PSub Turma B

(1) Observe que 3 vetores são LD se e somente se o produto misto deles for nulo.

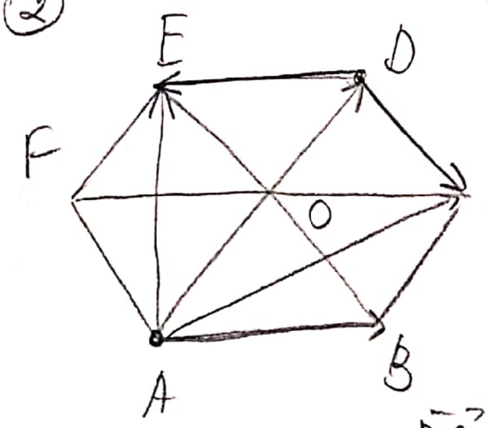
Denotando $\vec{b} = \vec{u} - \vec{v} + a\vec{w}$, $\vec{c} = \vec{u} - a\vec{v} + (10-3a)\vec{w}$

obtemos $(\vec{u} \times \vec{b}) \cdot \vec{c} = \det \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & a \\ 1 & -a & 10-3a \end{pmatrix} =$

$= 3a - 10 + a^2 = 0 \Rightarrow \left[\begin{array}{l} a = -5 \\ a = 2 \end{array} \right. \Rightarrow$

$\vec{u}, \vec{b}, \vec{c}$ são LD sse $\underline{a = -5 \text{ ou } a = 2}$.

(2)



E: $\vec{AE} = \vec{DE} - \vec{DA} = \vec{ED} - 2\vec{DO} =$

$= \vec{DE} - 2(\vec{DE} + \vec{DC}) = -\vec{DE} - 2\vec{DC} =$

$= (-1, -2) \Rightarrow \underline{E = (-1, -2)}$

C: $\vec{AC} = \vec{DC} - \vec{DA} = \vec{DC} - 2\vec{DO} =$

$= \vec{DC} - 2(\vec{DC} + \vec{DE}) = -\vec{DC} - 2\vec{DE} = (-2, -1)$

$\Rightarrow \underline{C = (-2, -1)}$

B: $\vec{AB} = -\vec{DE} = (-1, 0) \Rightarrow \underline{B = (-1, 0)}$

D: $\vec{AD} = -2\vec{DO} = -2(\vec{DE} + \vec{DC}) = (-2, -2) \Rightarrow \underline{D = (-2, -2)}$

(3) a)

$\vec{AC} = (0, 2, -2)$, $\vec{AB} = (-2, -1, -3)$ são vetores diretores do π

$\vec{n} = \vec{AC} \times \vec{AB}$ é vetor normal.

②

$$\vec{n} = \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 2 \\ -2 & -1 & -3 \end{pmatrix}\begin{matrix} \vec{i} & \vec{j} \\ 0 & 2 \\ -2 & -1 \end{matrix} = -6\vec{i} + 4\vec{j} + 4\vec{k} - 2\vec{i} = (-8, 4, 4) = (a, b, c)$$

$\pi: ax + by + cz = d \Rightarrow \pi: -8x + 4y + 4z = d$

Como $B = (-1, 0, -1) \in \pi$, obtemos $8 + 0 - 4 = \underline{4 = d} \Rightarrow$

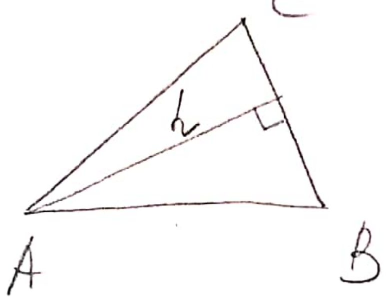
$\pi: -8x + 4y + 4z = 4$ ou $\underline{-2x + y + z = 1}$

b) $d(\pi, P)^{=(x_p, y_p, z_p)} = \dfrac{|a x_p + b y_p + c z_p - d|}{\sqrt{a^2 + b^2 + c^2}} = \dfrac{|-2 \cdot x + 1 \cdot 0 + 1 \cdot 1 - 1|}{\sqrt{4 + 1 + 1}} = 1$

