$$9-d)\quad \overbrace{x-\frac{x^3}{3!}}^{g(x)} < \underbrace{\operatorname{sen} x}_{f(x)} < \overbrace{x-\frac{x^3}{3!}+\frac{x^5}{5!}}^{h(x)}, \quad x>0$$

$$f(0)=g(0)=h(0)$$

$$g'(x)=1-\frac{x^2}{2} \Rightarrow g'(0)=1$$

$$f'(x)=\cos x \qquad f'(0)=1$$

$$g''(x)=-x \qquad \Rightarrow \quad g''(0)=0$$

$$f''(x)=-\operatorname{sen} x \qquad f''(0)=0$$

$$\left.\begin{array}{l} g'''(x)=-1 \\ f'''(x)=-\cos x \end{array}\right\} \quad -1\leq \cos x \leq 1 \to -\cos x \geq -1$$

$$\Downarrow$$

$$f'''(x)>g'''(x)$$

$$\Downarrow$$

$$f'''(x)-g'''(x)>0$$

$$\Downarrow$$

TVM

$$(f-g)'''(x)>0 \Rightarrow (f-g)'' \text{ é cresc.}$$

mas $(f''-g'')(0)=0 \Rightarrow f''(x)-g''(x)>0\ \forall x>0$

$$\Rightarrow f''(x)>g''(x),\ \forall x>0$$

$$\Rightarrow (f-g)''(x)>0,\ \forall x>0 \Rightarrow (f-g)'(x) \text{ é crescente}$$

mas $(f'-g')(0)=0 \Rightarrow f'(x)-g'(x)>0,\ \forall x>0$

$$\Rightarrow (f-g)'(x)>0,\ \forall x>0 \Rightarrow (f-g)(x) \text{ é crescente}$$

mas $(f-g)(0)=0 \Rightarrow (f-g)(x)>0,\ \forall x>0.$

$$\Rightarrow f(x)>g(x).$$

28. Seja $f(x) = \frac{4x+5}{x^2-1}$. Prove que f tem exatamente um ponto de inflexão e que esse ponto pertence ao intervalo $]-3,-2[$. Esboce o gráfico de f.

1) $D_f = \mathbb{R}\setminus\{-1,1\}$

2) $$f'(x) = \frac{4(x^2-1) - 2x(4x+5)}{(x^2-1)^2} = \frac{-4x^2-10x-4}{(x^2-1)^2}$$

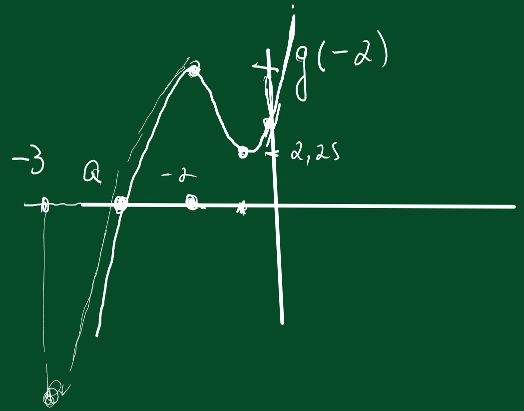
$$= \underbrace{\frac{-2}{(x^2-1)^2}}_{<0} \cdot (2x^2+5x+2)$$

$$\frac{-5 \pm 3}{4} \begin{cases} -2 \\ -1/2 \end{cases}$$

	-2	-1	$-1/2$	1
f'	$-$	$+$	$+$	$-$	$-$
f	↘	↗	↗	↘	↘

3) $$f''(x) = \frac{9}{(x-1)^3} - \frac{1}{(x+1)^3} = \frac{2\overbrace{(4x^3+15x^2+12x+5)}^{g(x)}}{(x^2-1)^3}$$

	a	-1	$-1/2$	1
f''	$-$	$+$	$-$	$-$	$+$
f	$\cap$	$\cup$	$\cap$	$\cap$	$\cup$



$$g'(x) = 12x^2+30x+12 = 6(2x^2+5x+2)$$

	-2	$-1/2$	
g'	$+$	$-$	$+$
g	↗	↘	↗

4) $\lim_{x\to \pm\infty} f(x) = 0$

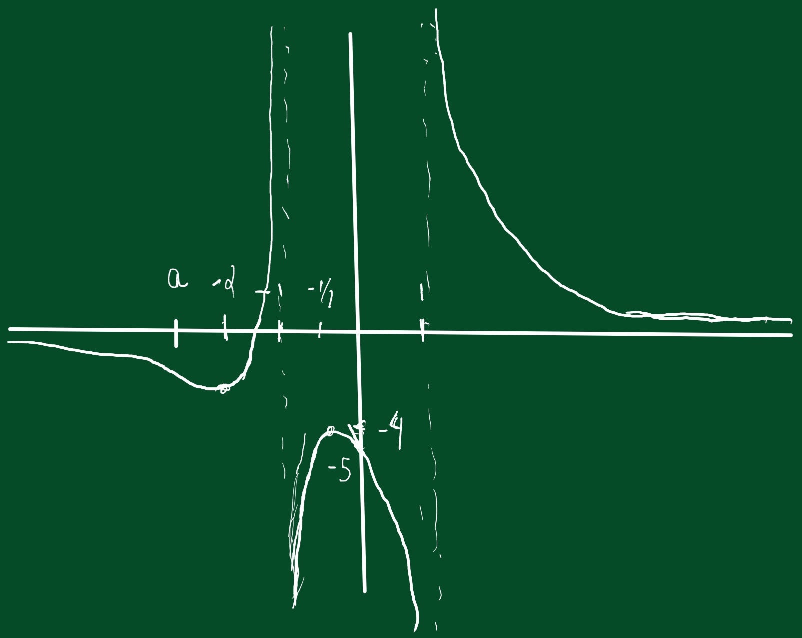
$\lim_{x\to -1^+} f(x) = -\infty = \lim_{x \to 1^-} f(x)$

