1. a) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3+27y^3+y^2x}{x^2+9y^2} = \lim\limits_{(x,y)\to(0,0)} x\cdot\frac{x^2}{x^2+9y^2} +$

$+ \lim\limits_{(x,y)\to(0,0)} 3y \frac{9y^2}{x^2+9y^2} + \lim\limits_{(x,y)\to(0,0)} x \frac{y^2}{x^2+9y^2} = 0$

pois $\frac{x^2}{x^2+9y^2} \leq 1$, $\frac{9y^2}{x^2+9y^2} \leq 1$, $\frac{y^2}{x^2+9y^2} \leq \frac{1}{9}$

e $x \to 0, y \to 0$

b) $\lim\limits_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$. O limite não existe pois:

- $x = 0, y \to 0, y \neq 0 \Rightarrow \lim\limits_{y\to 0} \frac{0}{y^6} = 0$
- $x = y^3, y \to 0, y \neq 0 \Rightarrow \lim\limits_{y \to 0} \frac{y^6}{2y^6} = \frac{1}{2}$

2. Aproxime $\sqrt[3]{8.02} + \sqrt{3.99}$. Consideremos a função:
$f(x,y) = \sqrt[3]{x} + \sqrt{y}$ Temos que $f(8+0.02, 4-0.01) =$
$= \sqrt[3]{8.02} + \sqrt{3.99}$

Por outro lado:
$f(8+0.02, 4-0.01) \approx f(8,4) + \frac{\partial f}{\partial x}(8,4)\cdot 0.02 + \frac{\partial f}{\partial y}(8,4)\cdot(-0.01)$

Temos que $f(8,4) = \sqrt[3]{8} + \sqrt{4} = 4$.

$\frac{\partial f}{\partial x} = \frac{1}{3} x^{-\frac{2}{3}}$, $\frac{\partial f}{\partial y} = \frac{1}{2} y^{-\frac{1}{2}} \Rightarrow \frac{\partial f}{\partial x}(8,4) = \frac{1}{3}\cdot 8^{-\frac{2}{3}} = \frac{1}{12}$

$\frac{\partial f}{\partial y}(8,4) = \frac{1}{2} 4^{-\frac{1}{2}} = \frac{1}{4}$.

Logo $\sqrt[3]{8.02} + \sqrt{3.99} \simeq 4 + \frac{1}{12} \cdot \frac{2}{100} + \frac{1}{4} \cdot \frac{(-1)}{100} =$ ②

$= 4 + \frac{2-3}{12 \cdot 100} = 4 - \frac{1}{1200}.$

3. Ache o plano π que passa por $(1,1,0)$ e $(0,1,2)$ e é tangente ao gráfico de $f(x,y) = xy + y$.

Sol $\pi: z = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$

Temos $\frac{\partial f}{\partial x} = y$, $\frac{\partial f}{\partial y} = x+1$, logo

$\pi: z = x_0 y_0 + y_0 + y_0(x-x_0) + (x_0+1)(y-y_0)$

Como $(1,1,0)$ e $(0,1,2) \in \pi$, obtemos:

$$\begin{cases} 0 = x_0y_0 + y_0 + y_0(1-x_0) + (x_0+1)(1-y_0) \\ 2 = x_0y_0 + y_0 + y_0(0-x_0) + (x_0+1)(1-y_0) \end{cases} \Rightarrow$$

$$\begin{cases} y_0 = -2 \\ 0 = -2x_0 - 2 - 2(1-x_0) + (x_0+1)(1+2) \end{cases} \Rightarrow$$

$0 = -2x_0 - 2 - 2 + 2x_0 + 3x_0 + 3 \Rightarrow x_0 = \frac{1}{3}$

$\Rightarrow \pi: z = -\frac{2}{3} - 2 - 2(x - \frac{1}{3}) + (\frac{1}{3}+1)(y+2) \Rightarrow$

$z = -\frac{2}{3} - 2 - 2x + \frac{2}{3} + \frac{4}{3}y + \frac{8}{3} \Rightarrow$

$z = \frac{2}{3} - 2x + \frac{4}{3}y.$

4. Veja Turma A

5. Seja $f: \mathbb{R}^2 \to \mathbb{R}$ derivável. Suponha que $f(2t^2+1, 4t^3-1) = t^2$, $t \in \mathbb{R}$ e $\frac{\partial f}{\partial x}(3,3) = 1$. Ache $\frac{\partial f}{\partial y}(3,3)$

Sol $f(\gamma(t)) = g(t)$, onde $\gamma(t) = (2t^2+1, 4t^3-1)$, $g(t) = t^2$. Pela regra de cadeia $g'(t) = \nabla f(\gamma(t)) \cdot \gamma'(t) =$

$= \frac{\partial f}{\partial x}(\gamma(t)) \cdot 4t + \frac{\partial f}{\partial y}(\gamma(t)) \cdot 12t^2 = 2t$. Seja $t = 1$

$\Rightarrow \frac{\partial f}{\partial x}(3,3) \cdot 4 + \frac{\partial f}{\partial y}(3,3) \cdot 12 = 2 \Rightarrow$

$$\frac{\partial f}{\partial y}(3,3) = \frac{1}{12}\left(2 - \frac{\partial f}{\partial x}(3,3) \cdot 4\right) = \frac{1}{12}(2-4) = -\frac{1}{6}.$$