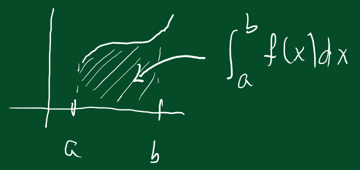
T.F.C.: $f:[a,b]\to\mathbb{R}$ e F é uma primitiva de f

$$\int_a^b f(x)\,dx = F(b)-F(a).$$



f contínua tem primitiva

como achar? $\left(F(x)=\int_a^x f(t)\,dt\right)$

Notação: $\int f(x)\,dx = \{F,\ \text{funções}: F'(x)=f(x)\}$

Imediatas:

1) $\int 0\,dx = k\in\mathbb{R}$. $\left(\int_a^b 0\,dx = k(b)-k(a) = k-k=0\right)$

2) $\int a\,dx = ax+k,\ k\in\mathbb{R}$

3) $\int x^{\alpha}\,dx = \dfrac{x^{\alpha+1}}{\alpha+1}+k,\ \alpha\neq -1.$

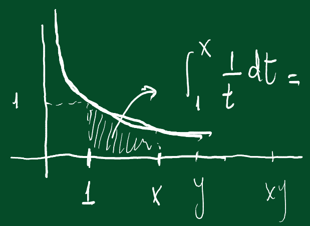
$\int x^2\,dx \underset{\alpha=2}{=} \dfrac{x^3}{3}+k$; $\int \dfrac{1}{x^4}\,dx \underset{\alpha=-4}{=} \dfrac{-1}{3x^3}+k.$

$v(t) = t^2-3t+1 \Rightarrow s(t) = \int v(t)\,dt = t^3/3 - 3t^2/2 + t + k$

$s(0)=1 \Rightarrow k=1 \Rightarrow s(t) = t^3/3 - 3t^2/2 + t + 1$

4) $\int \frac{1}{x}\,dx = \ln|x| + k.$

$$\left.\begin{array}{l} f(x)=\frac{1}{x},\ D_f=\mathbb{R}\setminus\{0\} \\ F(x)=\ln|x|,\quad x\neq 0 \end{array}\right\} \quad F(x)=\begin{cases}\ln x, & x>0\\ \ln -x, & x<0\end{cases} \Rightarrow F'(x)=\begin{cases}1/x, & x>0\\ 1/-x\cdot -1 = \frac{1}{x}, & x<0\end{cases}$$



$\int_1^x \frac{1}{t}\,dt = \ln|t|\Big|_1^x = \ln|x| - \overset{0}{\cancel{\ln|1|}} = \ln x.$

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$

$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}-\frac{0}{3} = 1/3$

5) $\int \cos x\,dx = \operatorname{sen} x + k$

6) $\int \operatorname{sen} x\,dx = -\cos x + k$

7) $\int e^x\,dx = e^x + k$ $\left(\int a^x\,dx = \dfrac{a^x}{\ln a} + k,\ a\neq 1,\ a>0\right)$

$(a^x)' = (e^{x\ln a})' = e^{x\ln a}\cdot\ln a = a^x\cdot\ln a$

8) $\int \sec^2 x\,dx = \operatorname{tg} x + k$

9) $\int \sec x\operatorname{tg} x\,dx = \sec x + k.$

10) $\int \dfrac{1}{1+x^2}\,dx = \operatorname{arctg} x + k$

Métodos p/ cálculo de primitivas:

Teorema: $f:[a,b]$ contínua e $g:[c,d] \to [a,b]$ de classe C^1.

$$\int_a^b f(x)\,dx = \int_c^d f(g(u)).g'(u)\,du.$$

$g(c)=a$
$g(d)=b$

Dem.: F primitiva de f $\left(\int_a^b f(x)\,dx = F(b)-F(a)\right)$

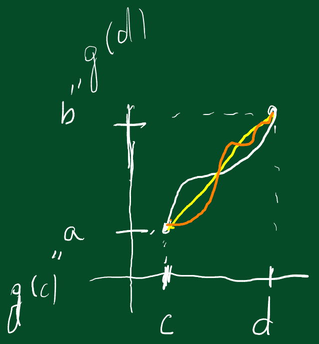
$F'=f$

primitiva de $f(g(u)).g'(u) = (F\circ g)'$ $\boxed{(h\circ g)' = h'(g(u)).g'(u)}$

$$\Rightarrow \int_c^d f(g(u)).g'(u)\,du = (F\circ g)(d) - (F\circ g)(c)$$

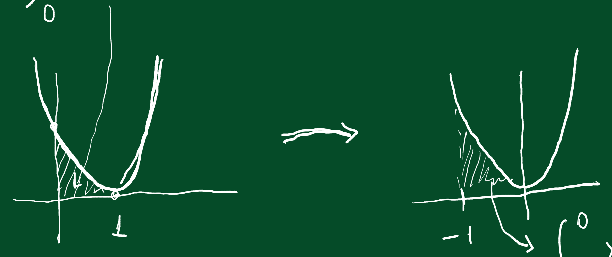
$$= F(g(d)) - F(g(c))$$

$$= F(b) - F(a) = \int_a^b f(x)\,dx \quad \#$$



Exemplos:

