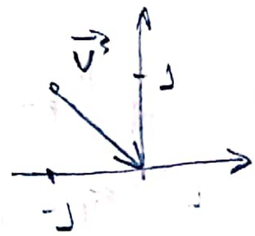
P3 - A

1. a) Temos $\frac{\partial f}{\partial \vec{u}} = \nabla f(-1,1)\cdot \vec{u} = \frac{\partial f}{\partial x}(-1,1)\cdot \frac{3}{5} + \frac{\partial f}{\partial y}(-1,1)\cdot\left(-\frac{4}{5}\right) = -2$

e $\frac{\partial f}{\partial \vec{v}} = \nabla f(-1,1)\cdot \vec{v} = \frac{\partial f}{\partial x}(-1,1)\cdot \frac{4}{5} + \frac{\partial f}{\partial y}(-1,1)\cdot \frac{3}{5} = 1$

$\Rightarrow \frac{\partial f}{\partial x}(-1,1) = \frac{-2}{5}$ e $\frac{\partial f}{\partial y}(-1,1) = \frac{11}{5}$

b) 

$\vec{v} = (1,-1) \Rightarrow$ versor é $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \frac{\partial f}{\partial \vec{u}}(-1,1) = \nabla f(-1,1)\cdot \vec{u} = \left(-\frac{2}{5}, \frac{11}{5}\right)\cdot\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$= \frac{-2-11}{5\sqrt{2}} = \frac{-13}{5\sqrt{2}}$

2. $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t} = (y^3 - 3x^2)\frac{\partial x}{\partial t} + 3y^2 x \frac{\partial y}{\partial t}$

$\Rightarrow \frac{\partial^2 f}{\partial t^2} = (y^3 - 3x^2)\frac{\partial^2 x}{\partial t^2} + \left(3y^2\frac{\partial y}{\partial t} - 6x\frac{\partial x}{\partial t}\right)\frac{\partial x}{\partial t} +$

$+ 3y^2 x\frac{\partial^2 y}{\partial t^2} + \left(3\cdot 2yx\frac{\partial y}{\partial t} + 3y^2\frac{\partial x}{\partial t}\right)\frac{\partial y}{\partial t}$

Como $\frac{\partial x}{\partial t} = se^t$, $\frac{\partial^2 x}{\partial t^2} = se^t$, $\frac{\partial y}{\partial t} = 2te^{-s}$, $\frac{\partial^2 y}{\partial t^2} = 2e^{-s} \Rightarrow$

$\frac{\partial^2 f}{\partial t^2} = (t^6 e^{-3s} - 3s^2 e^{2t})\cdot se^t + (3\cdot t^4 e^{-2s}(2te^{-s}) - 6se^t se^t)\cdot$

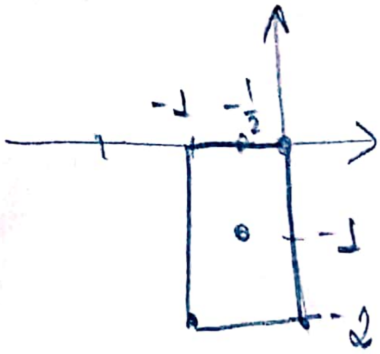
$\cdot se^t + 3t^4 e^{-2s} se^t \cdot 2e^{-s} + (6\cdot t^2 e^{-s} se^t \cdot 2te^{-s} + 3\cdot t^4 e^{-2s} se^t)\cdot$

$\cdot 2te^{-s} = t^6 s e^{-3s+t} - 3s^3 e^{3t} + 6t^5 s e^{-3s+t} - 6s^3 e^{3t} +$

$+ 6t^4 s e^{-3s+t} + 24t^4 s e^{-3s+t} + 6t^5 s e^{-3s+t} =$

$= t^6 s e^{-3s+t} - 9s^3 e^{3t} + 12t^5 s e^{-3s+t} + 30t^4 s e^{-3s+t}$

3, a)



Como retângulo é fechado e limitado, min e max são atingidos

1) Interior do retângulo:

$$\begin{cases} \frac{\partial f}{\partial x} = 2x + 1 = 0 \\ \frac{\partial f}{\partial y} = 2y + 2 = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{1}{2} \\ y = -1 \end{cases} \text{ é ponto crítico}$$

$$f(-\tfrac{1}{2}, -1) = 1 - 2 + \frac{1}{4} - \frac{1}{2} + 1 = -\frac{1}{4}$$

2) Fronteira: • $x = 0$, $-2 \leq y \leq 0$, $f(0,y) = y^2 + 2y + 1 = (y+1)^2 \geq 0$

$$\min f = f(0,-1) = 0$$
$$\max f = f(0,-2) = 1$$

• $x = -1$, $-2 \leq y \leq 0$: $f(-1,y) = y^2 + 2y + 1 = (y+1)^2$

$$\min f = f(-1,-1) = 0$$
$$\max f = f(-1,-2) = 1$$

• $y = 0$, $-1 \leq x \leq 0$: $f(x,0) = x^2 + x + 1$

$$\min f = f(-\tfrac{1}{2}, 0) = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$$
$$\max f = f(-1,0) = f(0,0) = 1$$

• $y = -2$, $-1 \leq x \leq 0$: $f(x,-2) = x^2 + x + 1$

$$\min f = f(-\tfrac{1}{2}, -2) = \frac{3}{4}$$
$$\max f = f(-1,-2) = f(0,-2) = 1$$

