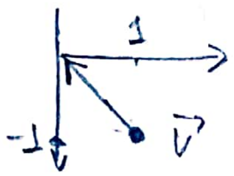
1. a) Temos $\frac{\partial f}{\partial \vec{u}} = \nabla f(1,-1)\cdot \vec{u} = \frac{\partial f}{\partial x}(1,-1)\left(-\frac{3}{5}\right) + \frac{\partial f}{\partial y}(1,-1)\cdot\frac{4}{5} = -1$

e $\frac{\partial f}{\partial \vec{v}} = \nabla f(1,-1)\cdot \vec{v} = \frac{\partial f}{\partial x}(1,-1)\cdot\frac{4}{5} + \frac{\partial f}{\partial y}(1,-1)\cdot\frac{3}{5} = 2$

$\Rightarrow \frac{\partial f}{\partial x}(1,-1) = \frac{11}{5}$ e $\frac{\partial f}{\partial y}(1,-1) = \frac{2}{5}$.

b) 

$\vec{v} = (-1,1) \Rightarrow$ versor é $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \frac{\partial f}{\partial \vec{u}} = \nabla f(-1,1)\cdot\vec{u} = (2,-1)\cdot\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{-2-1}{\sqrt{2}} = -\frac{3}{\sqrt{2}}$.

2. $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s} = 2xy^2\frac{\partial x}{\partial s} + (x^2\cdot 2y - 1)\frac{\partial y}{\partial s}$

$\Rightarrow \frac{\partial^2 f}{\partial s^2} = 2xy^2\frac{\partial^2 x}{\partial s^2} + \left(2y^2\frac{\partial x}{\partial s} + 2x\cdot 2y\frac{\partial y}{\partial s}\right)\frac{\partial x}{\partial s} +$

$+ (x^2\cdot 2y - 1)\frac{\partial^2 y}{\partial s^2} + \left(2x\cdot 2y\frac{\partial x}{\partial s} + 2x^2\frac{\partial y}{\partial s}\right)\frac{\partial y}{\partial s}$.

Como $\frac{\partial x}{\partial s} = te^s$, $\frac{\partial^2 x}{\partial s^2} = te^s$, $\frac{\partial y}{\partial s} = 2se^{-2t}$, $\frac{\partial^2 y}{\partial s^2} = 2e^{-2t}$.

$\Rightarrow \frac{\partial^2 f}{\partial s^2} = 2\cdot te^s\cdot s^4e^{-4t}\cdot te^s + \left(2\cdot s^4e^{-4t}\cdot te^s + 4\cdot te^s\cdot s^2e^{-2t}\cdot\right.$

$\left.\cdot 2se^{-2t}\right)\cdot te^s + (t^2e^{2s}\cdot 2\cdot s^2e^{-2t} - 1)\cdot 2e^{-2t} +$

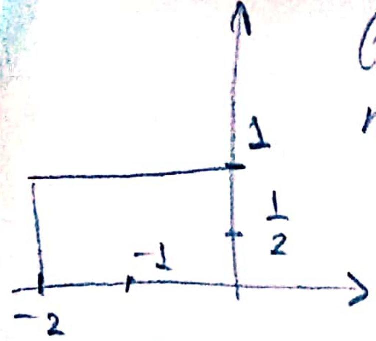
$+ \left(4\cdot te^s\cdot s^2e^{-2t}\cdot te^s + 2\cdot t^2e^{2s}\cdot 2se^{-2t}\right)2se^{-2t} =$

$= 2t^2s^4e^{-4t+2s} + 2s^4t^2e^{-4t+2s} + 8t^2s^3e^{-4t+2s} +$

$+ 4t^2s^2e^{-4t+2s} - 2e^{-2t} + 8t^2s^3e^{-4t+2s} + 8t^2s^2e^{-4t+2s}$

$= 4t^2s^4e^{-4t+2s} + 16t^2s^3e^{-4t+2s} + 12t^2s^2e^{-4t+2s} - 2e^{-2t}$

Como retângulo é fechado e limitado, max e min são atingidos



1) Interior do retângulo:

$$\begin{cases} \frac{\partial f}{\partial y} = 2x + 2 = 0 \\ \frac{\partial f}{\partial y} = -2y + 1 = 0 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = \frac{1}{2} \end{cases} \text{ é ponto crítico}$$

$$f(-1, \tfrac{1}{2}) = 1 - 2 - \frac{1}{4} + \frac{1}{2} - 1 = -2 + \frac{1}{4} = -\frac{7}{4}$$

2) Fronteira: • $x = 0$, $0 \leq y \leq 1$: $f(0,y) = -y^2 + y - 1 =$

$$= -(y - \tfrac{1}{2})^2 - \frac{3}{4}$$

$$\max f = f(0, \tfrac{1}{2}) = -\frac{3}{4}$$

$$\min f = f(0,0) = f(0,1) = -1$$

• $x = -2$, $0 \leq y \leq 1$: $f(-2, y) = -y^2 + y - 1 \Rightarrow$

$$\max f = f(-2, \tfrac{1}{2}) = -\frac{3}{4}$$

$$\min f = f(-2,0) = f(-2,1) = -1$$

• $y = 0$, $-2 \leq x \leq 0$: $f(x,0) = x^2 + 2x - 1 = (x+1)^2 - 2$

$$\min f = f(-1,0) = -2$$

$$\max f = f(-2,0) = f(0,0) = -1$$

• $y = 1$, $-2 \leq x \leq 0$: $f(x,1) = x^2 + 2x - 1 = (x+1)^2 - 2$

$$\min f = f(-1,1) = -2$$

$$\max f = f(-2,1) = -1$$

Logo, max global é $-\frac{3}{4}$ e min global é -2.

