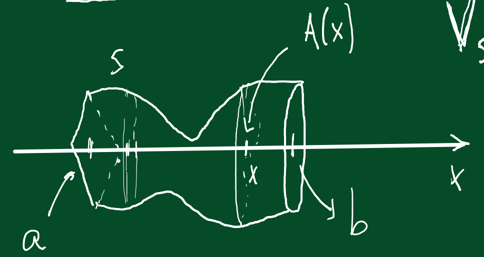
Volumes:



$V_S \approx \sum_{i=1}^{n} A(c_i).\Delta x_i \qquad P = \{x_i\}_{i=0}^{n}$

$\qquad\qquad \| $

$\qquad S(A, P, \{c_i\})$

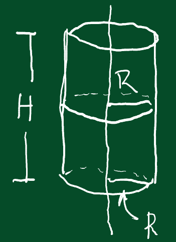
tomando limites temos

$$V_S = \int_a^b A(x)\,dx$$

$A : [a,b] \longrightarrow \mathbb{R}^+$ é contínua

$x \longmapsto A(x)$ área da "fatia x" de S

Exemplos: 1) cilindro:



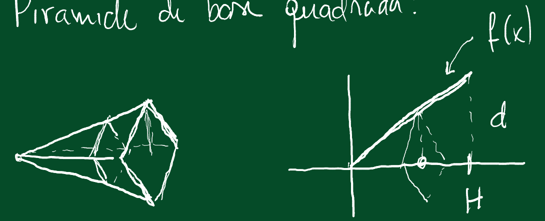
$A : [0,H] \longrightarrow \mathbb{R}^+$

$x \longmapsto \pi R^2$ (cte)

$$V = \int_0^H A(x)\,dx = \int_0^H \pi R^2 dx = \pi R^2 x \Big|_0^H$$

$$= \pi R^2 H.$$

2) Pirâmide de base quadrada:



$f(x) = \frac{d}{H} x$

$d = l\sqrt{2}$

$l = \frac{d}{\sqrt{2}}$

$A = l^2 = \frac{d^2}{2}$

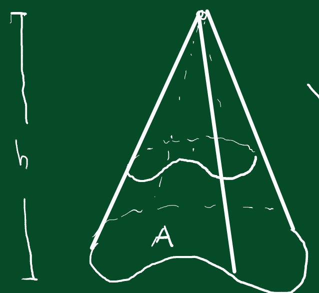
$D = 2d$

$l = D/\sqrt{2}$

$l^2 = \frac{D^2}{2} = 2d^2$

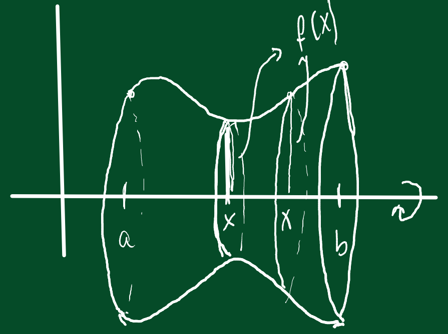
$$A(x) = \frac{(2f(x))^2}{2} = 2f^2(x) = \frac{2d^2x^2}{H^2}$$

$$V = \int_0^H \frac{2d^2x^2}{H^2}dx = \frac{2d^2}{H^2}\cdot\frac{x^3}{3}\Big|_0^H = \frac{\overbrace{2d^2}^{l^2} H}{3} = \frac{H}{3}.A_{base} = \frac{H}{3}.l^2.$$



$V = A \cdot \frac{h}{3}$

Volumes "de rotação":

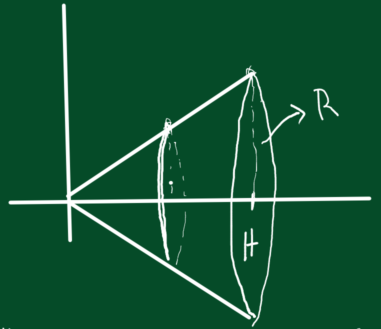


$$V = \int_a^b A(x)\,dx \Rightarrow V = \int_a^b \pi . f(x)^2\,dx$$

$$\Rightarrow \boxed{V = \pi \int_a^b f^2(x)\,dx}$$

Exemplos:

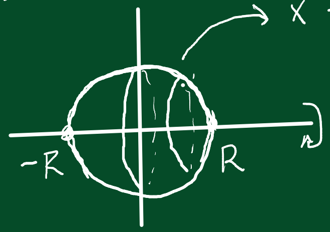
1) Cone:



$$f(x) = \frac{R}{H}x$$

$$V = \int_0^H \pi . \left(\frac{R^2x^2}{H^2}\right) dx = \frac{\pi R^2}{H^2} . \frac{x^3}{3}\Big|_0^H = \pi R^2 . \frac{H}{3}.$$

