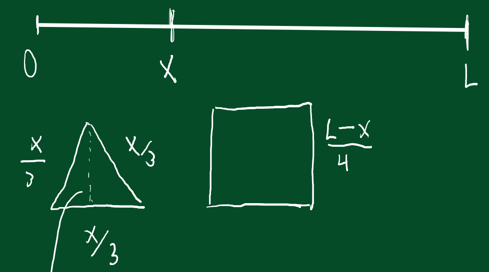
10. Um arame de comprimento L deve ser cortado em 2 pedaços, um para formar um quadrado e outro um triângulo equilátero. Como se deve cortar o arame para que a soma das áreas cercadas pelos 2 pedaços seja (a) máxima?; (b) mínima? Mostre que no caso (b) o lado do quadrado é 2/3 da altura do triângulo.



$$h = \sqrt{\frac{x^2}{9} - \frac{x^2}{36}} = \frac{\sqrt{3}\,x}{6}$$

$$A_\Delta = \frac{\sqrt{3}\,x^2}{36}$$

$$A_\square = \frac{(L-x)^2}{16}.$$

$$f(x) = \frac{(L-x)^2}{16} + \frac{\sqrt{3}\,x^2}{36}, \quad 0 \leqslant x \leqslant L$$

$$f'(x) = \frac{x-L}{8} + \frac{\sqrt{3}\,x}{18} = 0 \iff$$

$$\left(\frac{1}{8} + \frac{\sqrt{3}}{18}\right) x = L/8$$

$$\frac{9+4\sqrt{3}}{72} \cdot x = L/8$$

$$x = \frac{9L}{9+4\sqrt{3}} \in [0, L]$$

i) $f(0) = L^2/16$ $f(0) > f(L) \Rightarrow f(0)$ é max global

$f(L) = \sqrt{3}L^2/36$

$f(x) = \ldots\ldots$ min global

Técnicas de Integração: Frações Parciais

Primitivas p/ funções racionais, $f(x) = p(x)/q(x)$

p, q polinômios.

Teorema: Sejam $p(x), q(x)$ polinômios

com grau $q >$ grau p.

(i) $q(x) = (x - r_1)\ldots(x - r_k)$ então existem

$A_1, A_2, \ldots, A_k \in \mathbb{R}$ tq $\dfrac{p(x)}{q(x)} = \dfrac{A_1}{x - r_1} + \ldots + \dfrac{A_k}{x - r_k}$

(ii) $q(x) = (x - r_1)^{k_1} \ldots (x - r_j)^{k_j}$ então existem

$A_{11}, \ldots, A_{1k_1}, A_{21}, \ldots, A_{2k_2}, \ldots, A_{j1} \ldots A_{jk_j}$

tq $\dfrac{p(x)}{q(x)} = \dfrac{A_{11}}{x - r_1} + \dfrac{A_{12}}{(x - r_1)^2} + \ldots + \dfrac{A_{1k_1}}{(x - r_1)^{k_1}} +$

$\dfrac{A_{21}}{x - r_2} + \dfrac{A_{22}}{(x - r_2)^2} + \ldots + \dfrac{A_{2k_2}}{(x - r_2)^{k_2}} + \ldots + \dfrac{A_{jk_j}}{(x - r_j)^{k_j}}$

$$\frac{4x^2 + 2x + 1}{x^3 - x^2 - 2x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$\Downarrow$$

$$4x^2 + 2x + 1 = A\underbrace{(x-2)(x+1)}_{x^2 - x - 2} + Bx\overbrace{(x+1)}^{x^2 + x} + Cx\underbrace{(x-2)}_{x^2 - 2x}$$

$$\Downarrow$$

$$\begin{cases} A + B + C = 4 \\ -A + B - 2C = 2 \\ -2A = 1 \end{cases}$$

(iii) se $q(x) = (x - r)(ax^2 + bx + c)$ de grau 2 irredutível, $ax^2 + bx + c$, $b^2 - 4ac < 0$

$$\frac{p(x)}{q(x)} = \frac{Ax + B}{ax^2 + bx + c} + \frac{C}{x - r}$$

Exemplos:

