P Sub Turma A ①

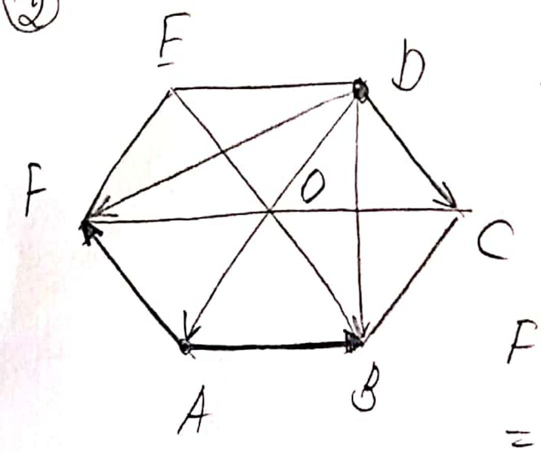
① Observe que 3 vetores são LD se e somente se o produto misto deles for nulo.

Denotando $\vec{b} = \vec{u} - a\vec{v} + \vec{w}$, $\vec{c} = a\vec{u} + (12 - 7a)\vec{v} + \vec{w}$

obtemos $(\vec{b} \times \vec{c}) \cdot \vec{w} = \det \begin{pmatrix} 1 & -a & 1 \\ a & 12-7a & 1 \\ 0 & 0 & 1 \end{pmatrix} = 12 - 7a + a^2 = 0$

$\Rightarrow \left[ \begin{array}{l} a = 3 \\ a = 4 \end{array} \right. \Rightarrow \vec{b}, \vec{c}, \vec{w}$ são LD sse $\underline{a = 3 \text{ ou } a = 4}$

②



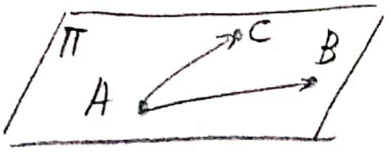

$B: \vec{DB} = \vec{AB} - \vec{AD} = \vec{AB} - 2\vec{AO} =$
$= \vec{AB} - 2(\vec{AB} + \vec{AF}) = -\vec{AB} - 2\vec{AF} =$
$= (-1, -2) \Rightarrow \underline{B = (-1, -2)}$

$F: \vec{DF} = \vec{AF} - \vec{AD} = \vec{AF} - 2\vec{AO} =$
$= \vec{AF} - 2(\vec{AB} + \vec{AF}) = -\vec{AF} - 2\vec{AB} =$
$= (-2, -1) \Rightarrow \underline{F = (-2, -1)}$

$C: \vec{DC} = -\vec{AF} = (0, -1) \Rightarrow \underline{C = (0, -1)}$

$A: \vec{DA} = -2\vec{AO} = -2(\vec{AB} + \vec{AF}) = (-2, -2) \Rightarrow \underline{A = (-2, -2)}$

③ a)




$\vec{AC} = (3, 0, 2)$, $\vec{AB} = (1, -1, -2)$ são vet-s diretores do $\pi$.

$\vec{n} = \vec{AC} \times \vec{AB}$ é vetor normal.

$\vec{n} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 2 \\ 1 & -1 & -2 \end{pmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 3 & 0 \\ 1 & -1 \end{matrix} = 2\vec{j} - 3\vec{k} + 2\vec{i} + 6\vec{j} =$

$= (2, 8, -3) = (a, b, c)$

$\pi: ax+by+cz=d \Rightarrow \pi: 2x+8y-3z=d$

Como $B=(-1,0,-1)\in\pi$, obtemos $2\cdot(-1)+8\cdot 0-3(-1)=$

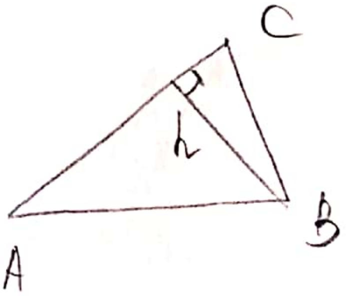
$=-2+3=\underline{1=d} \Rightarrow \underline{\pi: 2x+8y-3z=1}$

b) $d(\pi,P)^{=(x_p,y_p,z_p)} = \dfrac{|ax_p+by_p+cz_p-d|}{\sqrt{a^2+b^2+c^2}} = \dfrac{|2\cdot x+8\cdot 0-3\cdot 1-1|}{\sqrt{4+64+9}} = 1$