$\Rightarrow |-2x| = \sqrt{6} \Rightarrow 2x = \pm\sqrt{6} \Rightarrow$

$\Rightarrow \left[\begin{array}{l} x = \sqrt{6}/2 \\ x = -\sqrt{6}/2 \end{array}\right.$

c)



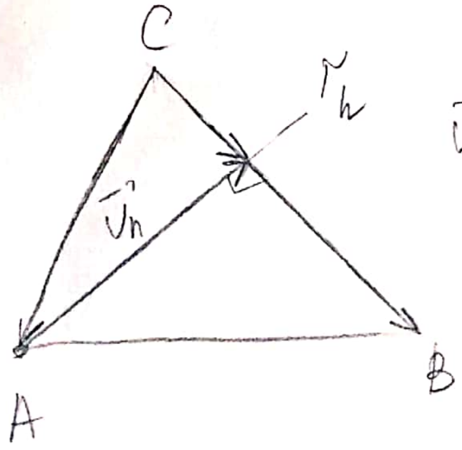
$S_{ABC} = \frac{1}{2} \cdot h \, \|\vec{BC}\| = \frac{1}{2} \|\vec{AB} \times \vec{AC}\| \Rightarrow$

$h = \dfrac{\|\vec{AB} \times \vec{AC}\|}{\|\vec{BC}\|}$.

$\|\vec{AB} \times \vec{AC}\| = \|(-8, 4, 4)\| = \sqrt{64 + 32} = \sqrt{96}$

$\|\vec{BC}\| = \|(2, 3, 1)\| = \sqrt{4 + 9 + 1} = \sqrt{14} \Rightarrow \underline{h = 4\sqrt{\frac{3}{7}}}$

d)



Seja $\vec{v}_h$ vetor diretor da r_h

$\vec{v}_h = \text{Proj}_{\vec{CB}} \vec{CA} - \vec{CA}$

$\text{Proj}_{\vec{CB}} \vec{CA} = \left(\dfrac{\vec{CB} \cdot \vec{CA}}{\|\vec{CB}\|^2} \right) \cdot \vec{CB} =$

$= \left(\dfrac{-(2,3,1) \cdot (0,-2,2)}{(4 + 9 + 1)} \right) (-2, -3, -1) =$

$= \dfrac{6 - 2}{14} (-2, -3, -1) = -\dfrac{2}{7} (2, 3, 1) \Rightarrow$

$\vec{v}_h = -\frac{2}{7}(2, 3, 1) - (0, -2, 2) = \frac{1}{7}(-4, -6 + 14, -2 - 14) = \frac{1}{7}(-4, 8, -16)$

$\Rightarrow r_h: X = A + t\vec{v}_h = (1,1,2) + \frac{t}{4}(-4,8,-16)$

④ Temos equações de mudança de Σ_2 para Σ_1:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\Sigma_1} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}_{\Sigma_1} + \begin{pmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{\Sigma_2} \Rightarrow$$

$$\begin{cases} x = y' + z' \\ y = 1 - x' + y' \\ z = 1 + x' + z' \end{cases} \Rightarrow \begin{cases} x = y' + z' \\ y + z = 2 + y' + z' \\ z = 1 + x' + z' \end{cases} \Rightarrow \begin{cases} x + y + z = 2 + 2y' \\ z' = y' - x \\ x' = z - 1 - z' \end{cases}$$

$$\Rightarrow \begin{cases} y' = -1 + \frac{x+y+z}{2} \\ z' = -1 + \frac{-x+y+z}{2} \\ x' = \frac{z+x-y}{2} \end{cases}$$

$$r: \begin{cases} x = -1 + t \\ y = 0 \\ z = 1 + 2t \end{cases} \Rightarrow \begin{cases} x' = \frac{1+2t-1+t}{2} \\ y' = -1 + \frac{-1+t+1+2t}{2} \\ z' = -1 + \frac{1-t+1+2t}{2} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x' = \frac{3}{2}t \\ y' = -1 + \frac{3}{2}t \\ z' = \frac{t}{2} \end{cases} \Rightarrow$$

$$r: \left[X = (0,-1,0) + t\left(\frac{3}{2}, \frac{3}{2}, \frac{1}{2}\right) \right]_{\Sigma_2}$$

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