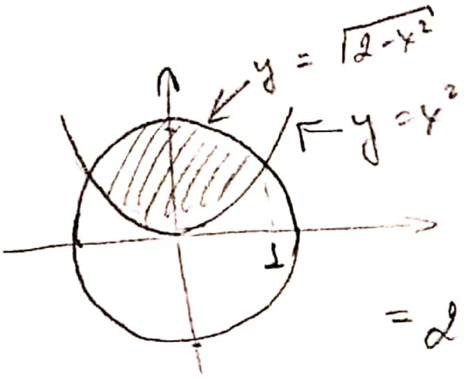
Volumes

2 f)

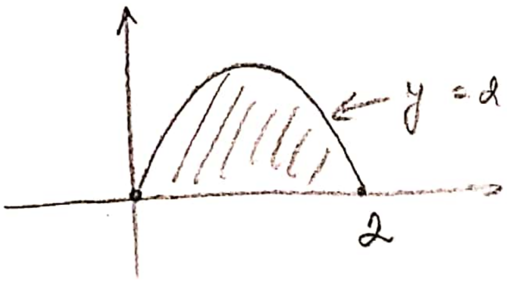


$$V = 2\pi \int_0^1 \left(x\sqrt{2-x^2} - x^3\right) dx$$

$$= 2\pi\left(-\frac{1}{2}\cdot\frac{2}{3}(2-x^2)^{\frac{3}{2}} - \frac{1}{4}x^4\right)\Big|_0^1 =$$

$$= 2\pi\left(-\frac{1}{3}\left(1-2^{\frac{3}{2}}\right) - \frac{1}{4}\right) = -2\pi\left(\frac{1}{4} + \frac{\left(1-2^{\frac{3}{2}}\right)}{3}\right)$$

6.



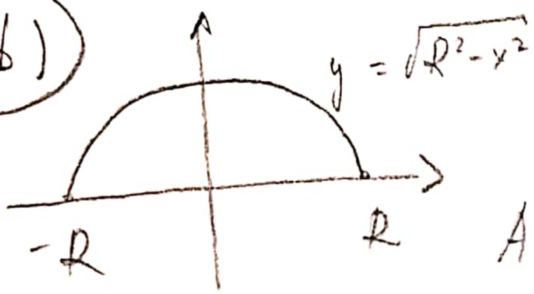
$$V = 2\pi\int_0^2 x(2x-x^2)\,dx =$$

$$= 2\pi\left(\frac{2}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^2 =$$

$$= 2\pi\left(\frac{2}{3}\cdot 8 - \frac{1}{4}\cdot 16\right) = 2\pi\left(\frac{16}{3} - 4\right) = 2\pi\cdot\frac{4}{3} = \frac{8\pi}{3}$$

Áreas de revolução:

1 b)



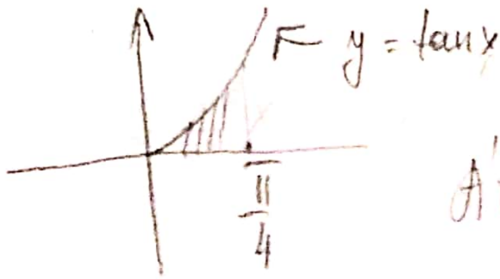
$$f'(x) = \frac{-2x}{2\sqrt{R^2-x^2}} = \frac{-x}{\sqrt{R^2-x^2}}$$

$$\text{Área} = 2\cdot 2\pi \int_0^R x\sqrt{1+\frac{x^2}{R^2-x^2}}\,dx$$

$$= 4\pi\int_0^R x\,\frac{R}{\sqrt{R^2-x^2}}\,dx = 4\pi R\cdot\frac{1}{-2}\sqrt{R^2-x^2}\cdot 2\,\Big|_0^R =$$

$$= 4\pi R^2$$

1. c)



$$f'(x) = \sec^2 x$$

$$\text{Área} = 2\pi\int_0^{\frac{\pi}{4}} \tan x\sqrt{1+\sec^2 x}\,dx$$

(2)

$$\int \sqrt{\sec^2 x+1}\ \tan x\, dx = \left| \sec^2 x = \tan^2 x + 1 \right| =$$

$$= \int \sec^2 x \cdot \frac{\tan x \sqrt{\tan^2 x + 2}}{\tan^2 x + 1}\, dx = \left| \begin{array}{l} u = \tan x, \\ du = \sec^2 x\, dx \end{array} \right| =$$

$$= \int \frac{u\sqrt{u^2+2}}{u^2+1}\, du = \left| \begin{array}{l} v = \sqrt{u^2+2} \\ dv = \frac{u}{\sqrt{u^2+2}}\, du \end{array} \right| = \int \frac{v^2}{v^2-1}\, dv =$$

$$= \int \frac{v^2-1+1}{v^2-1}\, dv = \int \left(1 + \frac{1}{v^2-1}\right) dv =$$

$$= v + \frac{1}{2}\left(\ln|v-1| - \ln|v+1|\right) + C =$$

$$= \sqrt{\tan^2 x + 2} + \frac{1}{2}\left(\ln\left|\sqrt{\tan^2 x + 2} - 1\right| - \ln\left|\sqrt{\tan^2 x + 2} + 1\right|\right)$$

$$+ C = \sqrt{\tan^2 x + 2} + \frac{1}{2} \ln\left(\left|\frac{\sqrt{\tan^2 x + 2} - 1}{\sqrt{\tan^2 x + 2} + 1}\right|\right)$$

