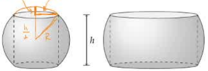
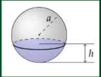
12. Um anel esférico é o sólido que permanece após a perfuração de um buraco cilíndrico através do centro de uma esfera sólida. Se a esfera tem raio R e o anel esférico tem altura h, prove o fato notável de que o volume do anel depende de h, mas não de R.

$R - h/2$ $\qquad r = \sqrt{R^2 - \frac{h^2}{4}} = \frac{\sqrt{4R^2 - h^2}}{2}$





10. $\pi h^2\left(a - \frac{h}{3}\right)$.

$y = \sqrt{a^2 - x^2}$ $\qquad x^2 + y^2 = a^2$

$$V = \int_{a-h}^{a} \pi.(a^2 - x^2)\,dx$$

$$= \pi.\left(a^2x - \frac{x^3}{3}\right)\Big|_{a-h}^{a}$$

$$= \pi\left(a^3 - \frac{a^3}{3} - \left(a^2(a-h) - \frac{(a-h)^3}{3}\right)\right)$$

$$V = V_{esf} - 2V_{cal} - V_{cil} = \frac{4}{3}\pi R^3 - \frac{\pi}{6}(R-h)^2 \cdot \left(3R - R + h/2\right) - \frac{\pi}{4}(4R^2 - h^2)h$$

$$V_{esf} = \frac{4}{3}\pi R^3$$

$$V_{cil} = \frac{\pi}{4}(4R^2 - h^2).h$$

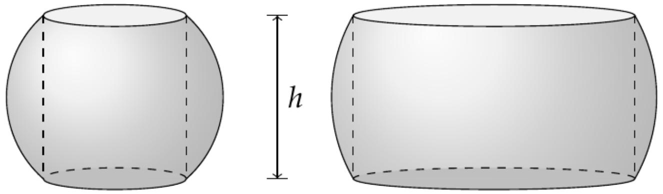
$$V_{cal} = \pi(R - h/2)^2 \cdot \left(R - \frac{R - h/2}{3}\right)$$

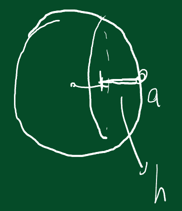
$$= \frac{4}{3}\pi R^3 - \frac{\pi}{6}(R^2 - 2Rh + h^2)(2R + h/2) + \pi R^2 - \frac{\pi h^3}{4}$$

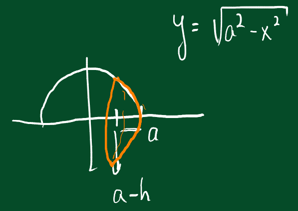
$$= \ldots$$

Ou $V'(R) = \ldots = 0$.

12. Um anel esférico é o sólido que permanece após a perfuração de um buraco cilíndrico através do centro de uma esfera sólida. Se a esfera tem raio R e o anel esférico tem altura h, prove o fato notável de que o volume do anel depende de h, mas não de R.







$$V = \int_{a-h}^{a} \pi (a^2 - x^2)\, dx$$

$$= \pi \left(a^2 x - x^3/3 \right) \Big|_{a-h}^{a}$$

$$= \pi \left(a^3 - a^3/3 - \left(a^2(a-h) - (a-h)^3/3 \right) \right).$$

Feito anteriormente (antes do bug).

11. Comp. de $y=\cosh x$, $-3\leq x\leq 4$

$$L=\int_{-3}^{4}\sqrt{1+(y')^2}\,dx=\int_{-3}^{4}\sqrt{1+\operatorname{senh}^2 x}\,dx=\int_{-3}^{4}\cosh x\,dx$$

$$=\operatorname{senh} x\Big|_{-3}^{4}=\operatorname{senh}4-\operatorname{senh}(-3)$$

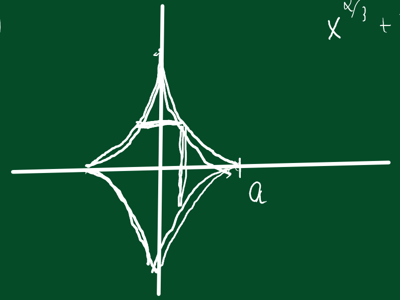
$$=\operatorname{senh}4+\operatorname{senh}3.$$

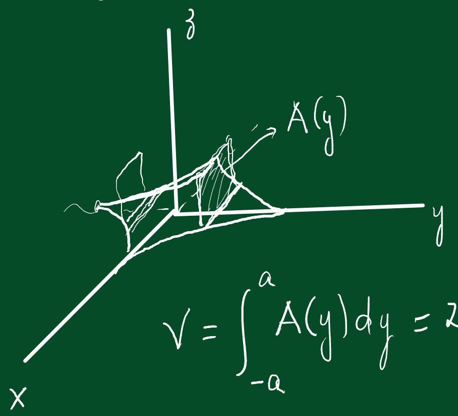
$$y(x)=\cosh x=\frac{e^{x}+e^{-x}}{2}\qquad \cosh^2 x-\operatorname{senh}^2 x=1$$

