Boundary regularity for area minimizing currents and a question of Almgren

Consider an area minimizing integral current T which has a smooth boundary Γ of multiplicity 1 in some smooth Riemannian ambient manifold Σ . If the current has codimension 1 a famous work of Hardt and Simon gives full regularity of T at the boundary. In higher codimension Allard's theory can be used to show regularity at points $p \in \Gamma$ where there is a uniformly convex barrier such that T lies locally on one side of it (cf. Hardt's work). While in some cases this guarantees the presence of some regular boundary point, in general the existence of even one of them is still open. In a recent joint work with Guido De Philippis, Jonas Hirsch and Annalisa Massaccesi we show that boundary regular points are always dense in Γ . This has some interesting consequences on the structure of the minimizer and in particular it allows us to answer positively to a question raised by Almgren at the end of his big regularity paper.