1) $\int \sec^2 x\, dx = tg\, x + K$

2) $\int tg^2 x\, dx = \int \sec^2 x - 1\, dx = tg^2 x - x + K$

$\sec^2 x = 1 + tg^2 x$

3) $\int tg\, x \sec^2 x\, dx = \int u\, du = \frac{u^2}{2} + K = \frac{tg^2 x}{2} + K.$ $\quad F(x) = \frac{tg^2 x}{2}$

$u = tg\, x$

$du = \sec^2 x\, dx$

ou $\int tg\, x \sec^2 x\, dx = \int \sec x \cdot tg\, x \sec x\, dx = \int u\, du = \frac{u^2}{2} + K = \frac{\sec^2 x}{2} + K$

$u = \sec x$

$du = \sec x\, tg\, x\, dx$

$G(x) = \frac{\sec^2 x}{2}$

$F(x) - G(x) = -\frac{1}{2}$

4) $\int tg^3 x\, dx = \int \frac{sen^3 x}{\cos^3 x}\, dx = \int \frac{-1 + u^2}{u^3}\, du = \int \frac{1}{u} - \frac{1}{u^3}\, du = \ln|u| + \frac{1}{2u^2} + K$

$= \ln|\cos x| + \frac{\sec^2 x}{2} + K.$

$sen^3 x = (1 - \cos^2 x)\, sen\, x$

$u = \cos x$

$du = -sen\, x\, dx$

ou $\int \tan^3 x\, dx = \int tg\, x\, tg^2 x\, dx = \int \tan x (\sec^2 x - 1)\, dx$

$= \frac{\tan^2 x}{2} + \ln|\cos x| + K.$

$\int tg\, x\, dx = \int \frac{sen\, x}{\cos x}\, dx = \int -\frac{1}{u}\, du = -\ln|\cos x| + K = \ln|\sec x| + K$

$u = \cos x$

$du = -sen\, x\, dx$

5) $\int \sec x\, dx = \int \frac{\sec x \cdot (\sec x + tg\, x)}{\sec x + tg\, x}\, dx = \int \frac{du}{u} = \ln|u| + K = \ln|\sec x + tg\, x| + K.$

$u = \sec x + tg\, x$

$du = (\sec x\, tg\, x + \sec^2 x)\, dx$

Integração por partes:

$$(f\cdot g)' = f'g + f\cdot g'$$

$$\Downarrow$$

$$K + f(x)g(x) = \int f'(x)g(x) + f(x).g'(x)\,dx$$

$$= \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$\Rightarrow \int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

Exemplos: 1) $\int x.\cos x\,dx = x.\operatorname{sen} x - \int \operatorname{sen} x.1\,dx = \underbrace{x\operatorname{sen} x + \cos x}_{F(x)} + K$

$f'(x) = \cos x \Rightarrow f(x) = \operatorname{sen} x$

$g(x) = x \Rightarrow g'(x) = 1$

$F'(x) = 1\cancel{\operatorname{sen} x} + x\cos x - \cancel{\operatorname{sen} x}$

$= x\cos x.$

2) $\int 1.\operatorname{atan} x\,dx = x\operatorname{atan} x - \int \frac{x}{1+x^2}\,dx = x\operatorname{atan} x - \int \frac{1}{2u}\,du = x\operatorname{atan} x - \frac{1}{2}\ln|u| + K$

$f'(x) = 1 \Rightarrow f(x) = x$

$g(x) = \operatorname{atan} x \Rightarrow g'(x) = \frac{1}{1+x^2}$

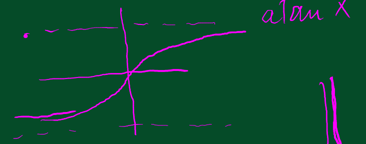
$u = 1 + x^2$

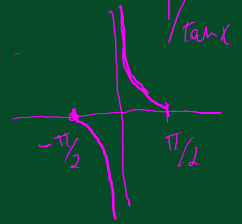
$du = 2x\,dx$

$= x\operatorname{atan} x - \frac{1}{2}\ln(1+x^2) + K$

$\tan(\operatorname{atan} x) = \operatorname{atan}(\tan x) = x$

$\frac{\cos x}{\operatorname{sen} x} = \frac{1}{\tan x}$





3) $\int x^2\operatorname{sen} x\,dx = -x^2\cos x + 2\int x\cos x\,dx = -x^2\cos x + 2(x\operatorname{sen} x + \cos x) + K.$

$f'(x) = \operatorname{sen} x \Rightarrow f(x) = -\cos x$

$g(x) = x^2 \Rightarrow g'(x) = 2x$

4) $\int e^x\cos x\,dx = e^x\cos x + \int e^x\operatorname{sen} x\,dx = e^x\cos x + e^x\operatorname{sen} x - \int e^x\cos x\,dx$

