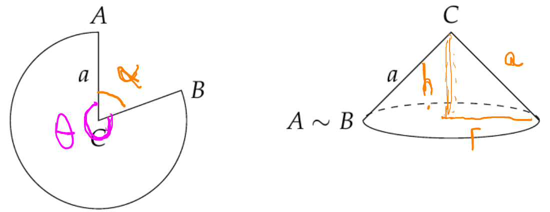
11. Um papel de filtro circular de raio a deve ser transformado em um filtro cônico cortando um setor circular e juntando as arestas CA e CB. Ache a razão entre o raio e a profundidade do filtro de capacidade máxima.



$a^2 = h^2 + r^2$

$$V_{cone} = \frac{A_b \cdot h}{3}$$

$$V(\theta) = \frac{\pi r^2 \cdot h}{3} = \frac{\pi \cdot \theta^2 \cdot a^2}{12\pi^2} \cdot \frac{a}{2\pi} \cdot \sqrt{4\pi^2 - \theta^2}, \quad \theta \in [0, 2\pi]$$

$$= \frac{a^3}{24\pi^2} \cdot \theta^2 \cdot \sqrt{4\pi^2 - \theta^2}$$

$$\frac{-\cancel{2}\theta}{\cancel{2}\sqrt{\quad}}$$

$$V'(\theta) = \frac{a^3}{24\pi^2}\left[2\theta \cdot \sqrt{4\pi^2 - \theta^2} - \frac{\theta^3}{\sqrt{4\pi^2 - \theta^2}}\right], \quad \theta \in]0, 2\pi[$$

$$V'(\theta) = 0 \iff 2\theta(4\pi^2 - \theta^2) = \theta^3 \Rightarrow 3\theta^3 = 2\theta \cdot 4\pi^2$$

$$\Rightarrow \theta = \frac{2\sqrt{2}\,\pi}{\sqrt{3}}$$

Arco AB: $2\pi - 2\pi a$

$\theta - l$

$$\Rightarrow \boxed{l = \theta \cdot a} \rightarrow l = 2\pi r \Rightarrow \theta \cdot a = 2\pi r$$

$$\Rightarrow \boxed{r = \frac{\theta \cdot a}{2\pi}}$$

$$h = \sqrt{a^2 - r^2} = \sqrt{a^2 - \frac{\theta^2 a^2}{4\pi^2}} \Rightarrow \boxed{h = \frac{a}{2\pi}\sqrt{4\pi^2 - \theta^2}}$$

∘) No bordo: $V(0) = V(2\pi) = 0$

∘) $V\left(\frac{2\sqrt{2}\,\pi}{\sqrt{3}}\right) = \ldots$ é máximo.

$$\Rightarrow r\left(\frac{2\sqrt{2}\,\pi}{\sqrt{3}}\right) = \frac{\sqrt{2}\,a}{\sqrt{3}}; \quad h\left(\frac{2\sqrt{2}\,\pi}{\sqrt{3}}\right) = \frac{a}{2\pi}\sqrt{4\pi^2 - \frac{8\pi^2}{3}} = \frac{a}{\sqrt{3}}.$$

$$\Rightarrow r/h = \sqrt{2}.$$

40. $\displaystyle\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

$x = \operatorname{tg} u$ ou $\underline{x = \operatorname{sen} u} \Rightarrow dx = \cos u \, du$

$$\| $$

$$\int \frac{\cos u \, du}{(1+\operatorname{sen}^2 u)\cos u} = \int \frac{du}{1+\operatorname{sen}^2 u} = \int \frac{\sec^2 u \, du}{\sec^2 u + \operatorname{tg}^2 u} =$$

$$= \int \frac{\sec^2 u \, du}{1 + 2\operatorname{tg}^2 u} \underset{z = \operatorname{tg} u}{=} \int \frac{dz}{1+2z^2}$$

$$dz = \sec^2 u \, du$$

$$= \int \frac{dz}{1+(\sqrt{2}z)^2} = \frac{1}{\sqrt{2}} \int \frac{dw}{1+w^2} = \frac{1}{\sqrt{2}} \operatorname{atan}(w) + k$$

$$w = \sqrt{2} z$$

$$dw = \sqrt{2}\, dz$$

$$= \frac{1}{\sqrt{2}} \operatorname{atan}(\sqrt{2} z) + k$$

$$= \frac{1}{\sqrt{2}} \operatorname{atan}(\sqrt{2} \operatorname{tg} u) + k$$

$$= \frac{1}{\sqrt{2}} \operatorname{atan}\left(\frac{\sqrt{2}\, x}{\sqrt{1-x^2}}\right) + k$$

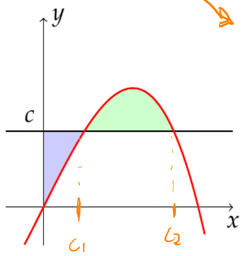
$$\operatorname{tg} u = \frac{\operatorname{sen} u}{\cos u} = \frac{x}{\sqrt{1-x^2}}$$

36. $\displaystyle\int \sqrt{a^2+b^2x^2}\, dx$

$$\sqrt{a^2+b^2x^2} = a\sqrt{1+\left(\frac{bx}{a}\right)^2} \qquad \frac{bx}{a} = \operatorname{tg} u$$

5. A reta horizontal $y = c$ intercepta a curva $y = 2x - 3x^3$ no primeiro quadrante como mostra a figura. Determine c para que as áreas das duas regiões sombreadas sejam iguais.