$\lim_{x\to -1^-} f(x) = +\infty = \lim_{x \to 1^+} f(x)$

5) $f(0) = -5$

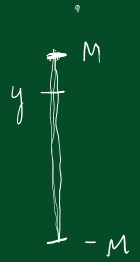
$f(x) = 0 \Leftrightarrow x = -5/4$

6) Assíntotas: $y = 0$



16. Dentre as alternativas abaixo, aquela que contém um polinômio que define uma função bijetora de $\mathbb{R}$ em $\mathbb{R}$ é:

a. $3x^5 - 5x^3 + 15x$.
b. $3x^5 - 5x^3 - 15x$.
c. $3x^5 + 5x^3 - 15x$.
d. $3x^5 - 5x^3$.
e. $5x^3 - 15x$.



•) sobrejetoras : $\lim_{x \to +\infty} f(x) = \pm\infty$ e $\lim_{x \to -\infty} f(x) = \mp\infty$

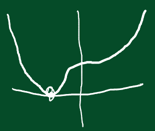
(f contínua)

•) injetora : $f'(x) > 0$ ou $f'(x) < 0$

$y = x^2$

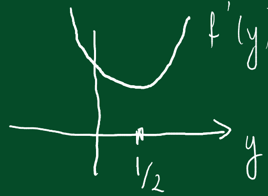
a. $f'(x) = 15x^4 - 15x^2 + 15$

$= 15y^2 - 15y + 15 > 0 \Rightarrow$ f é crescente.



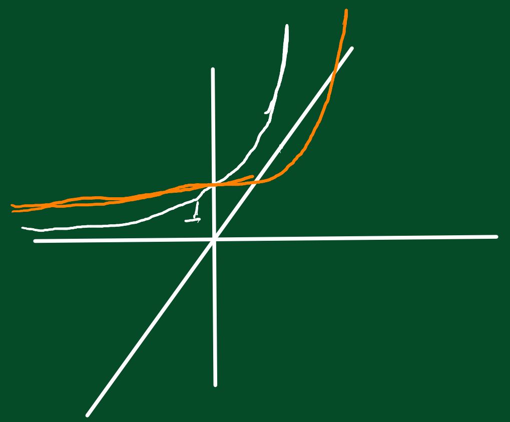
$f' > 0$

f'



$f'(y) = f'(x^2)$

20. Determine todos os números positivos a tais que a curva $y = a^x$ corta a reta $y = x$.



$$\left.\begin{array}{l} y = f(x) = a^x \\ y = x \end{array}\right\} \quad x = a^x \text{ tem soluções.}$$

$$\Downarrow$$

$$a = x^{1/x} = g(x), \quad x > 0$$

2) $g(x) = e^{\frac{1}{x}\ln x} \Rightarrow g'(x) = e^{\frac{1}{x}\ln x} \cdot \left(-\frac{1}{x^2}\ln x + \frac{1}{x^2}\right)$

$$= \underbrace{x^{1/x} \cdot \frac{1}{x^2}}_{>0} \cdot (1 - \ln x)$$

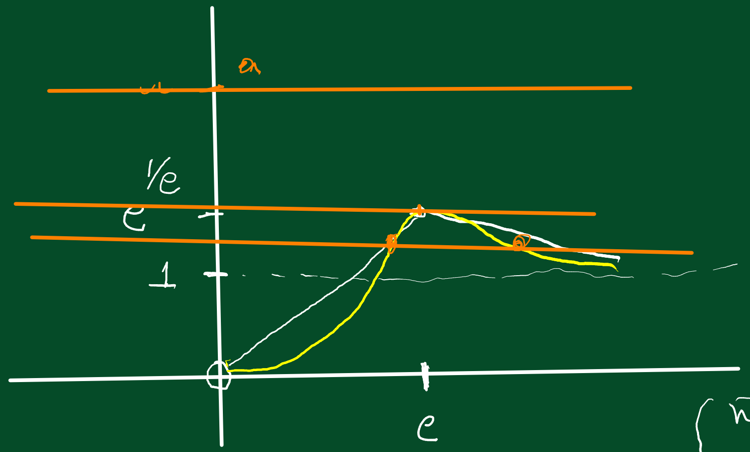
		e	
g'	+		−
g	↗		↘

3) Concavidade: ñ interessa

4) $\lim_{x\to 0^+} x^{1/x} = \lim_{x\to 0^+} e^{\frac{1}{x}\ln x} = e^{\overbrace{\lim_{x\to 0^+}\frac{\ln x}{x}}^{-\infty}} = 0$

$\lim_{x\to +\infty} x^{1/x} = \dots = e^{\lim_{x\to+\infty}\frac{\ln x}{x}} = e^{\lim_{x\to\infty}\frac{1}{x}} = e^0 = 1$

5) ñ há interceptos



$$g(x) = a \begin{cases} \text{ñ tem sol se } a > e^{1/e} \\ \text{tem 1 sol se } a = e^{1/e} \text{ ou } 0 < a \leq 1 \\ \text{tem 2 sols se } 1 < a < e^{1/e} \end{cases}$$