Finalmente $\text{Área} = 2\pi \left[\sqrt{\tan^2 x + 2} + \frac{1}{2} \cdot \ln\left(\frac{\sqrt{\tan^2 x + 2} - 1}{\sqrt{\tan^2 x + 2} + 1}\right)\right] \Bigg|_0^{\pi/4}$ = veja gabarito.

Curvas

1c $f'(x) = \frac{1}{x} \Rightarrow$ comprimento $= \int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx$

$$= \int_1^e \frac{\sqrt{1+x^2}}{x}\, dx = \int_1^e \frac{x\sqrt{1+x^2}}{x^2}\, dx$$

$$\int \frac{x\sqrt{1+x^2}}{x^2}\,dx = \left| \begin{matrix} x^2 = u \\ du = 2x\,dx \end{matrix} \right| = \frac{1}{2}\int \frac{\sqrt{1+u}}{u}\,du$$

$$= \left| \begin{matrix} \sqrt{1+u} = v \\ dv = \frac{1}{2}\frac{du}{\sqrt{1+u}} \end{matrix} \right| = \int \frac{v^2}{v^2-1}\,dv = \text{veja ex.}$$

$$\text{anterior} = v + \frac{1}{2}\left(\ln|v-1| - \ln|v+1|\right) + C$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right| + C$$

$$\text{Finalmente, comprimento} = \sqrt{1+x^2} + \frac{1}{2}\ln\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right|\Bigg|_1^e$$

$$= \sqrt{1+e^2} - \sqrt{2} + \frac{1}{2}\left(\ln\frac{\sqrt{e^2+1}-1}{\sqrt{e^2+1}+1} - \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)$$

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1 e) $$f'(x) = \frac{e^x+1}{e^x-1}\cdot\frac{e^x(e^x+1)-e^x(e^x-1)}{(e^x+1)^2} =$$

$$= \frac{2\cdot e^x}{(e^x+1)(e^x-1)} = \frac{2e^x}{e^{2x}-1}$$

$$f'(x)+1 = \frac{4e^{2x}+e^{4x}-2e^{2x}+1}{(e^{2x}-1)^2} = \left(\frac{e^{2x}+1}{e^{2x}-1}\right)^2$$

$$\text{Comprimento} = \int_a^b \frac{e^{2x}+1}{e^{2x}-1}\,dx = \int_a^b \frac{2\cdot e^{2x}-1}{e^{2x}-1}\,dx$$

$$= \int_a^b \frac{d(e^{2x}-1)}{e^{2x}-1} - \int_a^b dx = \left(\ln|e^{2x}-1| - x\right)\Big|_a^b =$$

$$= \ln\left|\frac{e^{2b}-1}{e^{2a}-1}\right| - b + a$$

2c) $x'(t) = \operatorname{sen} t \ , \ y'(t) = 1 - \cos t$

$$\text{comprimento} = \int_0^{\pi} \sqrt{\operatorname{sen}^2 t + 1 - 2\cos t + \cos^2 t}\, dt =$$

$$= \int_0^{\pi} \sqrt{2(1-\cos t)}\, dt = \int_0^{\pi} \sqrt{4 \operatorname{sen}^2 \frac{t}{2}}\, dt = 2\int_0^{\pi} \operatorname{sen} \frac{t}{2}\, dt$$

$$= 4\left(-\cos \frac{t}{2}\right)\Big|_0^{\pi} = \underline{4}$$

Coord-s polares

1e) $$\rho^2 = \frac{1}{1+\operatorname{sen}^2\theta} \ , \quad 0 \le \theta \le \pi$$

$$\rho = \pm\sqrt{\frac{1}{1+\operatorname{sen}^2\theta}}$$

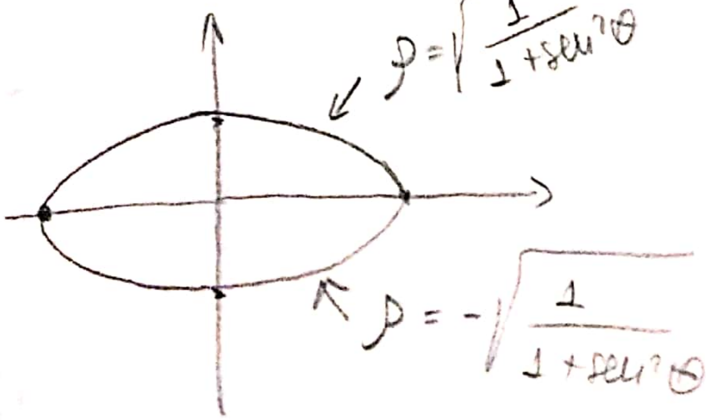
• $\rho = \sqrt{\dfrac{1}{1+\operatorname{sen}^2\theta}}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
ρ	1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{2}{3}$	1