$$y'(x)=\frac{e^{x}-e^{-x}}{2}=\operatorname{senh} x$$

3) $x^{2/3}+y^{2/3}=a^{2/3}$

$x,y\geq 0\Rightarrow y=(a^{2/3}-x^{2/3})^{3/2}$

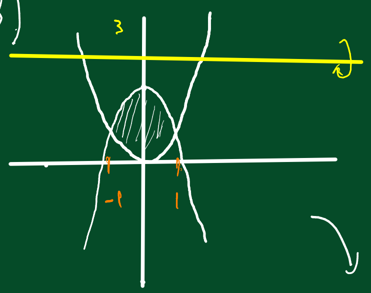


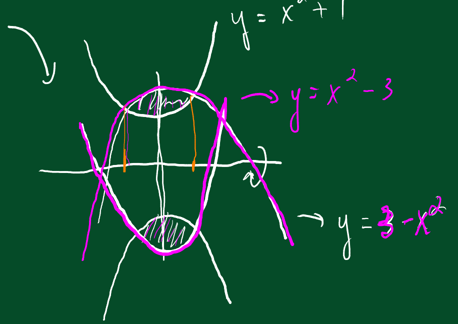


$$A(y)=\left(2\left(a^{2/3}-y^{2/3}\right)^{3/2}\right)^{2}$$
$$=4\left(a^{2/3}-y^{2/3}\right)^{3}$$

$$V=\int_{-a}^{a}A(y)\,dy=2\int_{0}^{a}A(y)\,dy$$

8)



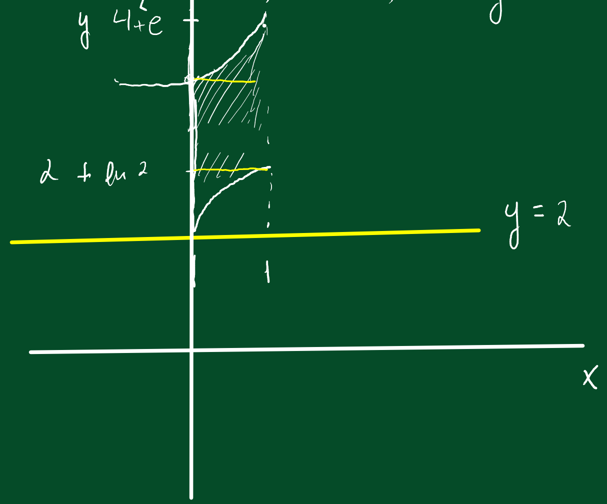


$$V=\int_{-1}^{1}\pi(3-x^2)^2-\pi(x^2+1)^2\,dx=\dots$$

$x^2=2-x^2$

$x=\pm 1$

7) $A=\{0\leq x\leq 1;\ \ln(x+1)+2\leq y\leq e^{x}+4\}$



$$V=\int_{0}^{1}\pi\left(e^{x}+2\right)^{2}dx-\int_{0}^{1}\pi\left(\ln(x+1)\right)^{2}dx$$

Secão 3:

$$\cos^2 x + \operatorname{sen}^2 x = 1$$

$$1 + tg^2 x = \sec^2 x$$

36) $$\int \sqrt{a^2+b^2x^2}\,dx = \int \sqrt{a^2\left(1+\frac{b^2x^2}{a^2}\right)}\,dx$$

$$= a\int \sqrt{1+\left(\frac{bx}{a}\right)^2}\,dx = a\cdot\int \sqrt{1+tg^2 u}\;\frac{a}{b}\cdot\sec^2 u\,du = \frac{a^2}{b}\cdot\int \sec^3 u\,du$$

$$\frac{b}{a}\cdot x = \tan u$$

$$\frac{b}{a}dx = \sec^2 u\,du$$

$$= \frac{a^2}{2b}\left(\sec u\cdot tg u + \ln(\sec u + tg u) + k\right)$$

$$= \frac{a^2}{2b}\left(\frac{b}{a}x\cdot\sqrt{1+\left(\frac{b}{a}x\right)^2} + \ln\left(\frac{b}{a}x + \sqrt{1+\left(\frac{b}{a}x\right)^2}\right)\right) + k$$

$$= \frac{x}{2b}\sqrt{a^2+b^2x^2} + \frac{a^2}{2b}\ln\left(bx + \sqrt{a^2+b^2x^2}\right) + k$$

$$|\sec x + tg x| = \sqrt{\frac{1+\operatorname{sen}x}{1-\operatorname{sen}x}}$$

$$\Uparrow$$

$$\ln\sqrt{\frac{1+\operatorname{sen}x}{1-\operatorname{sen}x}} = \ln|\sec x + tg x|$$

$$\Uparrow$$

$$\int \sec x\,dx = \int \frac{\cos x}{\cos^2 x}dx = \int \frac{\cos x}{1-\operatorname{sen}^2 x}dx = \int \frac{1}{1-u^2}du = \frac{1}{2}\int \frac{1}{1-u} + \frac{1}{1+u}\,du = \frac{1}{2}\left(\ln(1+u) - \ln(1-u)\right) + k = \ln\sqrt{\frac{1+\operatorname{sen}x}{1-\operatorname{sen}x}} + k$$

ou $$\int \sec x\,dx = \ln|\sec x + tg x| + k$$

$$\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u} = \frac{1}{2(1-u)} + \frac{1}{2(1+u)}$$