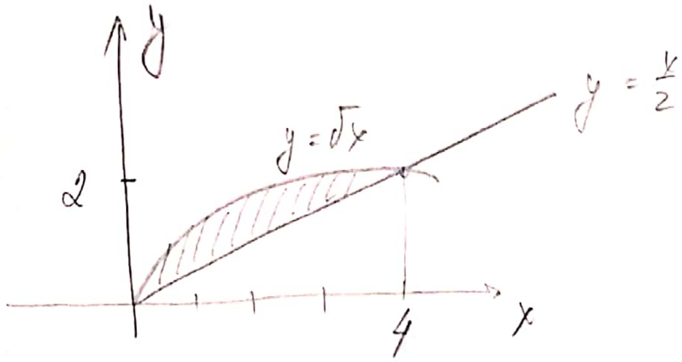
1. (2.0 pontos) Calcule o volume do solido obtido pela rotação em torno do eixo x do conjunto de todos os pares (x, y) tais que

$$0 \leq x \leq 4, \qquad \frac{x}{2} \leq y \leq \sqrt{x}.$$



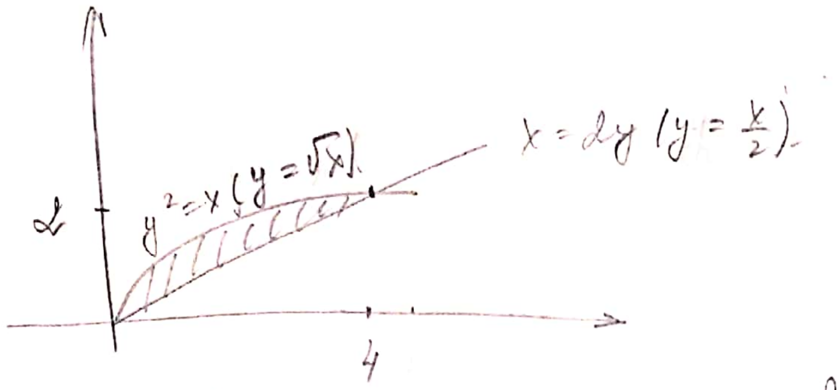
$$V = \pi \left[\int_0^4 \left((\sqrt{x})^2 - \left(\frac{x}{2}\right)^2 \right) dx \right] = \pi \int_0^4 \left(x - \frac{x^2}{4} \right) dx =$$

$$= \pi \left(\frac{x^2}{2} - \frac{x^3}{12} \right) \Big|_0^4 = \pi \left(\frac{16}{2} - \frac{64}{12} \right) = \pi \left(8 - \frac{16}{3} \right)$$

$$= \pi \left(\frac{24 - 16}{3} \right) = \frac{8\pi}{3}$$

2. (2.0 pontos) Calcule o volume do solido obtido pela rotação em torno do eixo y da região limitada pelas curvas

$$y^2 = x, \quad x = 2y.$$



$$V = \pi \left[\int_0^2 \left((2y)^2 - (y^2)^2 \right) dy \right] = \pi \int_0^2 (4y^2 - y^4)\, dy =$$

$$= \pi \left(\frac{4y^3}{3} - \frac{y^5}{5} \right) \Big|_0^2 = \pi \left(\frac{4 \cdot 8}{3} - \frac{32}{5} \right) =$$

$$= 32\pi \left(\frac{5-3}{15} \right) = \frac{64\pi}{15}$$

3. (2.0 pontos) Encontre o comprimento da curva:

$$x(t) = t^2, \quad y(t) = \frac{t^3}{3}, \qquad 1 \leq t \leq 2.$$

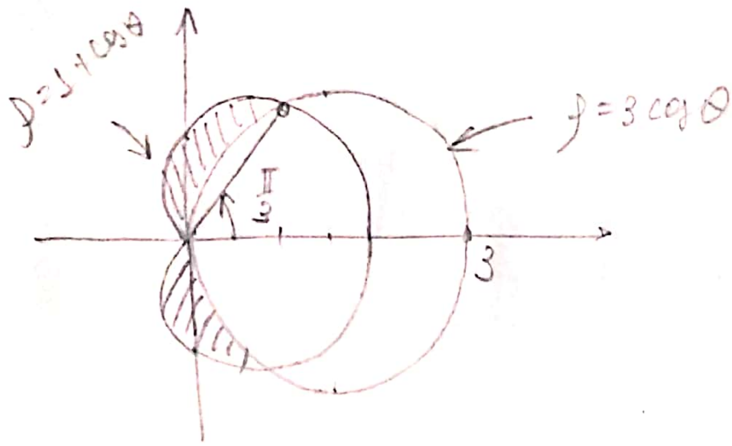
$x'(t) = 2t$, $y'(t) = t^2$ $\Rightarrow$

