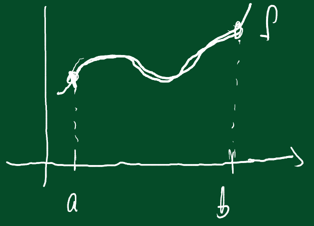
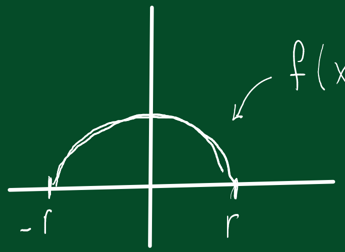
Comprimento de gráficos:



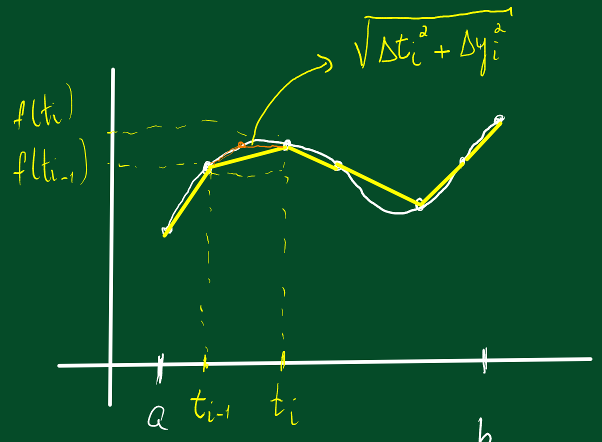
$f: [a,b] \to \mathbb{R}$ de classe C^1 (derivável com derivada contínua)

$L(f)$ comprimento do gráfico de f em $[a,b]$



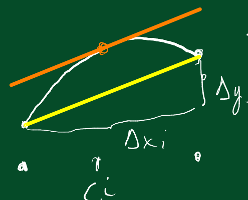
$f(x) = \sqrt{r^2 - x^2}$

$L(f) = \pi r.$



$\Delta t_i = \Delta x_i$

$$L \approx \sum_{i=1}^{n} \sqrt{\Delta t_i^2 + \Delta y_i^2} = \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot \Delta x_i \quad (*)$$



$$f'(c_i) = \frac{\Delta y_i}{\Delta x_i}$$

$$(*) = \sum_{i=1}^{n} \sqrt{1 + f'(c_i)^2}\, \Delta x_i = S(g, P, \{c_i\}) \text{ do TVM}$$

$\{t_i\}_{i=0}^{n}$

$g(x) = \sqrt{1 + f'(x)^2}.$

$(f(x) + k)' = f'(x)$

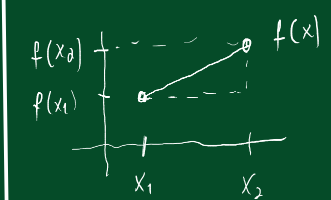
$(f(x-a))' = f'(x-a)$

Tomando limite com $|P| \to 0$

$$\lim_{|P| \to 0} S(g, P, \{c_i\}) = \int_a^b g(x)\,dx \Rightarrow \boxed{L(f) = \int_a^b \sqrt{1 + f'(x)^2}\, dx.}$$

Exemplos:

1) $f(x) = ax + b \Rightarrow f'(x) = a$



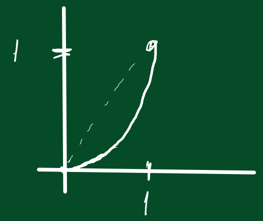
$$L(f) = \int_{x_1}^{x_2} \sqrt{1 + a^2}\, dx = \sqrt{1+a^2}\, x \Big|_{x_1}^{x_2}$$

$$= \sqrt{1+a^2}\,(x_2 - x_1)$$

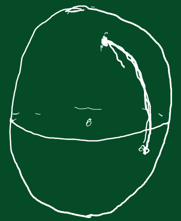
ou $L = \sqrt{(f(x_2) - f(x_1))^2 + (x_2 - x_1)^2} = \sqrt{(a(x_2 - x_1))^2 + (x_2 - x_1)^2}$

$$= (x_2 - x_1) \cdot \sqrt{a^2 + 1}.$$

2) $f(x) = x^2, \quad 0 \le x \le 1$



$f'(x) = 2x$



$$L(f) = \int_0^1 \sqrt{1+f'(x)^2}\,dx$$

$$= \int_0^1 \sqrt{1+4x^2}\,dx \qquad x = \frac{tg\,u}{2} \rightarrow u = atan(2x) \qquad 1 + tg^2 x = sec^2 x$$

$$dx = \frac{sec^2 u\,du}{2}$$

$$= \int_0^{atan(2)} \sqrt{1 + \cancel{4}\cdot\frac{tg^2 x}{\cancel{4}}}\;\frac{sec^2 u\,du}{2}$$