1) $\displaystyle\int \frac{x^4 + x^2 + 1}{x^3 - x^2 - 2x}\,dx$

$\parallel$

$\displaystyle\int x + 1 + \frac{4x^2 + 2x + 1}{x^3 - x^2 - 2x}\,dx$

$\parallel$

$\dfrac{x^2}{2} + x +$

$\dfrac{x^2}{2} + x + A\ln|x| + B\ln|x-2| + C.\ln|x+1|$

$p(x) = k(x)q(x) + r(x)$, grau $r <$ grau q

$$\Downarrow$$

$$\frac{p(x)}{q(x)} = k(x) + \frac{r(x)}{q(x)}$$

$$\begin{array}{rl} x^4 + x^2 + 1 & \underline{|\,x^3 - x^2 - 2x} \\ -(x^4 - x^3 - 2x^2) & x + 1 \\ \hline x^3 + 3x^2 + 1 & \\ -(x^3 - x^2 - 2x) & \\ \hline 4x^2 + 2x + 1 & \end{array}$$

$$\Downarrow$$

$$x^4 + x^2 + 1 = (x+1)(x^3 - x^2 - 2x) + 4x^2 + 2x + 1$$

2) $\displaystyle\int \frac{2x+1}{\underbrace{x^3-x^2-x+1}} dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} dx$

$(x-1).(x^2-1)$

$= A \ln|x-1| - \frac{B}{x-1} + C.\ln|x+1| + k.$

$(x-1)^2 (x+1)$

$x^3-x^2-x+1 \;\; | \; x-1$

$-(x^3-x^2) \qquad x^2-1$

$-x+1$

$-(x-1)$

0

$\displaystyle\int \frac{1}{1+x^2} dx$

$\parallel$

$\operatorname{atan} x + k$

$$\frac{2x+1}{x^3-x^2-x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$= \frac{A(x^2-1) + B(x+1) + C(x-1)^2}{(x-1)^2(x+1)}$$

3) $\displaystyle\int \frac{x^2+2x+3}{x^2+4x+13} dx$

$x^2+2x+3 \;\; | \; x^2+4x+13$

$-(x^2+4x+13) \qquad 1$

$-2x-10$

$\parallel$

$\displaystyle\int 1 - \frac{2x+10}{x^2+4x+13} dx = x - \int \frac{2x+10}{x^2+4x+13} = x - \int \underbrace{\frac{2x+4}{x^2+4x+13}}_{\mathrm{I}} + \underbrace{\frac{6}{x^2+4x+13}}_{\mathrm{II}} dx$

$= x - \ln(x^2+4x+13) - 2\operatorname{atan}\left(\frac{x+1}{3}\right) + k.$

I $\displaystyle\int \frac{2x+4}{x^2+4x+13} dx = \int \frac{du}{u} = \ln|u| + k = \ln(x^2+4x+13) + k.$

$u = x^2+4x+13$

$du = (2x+4)dx$

$u = \frac{x+2}{3} \Rightarrow du = \frac{1}{3}dx$

II $\displaystyle\int \frac{6}{x^2+4x+13} dx = 6\int \frac{1 \, dx}{(x+2)^2+9} = \frac{6}{9}\int \frac{1}{\left(\frac{x+2}{3}\right)^2+1} dx = \frac{6}{9}\int \frac{3du}{u^2+1}$

$= 2\operatorname{atan} u + k = 2\operatorname{atan}\left(\frac{x+1}{3}\right) + k$

4) $\int \sec x \, dx = \int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx$

$$= \int \frac{\cos x}{1 - \operatorname{sen}^2 x} dx = \int \frac{du}{1-u^2} =$$

$u = \operatorname{sen} x$

$du = \cos x \, dx$

frações parciais.