1) $\int_0^1 (x-1)^{2020}\,dx = ?$

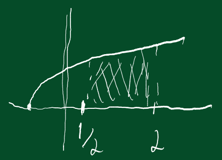


$$\int_{-1}^0 x^{2020}\,dx = \left.\frac{x^{2021}}{2021}\right|_{-1}^0 = 0 - \frac{(-1)^{2021}}{2021} = \frac{1}{2021}$$

$$\left.\begin{aligned} f(x) &= (x-1)^{2020} \\ g(u) &= u+1 \\ x \text{''} \quad g'(u) &= 1 \end{aligned}\right\} \quad \int_0^1 (x-1)^{2020}\,dx = \int_{-1}^0 u^{2020}.1\,du = \left.\frac{u^{2021}}{2021}\right|_{-1}^0 = \frac{1}{2021}.$$

$$f(g(u)) = (u+1-1)^{2020} = u^{2020}$$

2) $\displaystyle\int_{1/2}^{2} \sqrt{2x+1}\,dx = \int_{2}^{5} \sqrt{u}\cdot\frac{1}{2}\cdot du = \int_{2}^{5} \frac{u^{1/2}}{2}\,du = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\,u^{3/2}\Big|_{2}^{5} = \frac{5^{3/2}}{3} - \frac{2^{3/2}}{3}$



$2x+1=u$

$x = \dfrac{u-1}{2} = g(u)$

$g'(u) = 1/2$

$f(x) = \sqrt{2x+1}$

ou $\displaystyle\int_{1/2}^{2} \sqrt{2x+1}\,dx = \int_{2}^{5} \sqrt{u}\,\frac{du}{2} = \dots$

$2x+1=u$

$du = 2\,dx$

3) $\displaystyle\int_{0}^{\pi\ (\pi/2)} \cos x \cdot \operatorname{sen}^2 x\,dx = \int_{0}^{0\ (1)} u^2\cdot du = \frac{u^3}{3}\Big|_{0}^{0\ (1)} = 0 - 0 = 0\ \ (1/3)$

$u = \operatorname{sen} x$

$du = \cos x\,dx$

$f(g(x)).g'(x)$

$f(t) = t^2$

$g(x) = \operatorname{sen} x$

$\displaystyle\int_{0\ \leftarrow d}^{\pi\ \leftarrow c} f(g(x)).g'(x)\,dx = \int_{g(0)\ \to\ a=g(c)}^{g(\pi)\ \to\ b=g(d)} f(t)\,dt$

4) $\displaystyle\int_{0}^{\pi/2} \operatorname{sen}^3 x\,dx = \int_{0}^{\pi/2} \operatorname{sen} x \cdot \operatorname{sen}^2 x\,dx$

$\displaystyle= \int_{0}^{\pi/2} \operatorname{sen} x\,(1-\cos^2 x)\,dx = \int_{0}^{\pi/2} \operatorname{sen} x\,dx - \int_{0}^{\pi/2} \operatorname{sen} x \cos^2 x\,dx$ $-du$

$u = \cos x$

$du = -\operatorname{sen} x\,dx$

$\displaystyle= (-\cos x)\Big|_{0}^{\pi/2} + \int_{1}^{0} u^2\,du$

$\displaystyle= -0 - (-1) + \frac{u^3}{3}\Big|_{1}^{0} = 1 - 1/3 = 2/3$

5) $\displaystyle\int \frac{x}{1+x^4}\,dx = \int \frac{1}{1+u^2}\,\frac{du}{2} = \frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\cdot \operatorname{arctg} u + K$

$\displaystyle= \frac{1}{2}\operatorname{arctg}(x^2) + K.$

$u = x^2$

$du = 2x\,dx$

conferindo: $\displaystyle\left(\frac{1}{2}\operatorname{arctg}(x^2)\right)' = \frac{1}{2}\cdot\frac{1}{1+(x^2)^2}\cdot 2x$

6) $\int tg\,x\,dx = \int \frac{sen\,x}{cos\,x}\,dx = \int \frac{-du}{u} = -\int \frac{1}{u}\,du$

$= -\ln|u| + K$

$= -\ln|cos\,x| + K$

$= \ln|sec\,x| + K$

$-\int f(g(x)).g'(x)\,dx$

$f(u) = \frac{1}{u}$

$g(x) = cos\,x$

$u = cos\,x \quad (g(u) = arccos\,u)$

$du = -sen\,x\,dx$

$-\ln|cos\,x| = \ln|cos\,x|^{-1} = \ln\left|\frac{1}{cos\,x}\right| = \ln|sec\,x|.$

conferindo: $(\ln sec\,x)' = \frac{1}{\cancel{sec\,x}}.\cancel{sec\,x}.tg\,x$

Lição de casa:

- $\int sec^2 x\,dx$
- $\int tg^2 x\,dx$
- $\int tg\,x \cdot sec^2 x\,dx$