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b) $\frac{\partial f}{\partial x} = e^{x+y}(2x^2-y^2) + 4xe^{x+y} = 0$ ③

$\frac{\partial f}{\partial y} = e^{x+y}(2x^2-y^2) - 2ye^{x+y} = 0$

$\Rightarrow \begin{cases} 2x^2-y^2+4x=0 \\ 2x^2-y^2-2y=0 \end{cases} \Rightarrow \begin{cases} y=-2x \\ 2x^2-4x^2+4x=0 \end{cases}$

$\Rightarrow \begin{cases} y=-2x \\ -2x^2+4x=0 \end{cases} \Rightarrow \left[\begin{array}{l} \begin{cases} x=0 \\ y=0 \end{cases} \\ \begin{cases} x=2 \\ y=-4 \end{cases} \end{array} \right.$

$\frac{\partial^2 f}{\partial x^2} = e^{x+y}(2x^2-y^2) + 4xe^{x+y} + 4xe^{x+y} + 4e^{x+y}$

$\frac{\partial^2 f}{\partial y \partial x} = e^{x+y}(2x^2-y^2) - 2ye^{x+y} + 4xe^{x+y}$ $\Rightarrow$

$\frac{\partial^2 f}{\partial y^2} = e^{x+y}(2x^2-y^2) - 2ye^{x+y} - 2ye^{x+y} - 2e^{x+y}$

$H = e^{2(x+y)} \begin{vmatrix} (2x^2-y^2+8x+4) & (2x^2-y^2-2y+4x) \\ (2x^2-y^2-2y+4x) & (2x^2-y^2-4y-2) \end{vmatrix}$

$H(0,0) = \begin{vmatrix} 4 & 0 \\ 0 & -2 \end{vmatrix} = -8 < 0 \Rightarrow (0,0)$ é p-to de sela.

$H(2,-4) = e^{-4} \begin{vmatrix} (8-16+16+4) & (8-16+8+8) \\ (8-16+8+8) & (8-16+16-2) \end{vmatrix} =$

$= e^{-4}(12 \cdot 6 - 64) > 0$. Como $\frac{\partial^2 f}{\partial x^2}(2,-4) = 12 > 0 \Rightarrow$

$(2,-4)$ é p-to de min

4. Procuremos extremantes condicionados

$$\begin{cases} \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} \\ g(x,y) = 0 \end{cases}, \text{ onde } g(x,y) = x^2+y^2-2 : \begin{cases} 1 = \lambda \cdot 2x \\ -2 = \lambda \cdot 2y \\ x^2+y^2-2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{y}{x} = -2 \\ x^2+y^2-2=0 \end{cases} \Rightarrow \begin{cases} y = -2x \\ x^2+4x^2 = 2 \end{cases} \Rightarrow \begin{cases} x = \pm\sqrt{\frac{2}{5}} \\ y = -2x \end{cases}$$

$\Rightarrow \left(\sqrt{\frac{2}{5}}, -2\sqrt{\frac{2}{5}}\right)$ e $\left(-\sqrt{\frac{2}{5}}, 2\sqrt{\frac{2}{5}}\right)$ são extremantes

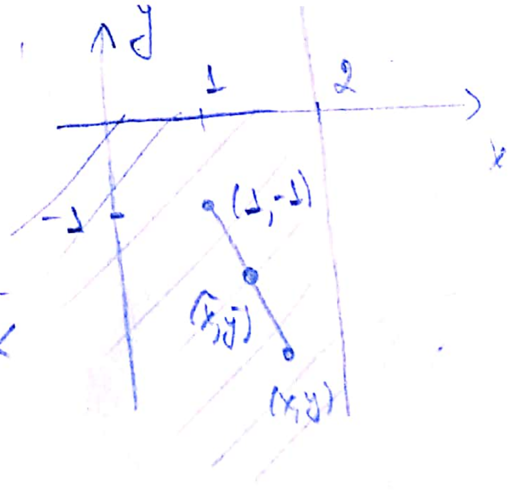
Temos $f\left(\sqrt{\frac{2}{5}}, -2\sqrt{\frac{2}{5}}\right) = \sqrt{\frac{2}{5}} + 4\sqrt{\frac{2}{5}} = \sqrt{10} \leftarrow$ max, $f\left(-\sqrt{\frac{2}{5}}, 2\sqrt{\frac{2}{5}}\right) = -\sqrt{\frac{2}{5}} - 4\sqrt{\frac{2}{5}} = -\sqrt{10}$ min.

5. Temos $|f(x,y) - P_1(x,y)| = |E_1(x,y)|$, onde

$$E_1(x,y) = \frac{1}{2}\left[\frac{\partial^2 f}{\partial x^2}(\bar{x},\bar{y})(x-1)^2 + 2\frac{\partial^2 f}{\partial y \partial x}(\bar{x},\bar{y})(x-1)(y+1) + \frac{\partial^2 f}{\partial y^2}(\bar{x},\bar{y})(y+1)^2\right]$$

$$= \frac{1}{2}\left[12\bar{x}(x-1)^2 + (6\bar{y}-2)(y+1)^2\right] = 6\bar{x}(x-1)^2 + (3\bar{y}-1)(y+1)^2,$$

onde $(\bar{x},\bar{y})$ fica no segmento $(1,-1)$ — (x,y). Temos $\bar{x} < 2$, $\bar{y} < 0$



$$\Rightarrow |f(x,y) - P_1(x,y)| = 6\bar{x}(x-1)^2 + (3\bar{y}-1)(y+1)^2 <$$
$$< 12(x-1)^2 - (y+1)^2$$