comprimento $= \int_1^2 \sqrt{(2t)^2 + (t^2)^2}\, dt =$

$= \int_1^2 \sqrt{4t^2 + t^4}\, dt = \int_1^2 t\sqrt{4+t^2}\, dt =$

$= \frac{1}{2} \cdot \frac{2}{3} (4+t^2)^{\frac{3}{2}} \Big|_1^2 = \frac{1}{3} \left(8^{\frac{3}{2}} - 5^{\frac{3}{2}} \right)$

4. (2.0 pontos) Encontre a área da região que está fora da curva $\rho = 3\cos\theta$ e dentro da curva $\rho = 1 + \cos\theta$.



Ponto de interseção:

$1 + \cos\theta = 3\cos\theta \Rightarrow$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$$\text{Área} = 2 \cdot \frac{1}{2}\left(\int_{\frac{\pi}{3}}^{\pi} (1+\cos\theta)^2 \, d\theta - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3\cos\theta)^2 \, d\theta\right) =$$

$$= \int_{\frac{\pi}{3}}^{\pi} (1 + 2\cos\theta + \cos^2\theta) \, d\theta - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9\cos^2\theta \, d\theta =$$

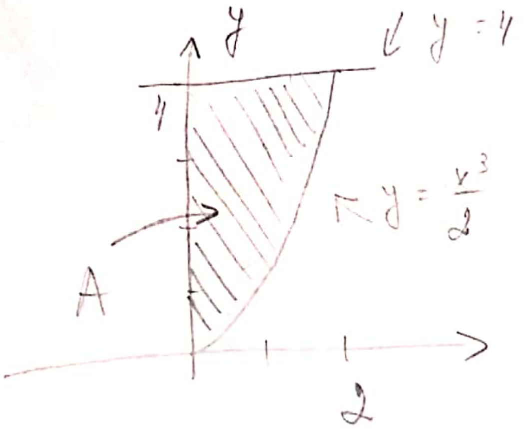
$$= (\theta + 2\,\text{sen}\,\theta)\Big|_{\frac{\pi}{3}}^{\pi} + \int_{\frac{\pi}{3}}^{\pi} \frac{1+\cos 2\theta}{2} \, d\theta - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{9}{2}(1 + \cos 2\theta) \, d\theta$$

$$= \frac{2\pi}{3} - \sqrt{3} + \frac{\theta}{2}\Big|_{\frac{\pi}{3}}^{\pi} + \frac{\text{sen}\, 2\theta}{4}\Big|_{\frac{\pi}{3}}^{\pi} - \frac{9}{2}\left(\theta + \frac{\text{sen}\, 2\theta}{2}\right)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{2\pi}{3} \cancel{-\sqrt{3}} + \frac{\pi}{3} - \frac{\sqrt{3}}{8} - \frac{9}{12}\pi + \frac{9\sqrt{3}}{8} = \underline{\frac{\pi}{4}}$$

5. (2.0 pontos) Determine o centro de massa da região:

$$A = \left\{(x, y) \in \mathbb{R}^2 \ : \ 0 \leq x \leq 2, \ \frac{x^3}{2} \leq y \leq 4\right\}.$$



$$\text{Área}(A) = \int_0^2 \left(4 - \frac{x^3}{2}\right) dx = \left(4x - \frac{x^4}{8}\right)\Big|_0^2 = 8 - \frac{16}{8} = 8 - 2 = 6$$

- $$\int_0^2 x\left(4 - \frac{x^3}{2}\right) dx = \left(2x^2 - \frac{x^5}{10}\right)\Big|_0^2 = 8 - \frac{32}{10} = \frac{40 - 16}{5} = \frac{24}{5}$$

- $$\int_0^2 \frac{1}{2}\left(4^2 - \left(\frac{x^3}{2}\right)^2\right) dx = \frac{1}{2}\left(16x - \frac{x^7}{4 \cdot 7}\right)\Big|_0^2 = \frac{1}{2}\left(32 - \frac{32}{7}\right) = \frac{96}{7}$$

Logo, $x_c = \frac{24}{5 \cdot 6} = \frac{4}{5}$, $y_c = \frac{96}{7 \cdot 6} = \frac{16}{7}$