$f'(x) = e^x \Rightarrow f(x) = e^x$ $\quad$ $f'(x) = e^x \Rightarrow f(x) = e^x$

$g(x) = \cos x \Rightarrow g'(x) = -\operatorname{sen} x$ $\quad$ $g(x) = \operatorname{sen} x \Rightarrow g'(x) = \cos x$

$\Rightarrow 2\int e^x\cos x\,dx = e^x(\cos x + \operatorname{sen} x) + K \Rightarrow \int e^x\cos x\,dx = \frac{e^x}{2}(\cos x + \operatorname{sen} x) + K.$

5) $\int \cos^2 x\, dx = \int \cos x \cdot \cos x\, dx = \operatorname{sen} x \cos x + \int \operatorname{sen}^2 x\, dx$

$f'(x) = \cos x \Rightarrow f(x) = \operatorname{sen} x$

$g(x) = \cos x \Rightarrow g'(x) = -\operatorname{sen} x$

$= \operatorname{sen} x \cos x + \int 1 - \cos^2 x\, dx$

$= \operatorname{sen} x \cos x + x - \int \cos^2 x\, dx$

$\Rightarrow \int \cos^2 x\, dx = \frac{x}{2} + \frac{\operatorname{sen} x \cos x}{2} + K.$

$u = 2x$

$du = 2dx$

ou $\int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2}\, dx = \frac{1}{2}\int 1 + \cos 2x\, dx$

$= \frac{1}{2}\left(x + \frac{\operatorname{sen}(2x)}{2} \right) + K$

$\operatorname{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$

$\cos^2 x - \operatorname{sen}^2 = \cos(2x)$

$\int_a^b f(x)\,dx$ $\quad (\sqrt{u})'$

$\|$ $\quad = \frac{1}{2\sqrt{u}}$

$-\int_b^a f(x)\,dx$

6) $\int \sec^3(x)\, dx = \int \sec x \cdot \sec^2 x = \sec x \tan x - \int \sec x \operatorname{tg}^2 x\, dx$ (*)

$\sec^2 x = 1 + \operatorname{tg}^2 x$

$f'(x) = \sec^2 x \Rightarrow f(x) = \tan x$

$g(x) = \sec x \Rightarrow g'(x) = \sec x \tan x$

(*) $= \sec x \tan x - \int \sec x (\sec^2 x - 1)\, dx = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

$\Rightarrow \int \sec^3 dx = \frac{\sec x \tan x}{2} + \frac{\ln|\sec x + \tan x|}{2} + K.$

7) $\int_0^{1/2} 1 \cdot \operatorname{arcsen} x\, dx = x \operatorname{arcsen} x \Big|_0^{1/2} - \int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\, dx = \frac{\pi}{12} + \int_1^{3/4} \frac{1}{\sqrt{u}} \frac{du}{2}$ (*)

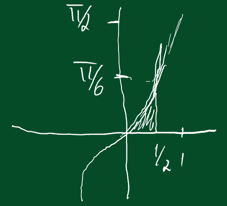
$f'(x) = 1 \Rightarrow f(x) = x$

$g(x) = \operatorname{arcsen} x \quad g'(x) = \frac{1}{\sqrt{1-x^2}}$

$u = 1 - x^2$

$du = -2x\, dx$

(*) $= \frac{\pi}{12} + \sqrt{u}\Big|_1^{3/4} = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1.$



Bônus: $\int \frac{1}{\sqrt{1-x^2}}\, dx = ?$

Cuidado: $\cancel{\int 1\cdot\frac{1}{x}\,dx} = 1 + \cancel{\int \frac{1}{x}\,dx} \Rightarrow 0 = 1$ ↯

$f'(x) = 1 \Rightarrow f(x) = x$

$g(x) = \frac{1}{x} \Rightarrow g'(x) = -\frac{1}{x^2}$

$$\underbrace{\int \frac{1}{x}\,dx - \int \frac{1}{x}\,dx}_{K \in \mathbb{R}} = 1 \checkmark$$

$$\cancel{\int_1^2 1\cdot\frac{1}{x}} = 1\Big|_1^2 + \cancel{\int_1^2 \frac{1}{x}\,dx}$$

$$0 = 1\Big|_1^2 = 1 - 1 = 0 \checkmark$$

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

$f'(x) \rightsquigarrow f(x) + c$

$$\int f'(x)g(x)\,dx = (f(x)+c)g(x) - \int (f(x)+c)g'(x)\,dx$$

$$= f(x)g(x) + c\cdot g(x) - \int f(x)g'(x)\,dx - \int c\,g'(x)\,dx$$

$$= f(x)g(x) - \int f(x)g'(x) + c\cancel{g}(x) - c\cdot\cancel{g}(x) - \underbrace{c.k}$$