• $\rho = -\sqrt{\dfrac{1}{1+\operatorname{sen}^2\theta}}$

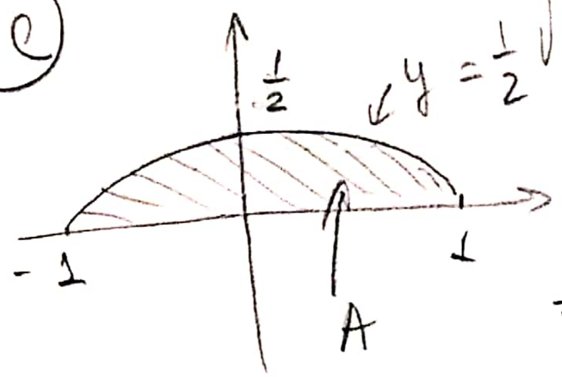
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
ρ	-1	$-\frac{2}{3}$	$-\frac{2}{5}$	$-\frac{1}{2}$	$-\frac{2}{5}$	$-\frac{2}{3}$	-1

$\| \ (1, \pi)$ $\qquad \| \ \left(\frac{2}{5}, \frac{2\pi}{3} + \pi\right)$



Centro de massa

1c



$$\text{Area}(A) = 2\int_0^1 \frac{1}{2}\sqrt{1-x^2}\,dx$$

$$= \int_0^1 \sqrt{1-x^2}\,dx =$$

$$= \frac{\text{arcsen}\, x + x\sqrt{1-x^2}}{2}\Big|_0^1 = \frac{\pi}{4}$$

$$\int_{-1}^{1} x\cdot\frac{1}{2}\sqrt{1-x^2}\,dx = 0 \Rightarrow x_c = 0!$$

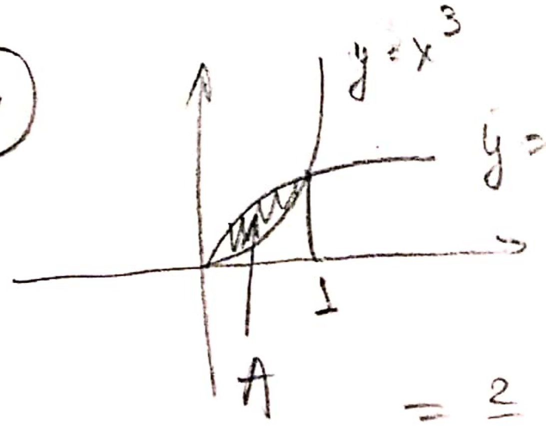
↑ função ímpar!

$$\int_{-1}^{1} \frac{1}{2}\cdot\frac{1}{2}\sqrt{1-x^2}\cdot\frac{1}{2}\sqrt{1-x^2}\,dx = \frac{1}{8}\int_{-1}^{1}(1-x^2)\,dx =$$

$$= \frac{1}{8}\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} = \frac{1}{8}\left(2-\frac{2}{3}\right) = \frac{1}{8}\cdot\frac{4}{3} = \frac{1}{6}$$

$$\Rightarrow y_c = \frac{\frac{1}{6}}{\text{area}(A)} = \frac{2}{3\pi} \Rightarrow \left(0, \frac{2}{3\pi}\right)$$

4



$$\text{Area}(A) = \int_0^1 (\sqrt{x} - x^3)\,dx$$

$$= \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^4}{4}\right)\Big|_0^1 =$$

$$= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$$

6.

- $\int_0^1 x \cdot (\sqrt{x} - x^3)\,dx = \left(\frac{2}{5} x^{\frac{5}{2}} - \frac{x^5}{5}\right)\Big|_0^1$

$= \frac{2}{5} - \frac{1}{5} = \frac{1}{5} \Rightarrow x_c = \frac{1}{5} / \frac{5}{12} = \frac{12}{25}$

- $\int_0^1 \frac{1}{2} (\sqrt{x} - x^3)(\sqrt{x} + x^3)\,dx = \frac{1}{2}\int_0^1 (x - x^6)\,dx =$

$= \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^7}{7}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{7}\right) = \frac{1}{2}\left(\frac{7-2}{14}\right) =$

$= \frac{5}{28} \Rightarrow y_c = \frac{5}{28} / \frac{5}{12} = \frac{12}{28} = \frac{3}{7}$

$\Rightarrow \underline{\left(\frac{12}{25}, \frac{3}{7}\right)}$