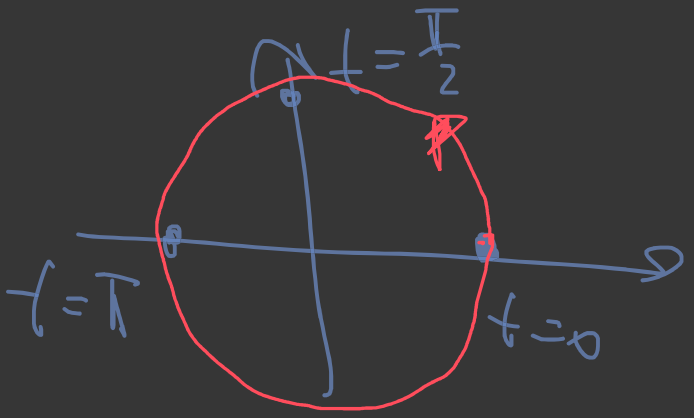
$$= \int -z^2\, dz = -\frac{z^3}{3} + C = -\frac{\cos^3 t}{3} + C$$

(*) $= \left(-8\cdot\left(-\frac{\cos^3 t}{3}\right) + 2t\cdot\frac{\operatorname{sen}^3 t}{3} + \frac{t^3}{3} \right)\Bigg|_0^{2\pi}$

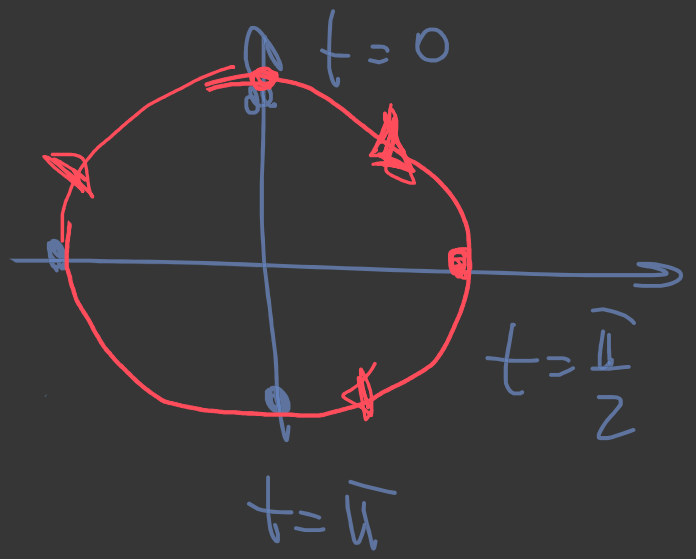
$$= \frac{(2\pi)^3}{3} = \frac{8\pi^3}{3}$$

