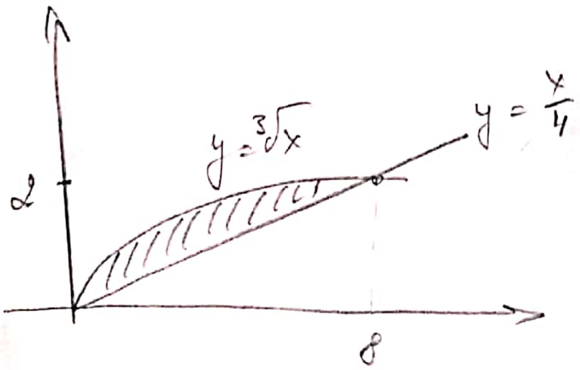
1. (2.0 pontos) Calcule o volume do solido obtido pela rotação em torno do eixo x do conjunto de todos os pares (x, y) tais que

$$0 \leq x \leq 8, \qquad \frac{x}{4} \leq y \leq \sqrt[3]{x}.$$



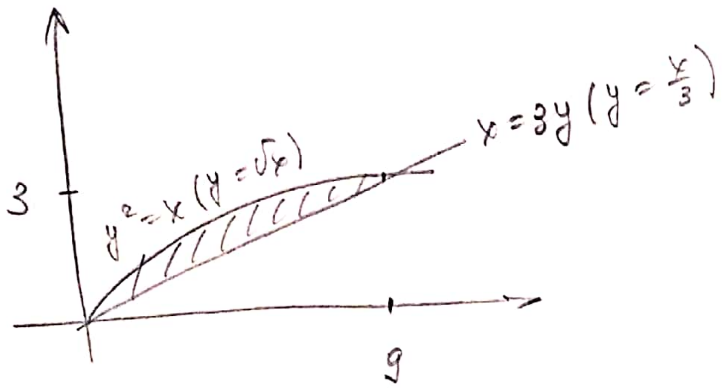
$$V = \pi \left[\int_0^8 \left(\left(\sqrt[3]{x}\right)^2 - \left(\frac{x}{4}\right)^2 \right) dx \right] = \pi \int_0^8 \left(x^{\frac{2}{3}} - \frac{x^2}{16} \right) dx$$

$$= \pi \left(\frac{3}{5} x^{\frac{5}{3}} - \frac{x^3}{3 \cdot 16} \right) \Big|_0^8 = \pi \left(\frac{3}{5} \cdot 32 - \frac{8^3}{3 \cdot 16} \right) =$$

$$= \pi \cdot 32 \left(\frac{3}{5} - \frac{1}{3} \right) = \pi \cdot 32 \cdot \frac{9-5}{15} = \frac{128\pi}{15}$$

2. (2.0 pontos) Calcule o volume do solido obtido pela rotação em torno do eixo y da região limitada pelas curvas

$$y^2 = x, \quad x = 3y.$$



$$V = \pi \left[\int_0^3 \left((3y)^2 - (y^2)^2 \right) dy \right] = \pi \int_0^3 (9y^2 - y^4)\, dy =$$

$$= \pi \left(3y^3 - \frac{y^5}{5} \right) \Big|_0^3 = 81\pi \left(1 - \frac{3}{5} \right) = 81\pi \cdot \frac{2}{5} =$$

$$= \frac{162}{5}\pi$$

3. (2.0 pontos) Encontre o comprimento da curva:

$$x(t) = \frac{t^2}{2}, \quad y(t) = t^3, \qquad 1 \leq t \leq 2.$$

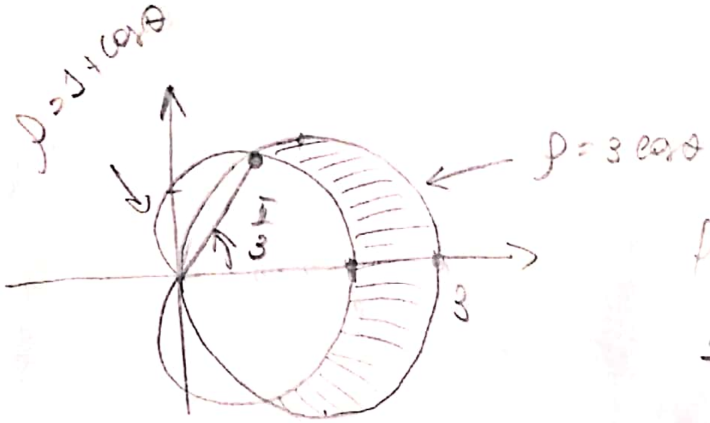
$x'(t) = t$, $y'(t) = 3t^2$ $\Rightarrow$

comprimento $= \int_1^2 \sqrt{t^2 + (3t^2)^2}\, dt = \int_1^2 \sqrt{t^2 + 9t^4}\, dt$