Logo, max global é 1 e min global é $-\frac{1}{4}$

b) $\frac{\partial f}{\partial x} = e^{x+y}(x^2-2y^2) + e^{x+y}2x = 0$

$\frac{\partial f}{\partial y} = e^{x+y}(x^2-2y^2) - 4y\, e^{x+y} = 0$

$\Rightarrow \begin{cases} x^2-2y^2+2x=0 \\ x^2-2y^2-4y=0 \end{cases} \Rightarrow \begin{cases} x=-2y \\ 4y^2-2y^2-4y=0 \end{cases} \Rightarrow$

$\begin{cases} x=-2y \\ y^2-2y=0 \end{cases} \Rightarrow \left[\begin{array}{l} \begin{cases} x=0 \\ y=0 \end{cases} \\ \begin{cases} x=-4 \\ y=2 \end{cases} \end{array}\right.$

$\frac{\partial^2 f}{\partial x^2} = e^{x+y}(x^2-2y^2) + e^{x+y}2x + e^{x+y}2x + 2e^{x+y}$

$\frac{\partial^2 f}{\partial y \partial x} = e^{x+y}(x^2-2y^2) - 4y\,e^{x+y} + e^{x+y}2x$

$\frac{\partial^2 f}{\partial y^2} = e^{x+y}(x^2-2y^2) - 4y\,e^{x+y} - 4e^{x+y} - 4y\,e^{x+y}$

$$H = \begin{vmatrix} e^{x+y}(x^2-2y^2+4x+2) & e^{x+y}(x^2-2y^2+2x-4y) \\ e^{x+y}(x^2-2y^2+2x-4y) & e^{x+y}(x^2-2y^2-8y-4) \end{vmatrix}$$

$H(0,0) = \begin{vmatrix} 2 & 0 \\ 0 & -4 \end{vmatrix} = -8 < 0 \Rightarrow (0,0)$ é ponto de sela

$$H(2,-4) = \begin{vmatrix} e^{-2}(16-8-16+2) & e^{-2}(16-8+8-8) \\ e^{-2}(16-8+8-8) & e^{-2}(16-8-16-4) \end{vmatrix}$$

$= (-6\cdot(-12) - 64)\,e^{-4} > 0$ Como $\frac{\partial^2 f}{\partial x^2}(-4,2) = -6 < 0$

$(-4,2)$ é p-to de max

4. Procuremos extremantes condicionados:

$$\begin{cases} \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} \\ g(x,y) = 0 \end{cases}, \text{ onde } g(x,y) = x^2+y^2-2: \begin{cases} 2 = \lambda \cdot 2x \\ -1 = \lambda \cdot 2y \\ x^2+y^2-2=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{x}{y} = -2 \\ x^2+y^2-2=0 \end{cases} \Rightarrow \begin{cases} x=-2y \\ y^2+4y^2=2 \end{cases} \Rightarrow \begin{cases} y = \pm\sqrt{\frac{2}{5}} \\ x=-2y \end{cases} \Rightarrow$$

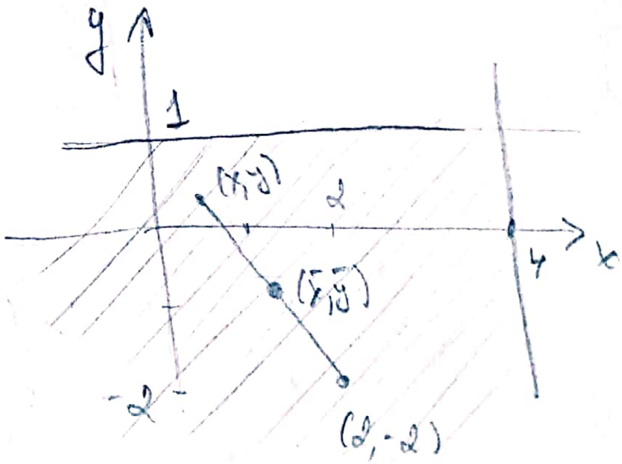
$\left(-2\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}}\right)$ e $\left(2\sqrt{\frac{2}{5}}, -\sqrt{\frac{2}{5}}\right)$ são extremantes.

Temos $f\left(-2\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}}\right) = \underset{\uparrow \min}{-\sqrt{10}}$, $f\left(2\sqrt{\frac{2}{5}}, -\sqrt{\frac{2}{5}}\right) = \sqrt{10} \leftarrow \max.$

5. Temos $|f(x,y) - P_1(x,y)| = |E_1(x,y)|$, onde

$$E_1(x,y) = \frac{1}{2}\left[\frac{\partial^2 f}{\partial x^2}(\bar{x},\bar{y})(x-2)^2 + 2\frac{\partial^2 f}{\partial y \partial x}(\bar{x},\bar{y})(x-2)(y+2) + \frac{\partial^2 f}{\partial y^2}(\bar{x},\bar{y})(y+2)^2\right]$$

$= \frac{1}{2}\left[12\bar{x}(x-2)^2 + (6\bar{y}-2)(y+2)^2\right]$, onde $(\bar{x},\bar{y})$ fica no segmento $(2,-2)$ —— (x,y).



Temos $\bar{x}<4$, $\bar{y}<1$ $\Rightarrow$

$$|f(x,y) - P_1(x,y)| = 6\bar{x}(x-2)^2 + (3\bar{y}-1)(y+2)^2 <$$
$$< 24(x-2)^2 + 2(y+2)^2;$$