Obs Lembre que a curva γ pode ter os sentidos de percurso diferentes

Por ex, 1) $\gamma(t) = (\cos t, \operatorname{sen} t)$, $t \in [0, 2\pi]$



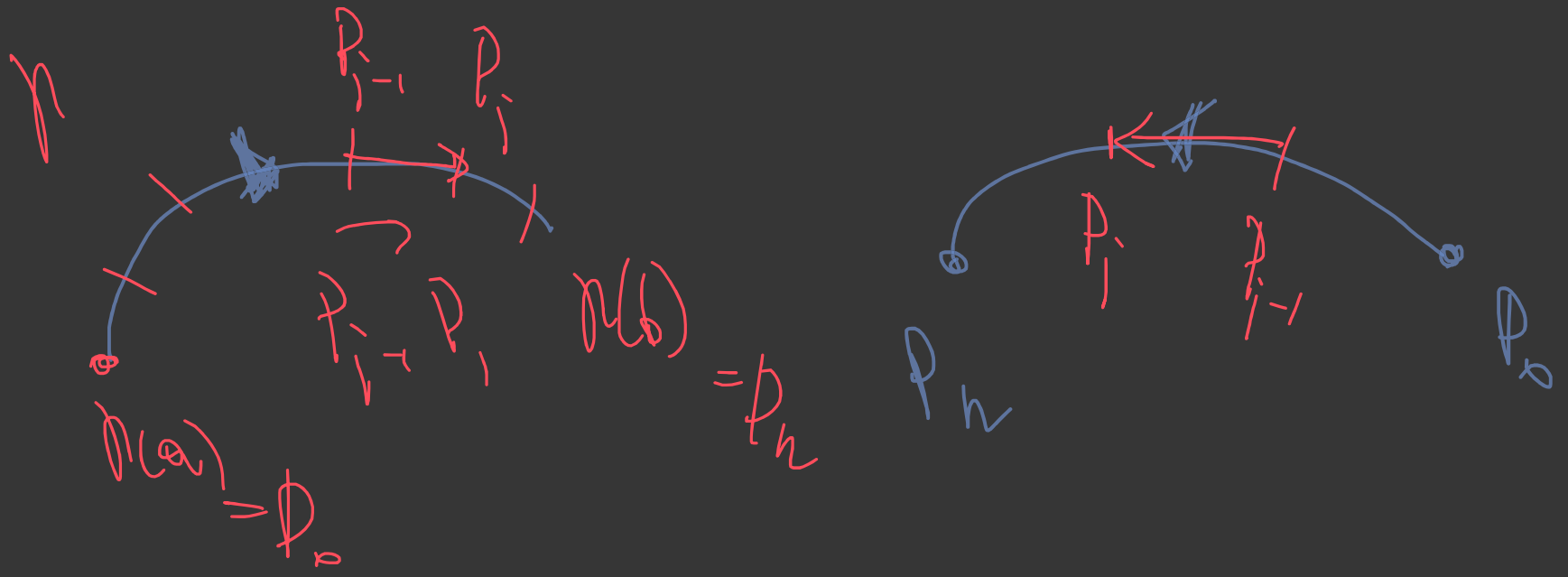
2) $\gamma(t) = (\operatorname{sen} t, \cos t)$, $t \in [0, 2\pi]$



Obs Seja γ uma curve com sentido de percurso fixo $\Rightarrow$
- γ vai denotar a curve com o sentido

di percorso opposto $\Rightarrow$

$$\int_{\gamma} \vec{F} \cdot d\vec{r} = - \int_{-\gamma} \vec{F} \cdot d\vec{r}$$



Ex 3 Ache $\int_{\gamma} \vec{F} \cdot d\vec{r}$, onde γ é

circunferência de raio 2 percorrida

no sentido horário e

$$\vec{F}(x,y) = \frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j} = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$$

Observe que a parametrização de

curva pode ser o seguinte!

$$\gamma(t) = (\overbrace{2\operatorname{sen} t}^{x(t)}, \overbrace{2\cos t}^{y(t)}) \quad (x^2(t) + y^2(t) = 4), \quad t \in [0, 2\pi]$$

Temos que $\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\gamma(t)) \cdot \gamma'(t)\, dt$

- $\vec{F}(\gamma(t)) = \vec{F}(x(t), y(t)) = \left(\dfrac{-2\cos t}{4\cos^2 t + 4\operatorname{sen}^2 t}, \dfrac{2\operatorname{sen} t}{4\cos^2 t + 4\operatorname{sen}^2 t} \right)$

$$= \left(-\frac{\cos t}{2}, \frac{\operatorname{sen} t}{2} \right)$$

- $\gamma'(t) = (2\cos t, -2\operatorname{sen} t) \Longrightarrow$

- $\vec{F}(\gamma(t)) \cdot \gamma'(t) = \left(-\frac{\cos t}{2}, \frac{\operatorname{sen} t}{2}\right) \cdot (2\cos t, -2\operatorname{sen} t)$

$= -\cos^2 t - \operatorname{sen}^2 t = -1$

Finalmente, $\int_{\gamma} \vec{F} \cdot d\vec{r} = \int_0^{2\pi} -1 \, dt = -2\pi$