2) Esfera:



$$x^2 + y^2 = R^2 \Rightarrow y = \sqrt{R^2 - x^2}$$

$$V(R) = \frac{4}{3}\pi R^3$$

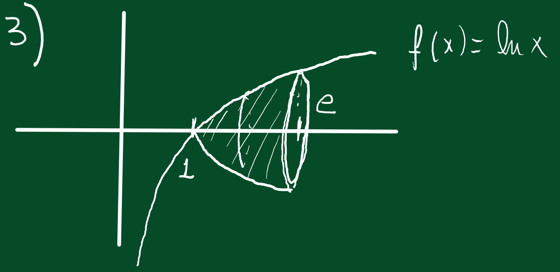
$$V'(R) = 4\pi R^2 = A(R)$$

$$V = \int_{-R}^{R} \pi\, y^2(x)\,dx = \pi \int_{-R}^{R} (R^2 - x^2)\,dx$$

$$= \pi\left(R^2x - x^3/3\right)\Big|_{-R}^{R} = \pi\left(R^3 - R^3/3 - \left(-R^3 + R^3/3\right)\right)$$

$$= \pi\left(2R^3 - 2R^3/3\right) = \frac{4}{3}\pi R^3.$$

3)



$$V=\int_1^e \pi \cdot \ln^2 x\,dx = \pi\left(x\ln^2 x - 2x\ln x + 2x\Big|_1^e\right)$$

$$=\pi(e-2).$$

$$\int \ln^2 x\,dx = \int 1\cdot \ln^2 x\,dx = x\cdot \ln^2 x - 2\int \ln x\,dx = x\ln^2 x - 2(x\ln x - x) + C$$

$$= x\ln^2 x - 2x\ln x + 2x + C.$$

$f'(x)=1 \Rightarrow f(x)=x$

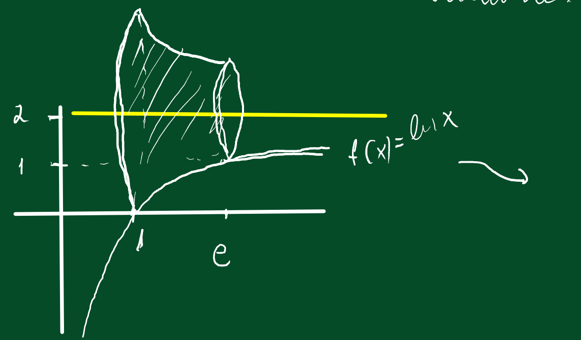
$g(x)=\ln^2 x \Rightarrow g'(x)=\dfrac{2\ln x}{x}$

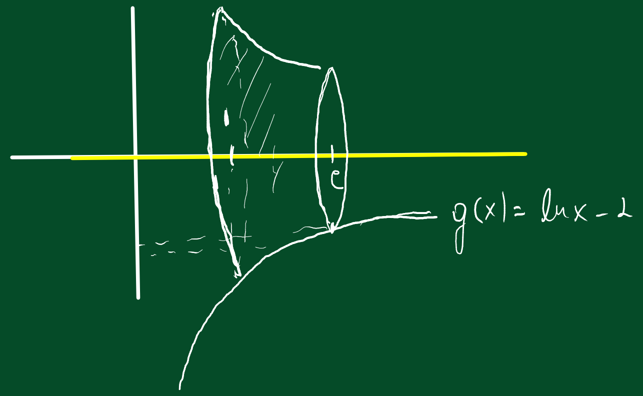
$$\int \ln x\,dx = \int 1\cdot \ln x\,dx = x\ln x - \int 1\,dx = x\ln x - x + C$$

$f'(x)=1 \Rightarrow f(x)=x$

$g(x)=\ln x \Rightarrow g'(x)=\dfrac{1}{x}$

Variante:



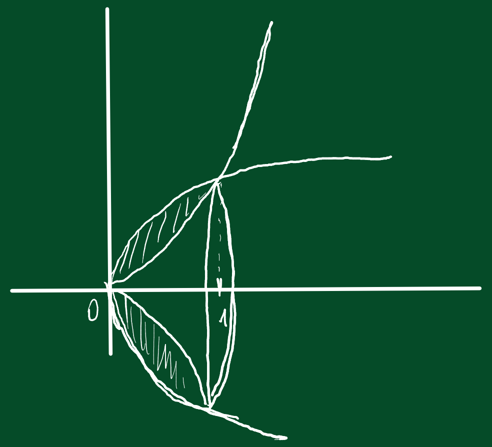


$$V=\int_1^e \pi\, g^2(x)\,dx = \pi\int_1^e (\ln x - 2)^2\,dx$$

$$=\pi\int_1^e \ln^2 x\,dx - 4\pi\int_1^e \ln x\,dx + 4\pi\int_1^e 1\,dx$$

$$=\pi(e-2) - 4\pi + 4\pi(e-1).$$

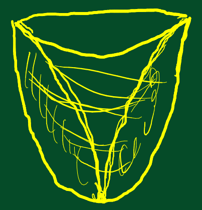
4) Volume de rotação da região entre $y = x^2$ e $y = \sqrt{x}$ limitada



$f(x) = x^2$; $g(x) = \sqrt{x}$

$f(a) = g(a) \Rightarrow a^2 = \sqrt{a}$

$\Rightarrow a = 0$ ou $a = 1$



$$V = \pi \int_0^1 (\sqrt{x})^2 \, dx - \pi \int_0^1 (x^2)^2 \, dx$$

$$= \pi \int_0^1 g^2(x) - f^2(x) \, dx = \pi \int_0^1 x \, dx - \pi \int_0^1 x^4 \, dx$$

$$= \frac{\pi}{2} - \pi/5 .$$