$= \int_1^2 t\sqrt{1 + 9t^2}\, dt = \frac{1}{18} \cdot \frac{2}{3} (1 + 9t^2)^{\frac{3}{2}} \Big|_1^2 =$

$= \frac{1}{27} \left(37^{\frac{3}{2}} - 10^{\frac{3}{2}}\right)$

4. (2.0 pontos) Encontre a área da região que está dentro da curva $\rho = 3\cos\theta$ e fora da curva $\rho = 1 + \cos\theta$.



Ponto de intersecção:

$1 + \cos\theta = 3\cos\theta \Rightarrow$

$\cos\theta = \frac{1}{2} \Rightarrow$

$\theta = \frac{\pi}{3}$

$$\text{Área} = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} \left((3\cos\theta)^2 - (1+\cos\theta)^2 \right) d\theta =$$

$$= \int_0^{\frac{\pi}{3}} \left(9\cos^2\theta - 1 - 2\cos\theta - \cos^2(\theta) \right) d\theta =$$

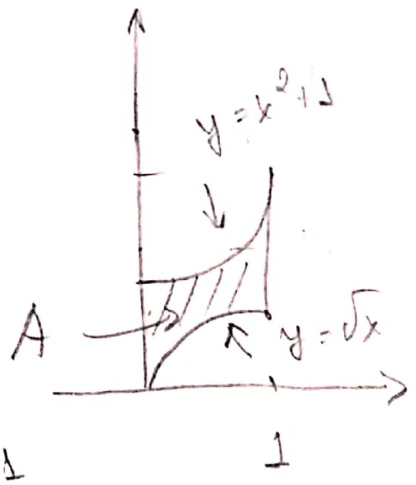
$$= \left(-\theta - 2\,\text{sen}\,\theta \right) \Big|_0^{\frac{\pi}{3}} + \int_0^{\frac{\pi}{3}} 8 \cdot \frac{1 + \cos 2\theta}{2} \, d\theta =$$

$$= -\frac{\pi}{3} - \sqrt{3} + 4\left(\theta + \frac{\text{sen}\, 2\theta}{2} \right) \Big|_0^{\frac{\pi}{3}} =$$

$$= -\frac{\pi}{3} - \sqrt{3} + \frac{4\pi}{3} + \frac{\sqrt{3}}{4} = \underline{\pi - \frac{3\sqrt{3}}{4}}$$

5. (2.0 pontos) Determine o centro de massa da região:

$$A = \{(x,y) \in \mathbb{R}^2 \ : \ 0 \le x \le 1, \ \sqrt{x} \le y \le x^2 + 1\}.$$



$$\text{Área}(A) = \int_0^1 ((x^2+1) - \sqrt{x})\,dx$$

$$= \left(\frac{x^3}{3} + x - \frac{2}{3}x^{\frac{3}{2}}\right)\Big|_0^1 =$$

$$= \frac{1}{3} + 1 - \frac{2}{3} = \frac{2}{3}$$

- $\displaystyle\int_0^1 x\,(x^2+1-\sqrt{x})\,dx = \left(\frac{x^4}{4} + \frac{x^2}{2} - \frac{2}{5}x^{\frac{5}{2}}\right)\Big|_0^1$

$$= \frac{1}{4} + \frac{1}{2} - \frac{2}{5} = \frac{5+10-8}{20} = \frac{7}{20}$$

- $\displaystyle\int_0^1 \frac{1}{2}\left((x^2+1)^2 - (\sqrt{x})^2\right)dx = \frac{1}{2}\int_0^1 (x^4 + 2x^2 + 1 - x)\,dx$

$$= \frac{1}{2}\left(\frac{x^5}{5} + \frac{2}{3}x^3 + x - \frac{x^2}{2}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{1}{5} + \frac{2}{3} + 1 - \frac{1}{2}\right)$$

$$= \frac{1}{2} \cdot \frac{6+20+15}{30} = \frac{41}{60}$$

Logo, $x_c = \frac{7}{20} \cdot \frac{3}{2} = \frac{21}{40}$, $y_c = \frac{41}{60} \cdot \frac{3}{2} = \frac{41}{40}$