$\Rightarrow |2x-4| = \sqrt{77} \Rightarrow 2x-4 = \pm\sqrt{77} \Rightarrow$

$$\left[\begin{array}{l} x=(\sqrt{77}+4)/2 \\ x=(-\sqrt{77}+4)/2 \end{array}\right.$$

c)



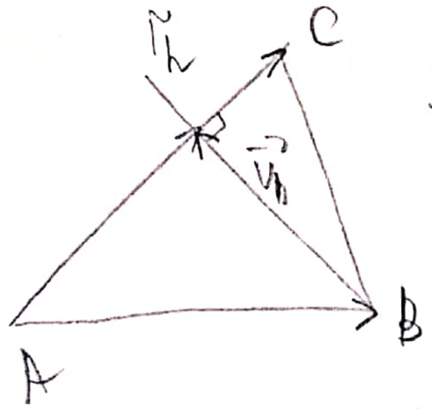

$S_{ABC} = \frac{1}{2} h \|\vec{AC}\| = \frac{1}{2}\|\vec{AB}\times\vec{AC}\| \Rightarrow$

$h = \dfrac{\|\vec{AB}\times\vec{AC}\|}{\|\vec{AC}\|}$.

$\|\vec{AB}\times\vec{AC}\| = \|(2,8,-3)\| = \sqrt{77}$ e

$\|\vec{AC}\| = \|(3,0,2)\| = \sqrt{13} \Rightarrow h = \sqrt{\dfrac{77}{13}}$

d)




Seja $\vec{v_h}$ vetor diretor da reta $r_h$

$\vec{v_h} = \text{Proj}_{\vec{AC}}\vec{AB} - \vec{AB}$.

$\text{Proj}_{\vec{AC}}\vec{AB} = \left(\dfrac{\vec{AC}\cdot\vec{AB}}{\|\vec{AC}\|^2}\right)\cdot\vec{AC} =$

$= \left(\dfrac{(3,0,2)\cdot(1,-1,-2)}{13}\right)(3,0,2) = -\dfrac{1}{13}(3,0,2) \Rightarrow$

$\vec{v_h} = -\frac{1}{13}(3,0,2)-(1,-1,-2) = -\frac{1}{13}(16,-13,-24) \Rightarrow$

$\underline{r_h: X = B+t\cdot\vec{v_h} = (-1,0,-1) - \frac{t}{13}(16,-13,-24)}$

(11) Temos equações de mudança de $\Sigma_2$ para $\Sigma_1$.



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\Sigma_1} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}_{\Sigma_1} + \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{\Sigma_2} \Rightarrow$$

$$\Rightarrow \begin{cases} x = -1 + y' + z' \\ y = x' + z' \\ z = 1 + x' + y' \end{cases} \Rightarrow \begin{cases} x = -1 + y' + z' \\ y - z = -1 + z' - y' \\ y = x' + z' \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x + y - z = -2 + 2z' \\ y' = 1 + x - z' \\ x' = y - z' \end{cases} \Rightarrow \begin{cases} z' = 1 + \frac{x+y-z}{2} \\ y' = \frac{x-y+z}{2} \\ x' = -1 + \frac{-x+y+z}{2} \end{cases}$$

$$r: \begin{cases} x = 1 \\ y = -t \\ z = 1 + 2t \end{cases} \Rightarrow \begin{cases} x' = -1 + \frac{-1-t+1+2t}{2} \\ y' = \frac{1+t+1+2t}{2} \\ z' = 1 + \frac{1-t-1-2t}{2} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x' = -1 + \frac{t}{2} \\ y' = 1 + \frac{3t}{2} \\ z' = 1 - \frac{3}{2}t \end{cases} \Rightarrow$$

$$r: \left[ X = (-1, 1, 1) + t\left(\frac{1}{2}, \frac{3}{2}, \frac{-3}{2}\right) \right]_{\Sigma_2}$$