$y = f(x) \Rightarrow \int f(x)\,dx = x^2 - \frac{3x^4}{4} + K$



$$\underset{\underset{c}{\|}}{f(c_1)} = f(c_2) \Rightarrow \boxed{2c_1 - 3c_1^3 = 2c_2 - 3c_2^3.}$$

$$\int_0^{c_1} c - f(x)\,dx = \int_{c_1}^{c_2} f(x) - c\,dx$$

$$\Downarrow$$

$$\cancel{c_1}c - \cancel{c_1^2} + \frac{3\cancel{c_1^4}}{4} = c_2^2 - \frac{3c_2^4}{4} - \cancel{c_1^2} + \frac{3\cancel{c_1^4}}{4} - c(c_2 - \cancel{c_1}).$$

$$c_2^2 - \frac{3c_2^4}{4} - cc_2 = 0 \Rightarrow c = c_2 - \frac{3}{4}c_2^3$$

$$\Rightarrow 4c = 4c_2 - 3c_2^3$$

$$\Rightarrow 4f(c_2) = c_2(4 - 3c_2^2)$$

$$\Rightarrow 8c_2 - 12c_2^3 = 4c_2^2 - 3c_2^3 \Rightarrow 8 - 12c_2^2 = 4c_2 - 3c_2^2$$

$$\Rightarrow c_2 = \ldots$$

37. $\displaystyle\int \frac{1}{\sqrt{a^2+b^2x^2}}\,dx = \frac{1}{a}\int \frac{1}{\sqrt{1+\left(\frac{bx}{a}\right)^2}}\,dx$ $\qquad \frac{bx}{a} = \operatorname{tg} u \Downarrow \qquad \frac{b}{a}\,dx = \sec^2 u\,du$

$$= \frac{1}{b}\int \frac{\sec^2 u\,du}{\sec u} = \frac{1}{b}\int \sec u\,du$$

$$= \frac{1}{b}\ln|\sec u + \operatorname{tg} u| + k$$

$$= \frac{1}{b}\ln\left|\sqrt{1+\left(\frac{bx}{a}\right)^2} + \frac{bx}{a}\right| + k.$$

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22) $\displaystyle\int e^{x^3}\cdot x^2\,dx = \int \frac{e^u}{3}\,du = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + k$

$$= \frac{e^{x^3}}{3} + k$$

$u = x^3$

$du = 3x^2\,dx$

39) $\displaystyle\int \sqrt{3-2x-x^2}\,dx = \int \sqrt{4-(1+x)^2}\,dx = 2\int \sqrt{1-\left(\frac{x+1}{2}\right)^2}\,dx$

$\frac{x+1}{2} = \operatorname{sen} u \Rightarrow \dots$

38) $\displaystyle\int \sqrt{x^2-2x+2}\,dx = \int \sqrt{(x-1)^2+1}\,dx$

$\sec^2 u = 1 + \operatorname{tg}^2 u$

$\cosh^2 u - \operatorname{senh}^2 u = 1$

$x - 1 = \operatorname{tg} u$

$dx = \sec^2 u\,du$

$$= \int \sec^3 u\,du$$

$$= \frac{\sec u\,\operatorname{tg} u}{2} + \frac{1}{2}\ln|\sec u + \operatorname{tg} u|$$

$$\frac{\sqrt{1+(x-1)^2}\,(x-1)}{2} + \frac{1}{2}\ln\left|\sqrt{} + (x-1)\right| + k$$

$\cosh u = \dfrac{e^u + e^{-u}}{2}$

$x - 1 = \operatorname{senh} u$

$dx = \cosh u\,du$

$$= \int \cosh^2 u\,du$$

$$= \int \frac{e^{2u} + 2 + e^{-2u}}{4}\,du$$

$$= \frac{e^{2u}}{8} + \frac{u}{2} - \frac{e^{-2u}}{8} + K$$

$$= \frac{e^{2u} - e^{-2u}}{2}\cdot\frac{1}{4} + u + k$$

$$= \frac{\operatorname{senh}(2u)}{4} + u + k = \text{em } x.$$