$$= \int_0^{atan(2)} \frac{sec^3 u\cdot du}{2} = \frac{sec(atan(2))\,tg(\overset{2}{\cancel{atan(2)}})}{4} + \frac{ln\left(sec(atan(2)) + tg(\overset{2}{\cancel{atan(2)}})\right)}{4} - 0$$

$$= \frac{\sqrt5}{2} + \frac{ln(2+\sqrt5)}{4} = \frac{\sqrt5 + ln\left(\sqrt{2+\sqrt5}\right)}{2}.$$

$$\int sec^3 u\,du = \int \underset{f}{sec\,u}\cdot \underset{g'}{sec^2 u}\,du = sec\,u\cdot tg\,u - \int sec\,u\cdot tg\,u\cdot tg\,u\,du$$

$$= sec\,u\,tg\,u - \int sec\,u\,tg^2(u)\,du$$

$$= sec\,u\,tg\,u - \int sec^3 u - sec\,u\,du$$

$$\Rightarrow 2\int sec^3 u\,du = sec\,u\,tg\,u + \int sec\,u\,du \Rightarrow \int sec^3 u\,du = \frac{sec\,u\,tg\,u}{2} + \frac{ln|sec\,u + tan\,u|}{2} + K$$

$$2 = tg(atan\,2) = \frac{sen(atan\,2)}{cos(atan\,2)} = \frac{\sqrt{1-cos^2(atan\,2)}}{cos(atan\,2)}$$

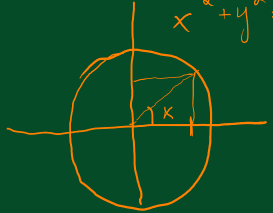
$$\Rightarrow 4\,cos^2(atan\,2) = 1 - cos^2(atan\,2) \Rightarrow cos(atan\,2) = \frac{\sqrt5}{5}$$

$$\Rightarrow sec(atan\,2) = \sqrt5$$

Lista 3: 11) $\underbrace{cosh\,x}_{f(x)} = \frac{e^x+e^{-x}}{2} \Rightarrow f'(x) = \frac{e^x - e^{-x}}{2} = senh\,x$

$$cosh^2 x - senh^2 x = 1$$

$x^2 - y^2 = 1$



$x^2 + y^2 = 1$

Máximos e mínimos:

1) Existe $x>0$ tq a soma dele e seu inverso é mínima? E se $x \in [2,3]$?

∘) $f(x) = x + \frac{1}{x}$, $x>0$ tem mínimo?

$f'(x) = 1 - \frac{1}{x^2} = 0 \iff x=1$ ou $\cancel{x=-1}$ é crítico.

	1	
f'	$-$	$+$
f	$\searrow$	$\nearrow$

$\Rightarrow$ 1 é mínimo local. $\lim_{x\to 0^+} f(x) = +\infty$

$\lim_{x\to +\infty} f(x) = +\infty$.

1 é mínimo global.

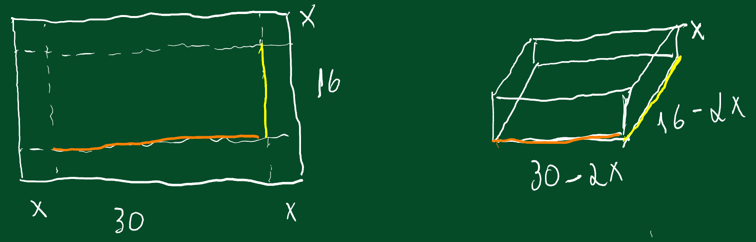
∘) $2 \leq x \leq 3$, $f(x)$ tem mínimo? Weierstraß!

∘) $f'(x)=0$ ñ tem solução.

∘) $f(2) = 5/2 \rightarrow 2$ é mínimo global.

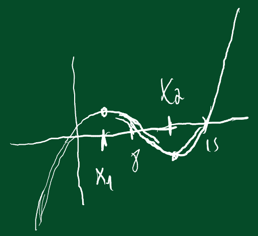
$f(3) = 10/3$

2)



Determine x tq o volume seja máximo

$V(x) = x(16-2x)(30-2x)$, $0 \leq x \leq 8$



∘) $V(0) = 0 = V(8)$

∘) $V'(x) = (16-2x)(30-2x) - 2x(30-2x) - 2x(16-2x)$

$= 4\big((8-x)(15-x) - x(15-x) - x(8-x)\big)$

$= 4(3x^2 - 46x + 120) = 0 \Rightarrow x = \frac{10}{3}$ ou $x \neq 12$

3) Mario Paper:

https://www.youtube.com/watch?v=qIw-EVmnr2Q&t=0s